Generating Year-Equivalent Databases of Fractally Synthesised Rain Rate Fields

Sarah Callaghan
STFC – Rutherford Appleton Laboratory
Radio Communications Research Unit, Chilton
Didcot, OX11 0QX, United Kingdom
Email: s.a.callaghan@rl.ac.uk

Introduction

- Bandwidth-hungry devices (such as 3G mobile) create pressure to improve spectrum efficiency and open up new frequencies to commercial exploitation.
- Radio systems operating at frequencies above 10 GHz are adversely affected by rain and cloud.
- This attenuation cannot be compensated for effectively and cost-efficiently through the use of fade margin alone.
- Hence, the use of Fade Mitigation Techniques (FMT)

One popular type of FMT is spatial diversity, which takes advantage of rain changing in time and space. To optimise systems using this FMT, we need to know about this rain field variation.

Measured rain radar data

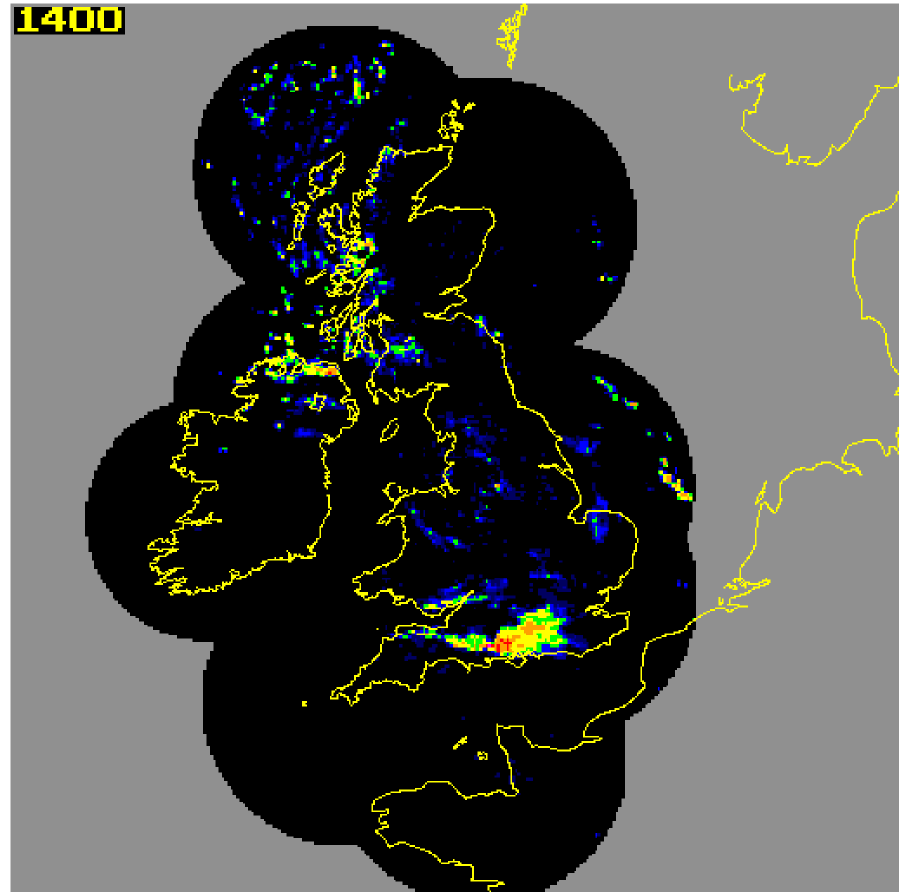
The Met Office have a database of rain radar measurements available which is composite rain radar data on the Cartesian national grid.

Space resolution: 1/2/5km hybrid
Time resolution: 5 minutes

Database starts in April 2004.

Before that, composite rain radar data is available on a space resolution of 5 km * 5 km and at a time resolution of 15 minutes.

Unfortunately, this temporal/spatial resolution is not high enough for a lot of radio communications applications.



Example radar map of the UK derived from the Met Office Nimrod radar data

Science & Technology Facilities Council

Requirements for a rain model for use by radio system engineers

A physically-based rain model should:

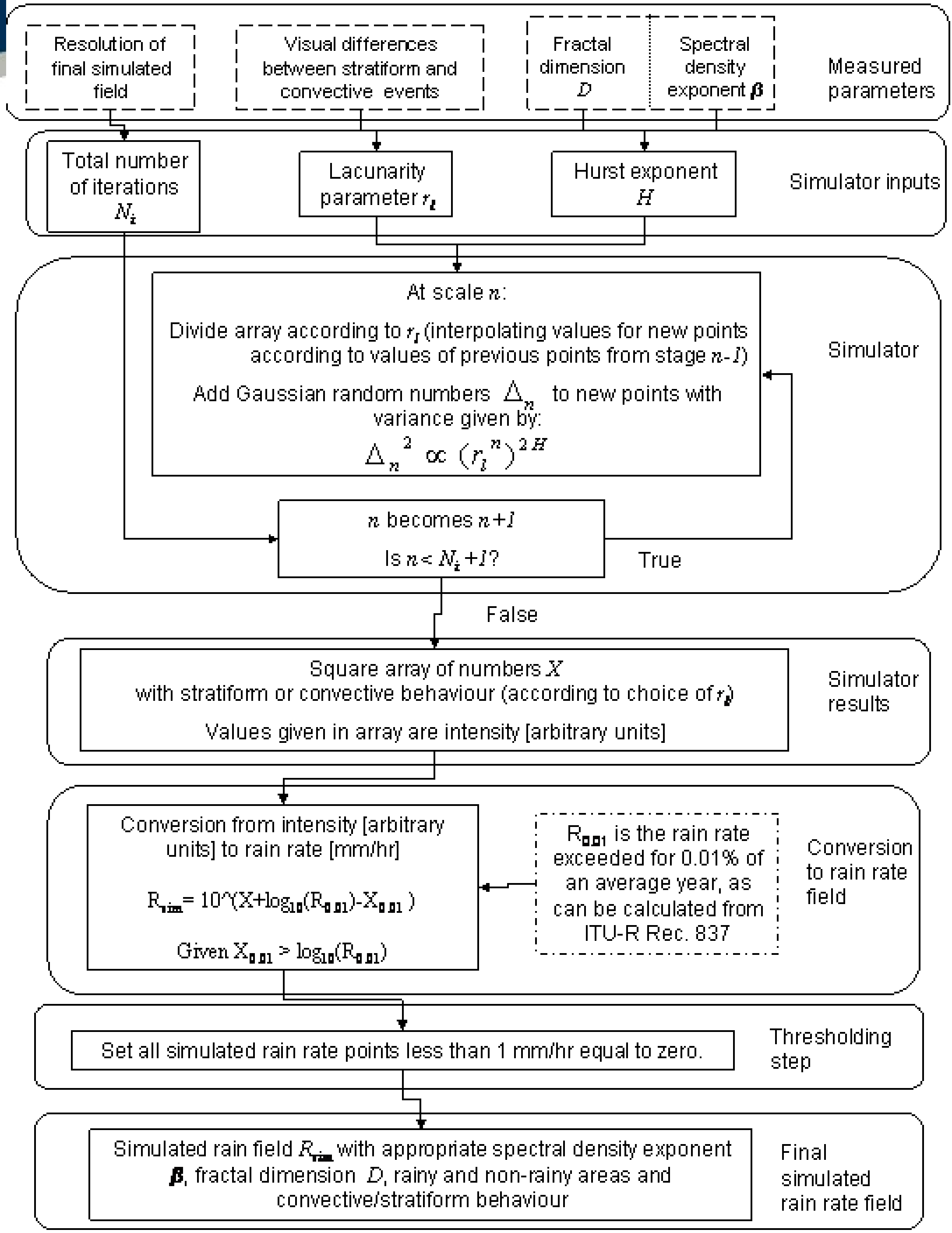
• have a time resolution of 1 s
• have a spatial resolution of about 100 m
• be able to take inputs from a weather model
• be suitable for use in spectrum management and simulation software
• be capable of generating databases which replicate average annual statistics

Bacon, 2005 (http://www.rainmap.rl.ac.uk/pdfs/jan2005_workshop/Bacon.pdf)

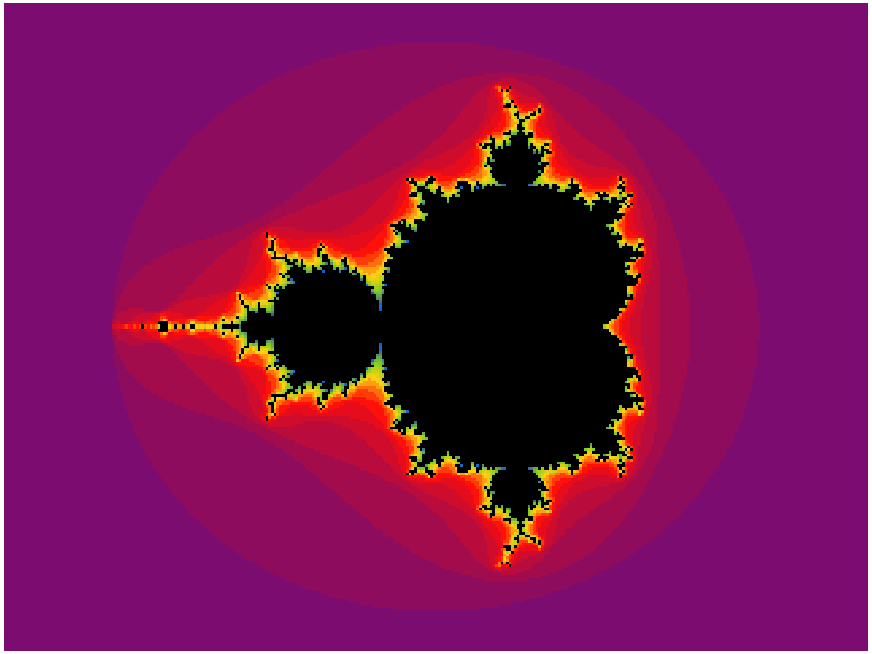
Rain Field Simulator

The fractal nature of rain fields is well documented in the literature.

The simulator presented here is based on the Voss successive random additions algorithm for generating fractional Brownian motion in two dimensions. This is an additive discrete cascade, producing log, monofractal fields.



What is a fractal?



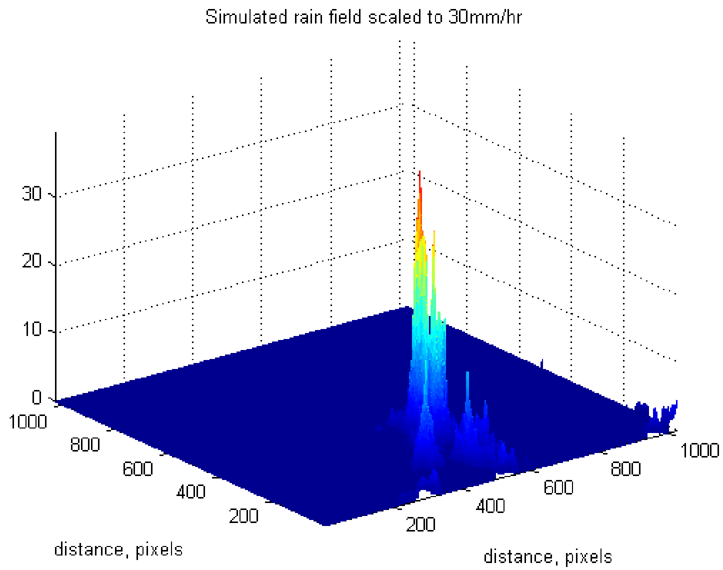
Mandelbrot set

Romanesco broccoli

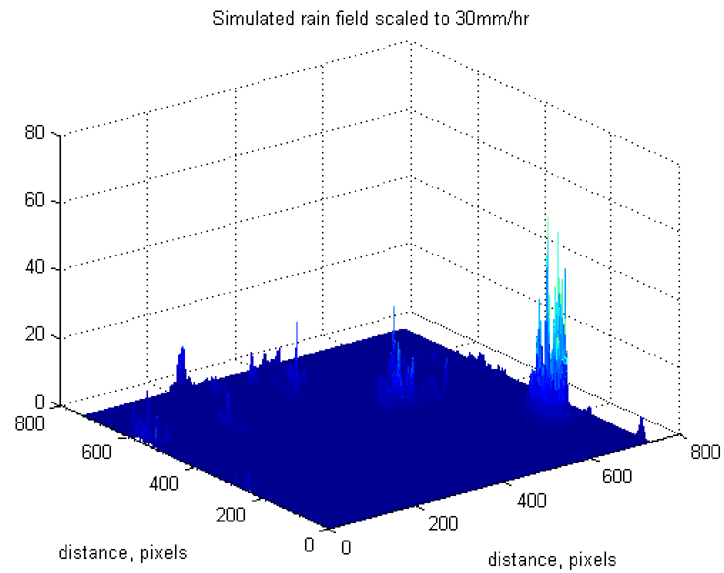
A fractal is an object that is self-similar on many different scales. Fractals can be exactly self similar, or statistically self-similar.

Real world examples: trees, ferns, broccoli.

Example simulated rain fields



Example simulated stratiform rain field.



Example simulated convective rain field.

Each run of the simulator produces a single realisation of a stratiform-like or a convective-like rain field. These realisations are independent of each other, hence there is no temporal component to the model (work is ongoing).

However, given enough synthetic fields we can create simulated statistics for an average year.

Proportion of stratiform/convective events in a year (1)

To create simulated statistics for an average year, we need to know the proportion of stratiform and convective events in a year.

Rec. 837 has data files associated with it containing parameters such as grids of latitude, longitude, etc.

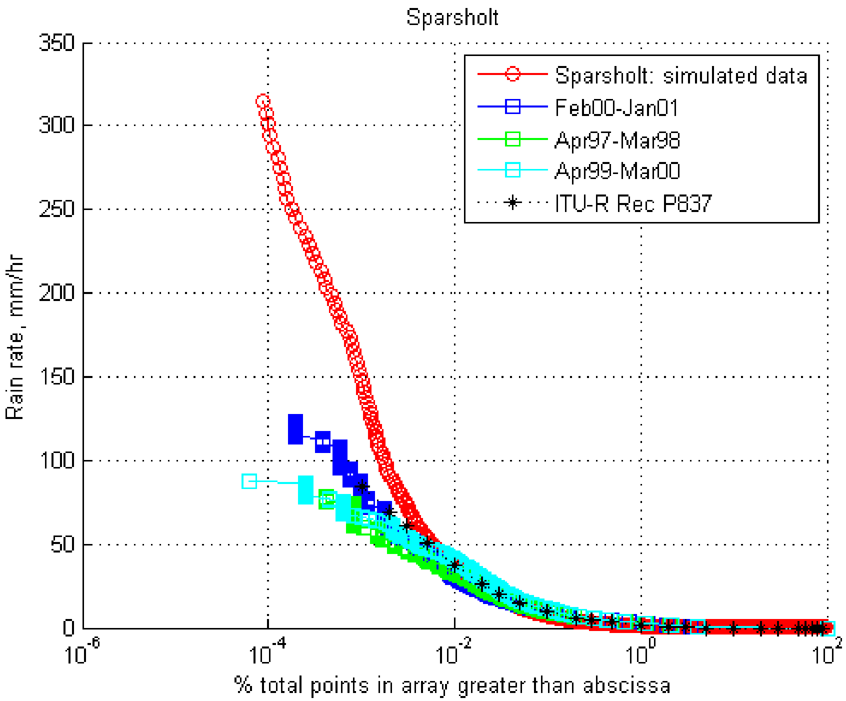
Two of these parameters, M_c and M_s, aren't defined in the recommendation text, but a bit of digging reveals that:

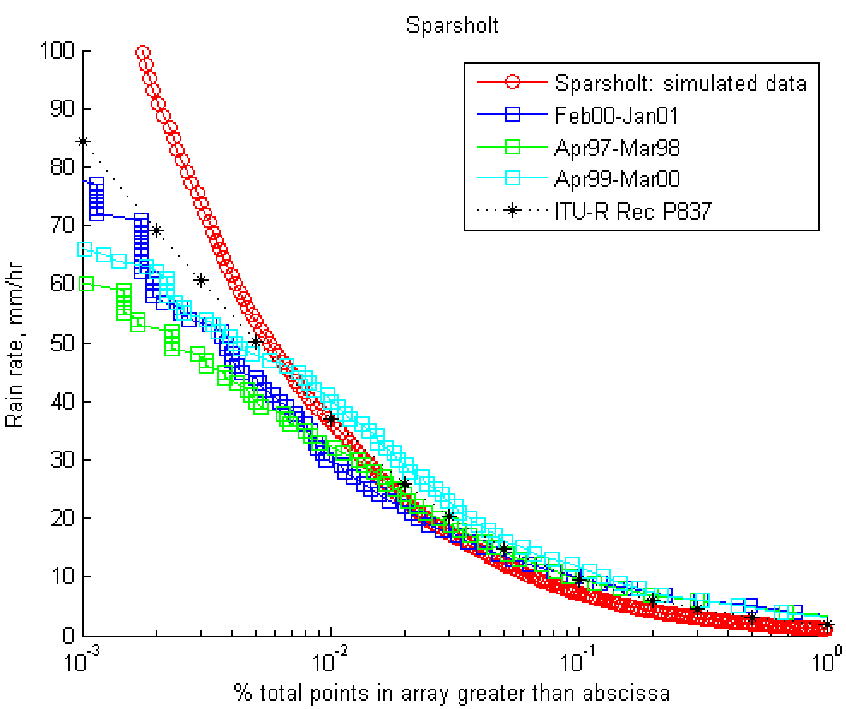
M_c = annual average convective rainfall amount (mm)
M_s = annual average stratiform rainfall amount (mm)

For Chilbolton, M_c =120mm and M_s=593mm.

In other words, 16.8% of the total rain accumulation rain falling at Chilbolton is convective and 83.2% is stratiform.

Use these values to determine the mix of stratiform/convective events to create an average year.





Proportion of stratiform/convective events in a year (2)

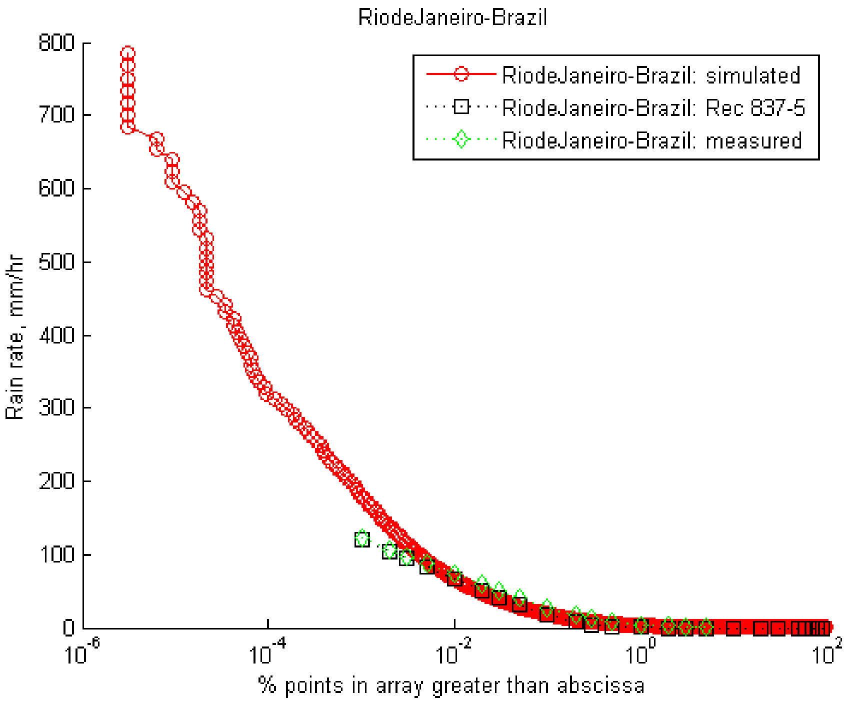
Assuming spatio-temporal equivalence, we use the stratiform and convective rain fields in proportion as given by M_c and M_s.

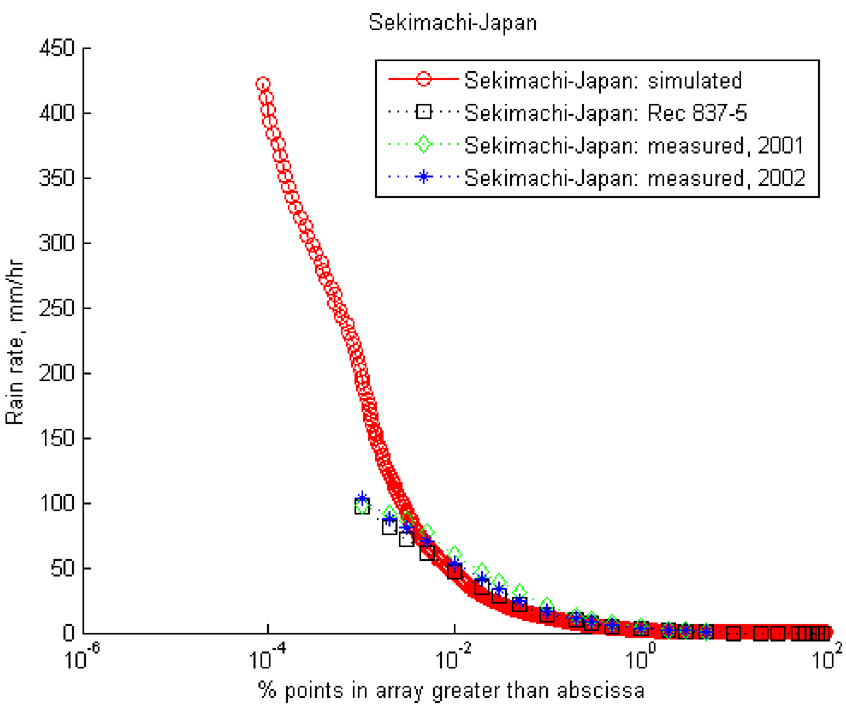
Simulated gauge curve is scaled to $R_{0.01}$

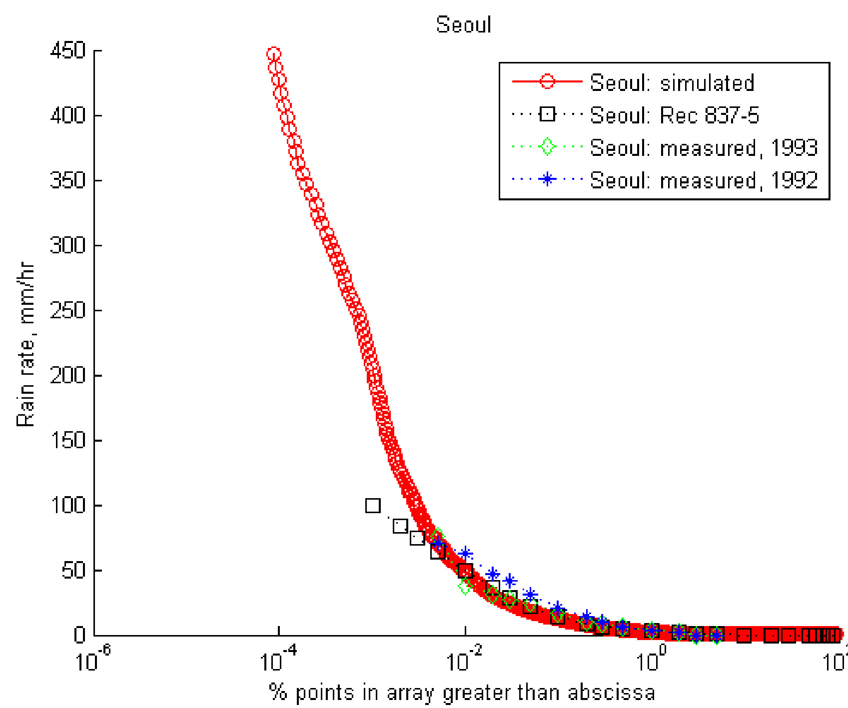
This gives good agreement along most of the length of the 2 curves.

This also suggests that if we simulated an entire year's worth of 2D rain fields in the proportions given by M_c and M_s and scaled them to $R_{0.01}$ then the resulting statistics would be consistent with the ITU model.

Results of simulations for different locations (1)

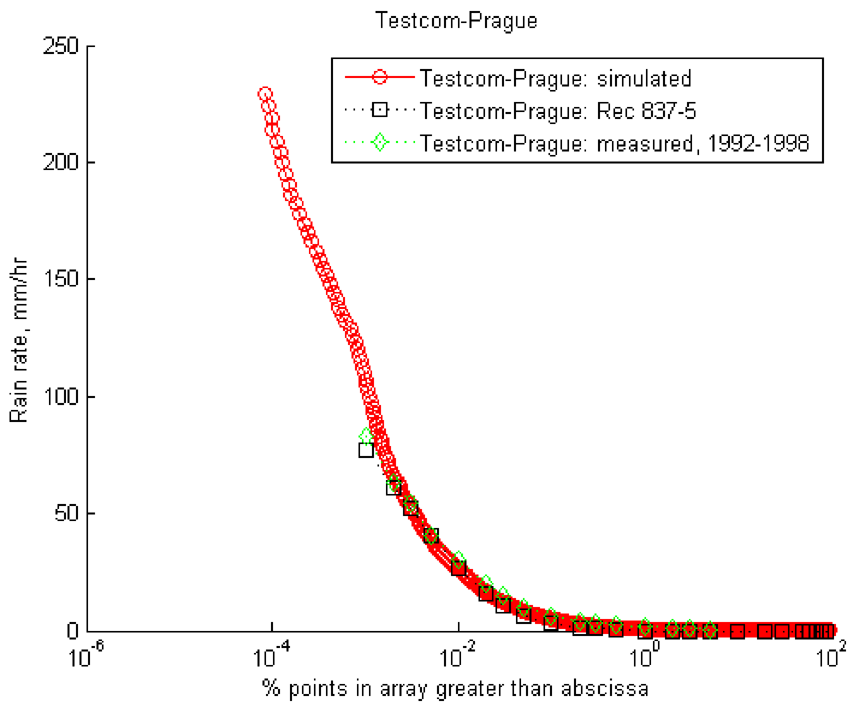


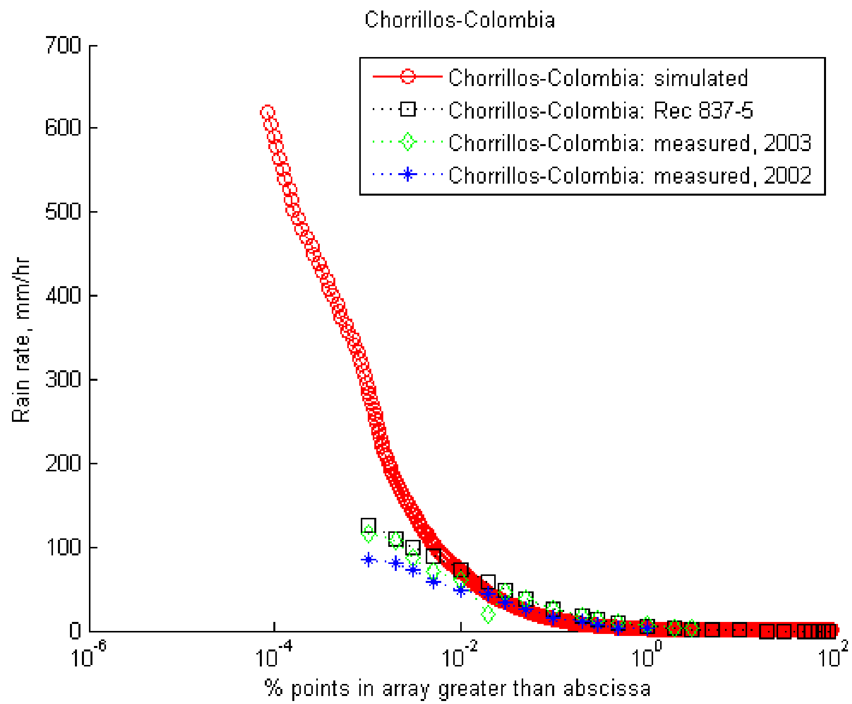




Science & Technology Facilities Council

Results of simulations for different locations (2)





Moving to annual statistics

We want to test the statistics produced by the simulated fields, especially with reference to the spatial correlation.

- Assume that each simulated field is a snapshot at any given minute:
  - Need 43,200 arrays for a month (30 days)
  - Need 525,600 arrays for a year

Considering that each convective array is 8,328 KB and each stratiform array is 16,419 KB, that's a lot of data.

525,600 arrays would weigh in at ~6,150 GB and would take ~ 40 days to create.

That's a lot of simulation!

Simulating the tail of the annual distribution

Most radio system designers use ITU-R recommendations to produce the rain cumulative distribution curves that are used to determine system availability. We want to create a subset of 2D events which will accurately reproduce the tail of the annual ITU distribution and still keep accurate spatial statistics.

We therefore use 526 simulated arrays (equivalent to 0.1% of a year - assuming each array is equivalent to 1 radar snapshot/minute). We can then fit this dataset to the ITU Rec. 837 values in the range 0.1% to 0.001%

To fit to the tail of the annual ITU distribution, we need two new parameters, a and b.

$$R_{sim} = 10^{(\frac{V}{a}+b)}$$

where:

R_{sim} is the simulated rain array (mm/hr)
V is the data array produced by the simulator

Simulating the tail of the annual distribution – a and b

a is given by:

$$a = \frac{std(R_{GA} \geq m_{GA})}{std(\log_{10}(I_{R \leq 0.1}))}$$

where:

R_{GA} is the simulated rain gauge data extracted from V (equivalent to log values)
$I_{R \leq 0.1}$ is the rainrate exceeded for percentages of time less than 0.1% (from Rec 837)
m_{GA} is the mean of R_{GA}
std is the standard deviation

The parameter a modifies the standard deviation of the simulated distribution to fit in with the (tail of the) ITU distribution.

Finally, the simulated curve needs to be offset to match the ITU curve.

Hence, the parameter b:

$$b = \left|\log_{10}(I_{R=0.1})\right|$$

Where $I_{R=0.1}$ is the rainrate exceeded for 0.1% of the time (from Rec 837)

Simulating the tail of the annual distribution – sample a and b values

Location	Latitude (- for deg. S)	Longitude (- for deg. W)	Mc (from Rec. 837-5)	Ms (from Rec. 837-5)	a	b
Chilbolton (England)	51.1333	-1.4333	158.23	581.29	3.2266	0.9106
Cairo (Egypt)	30.05	31.25	12.7983	45.8753	1.6580	0.1467
Prague (Czech Republic)	50.1	14.4333	149.2240	491.2206	2.3112	0.9372
Buenos Aires (Argentina)	-34.6667	-58.5	164.8082	868.3786	2.5511	1.0789
Jakarta (Indonesia)	-6.1333	106.75	1580.3	820.9559	4.4872	1.6714
Delhi (India)	28.6667	77.2333	322.6315	323.6087	2.5390	1.1243

Simulating the tail of the annual distribution - results

Results after scaling with a and b.
For Chilbolton data, a=3.2266 and b=0.9106

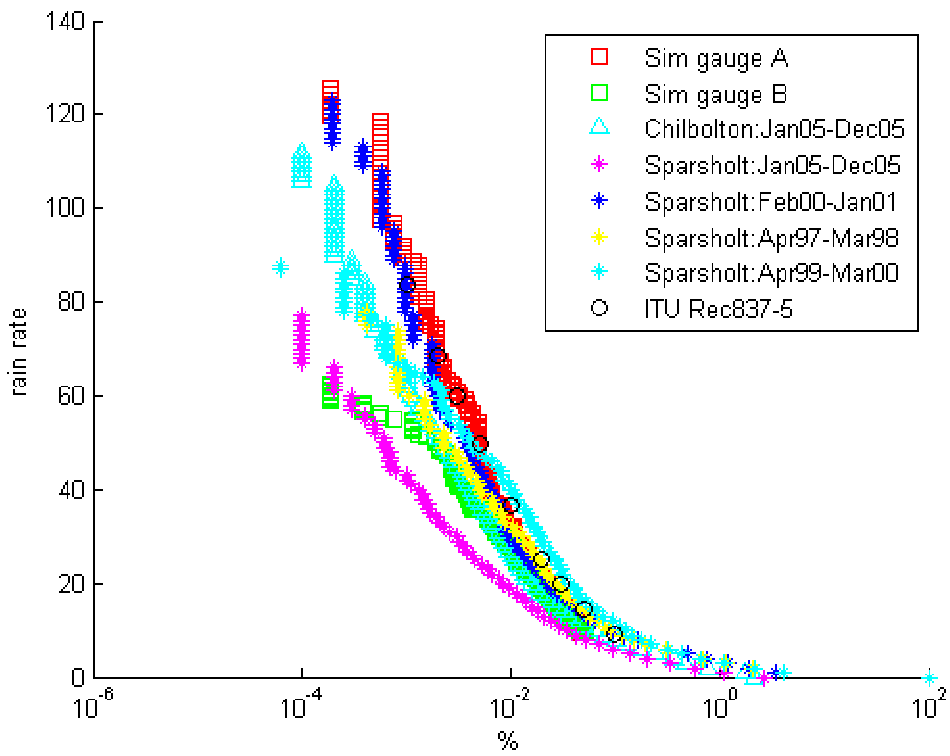
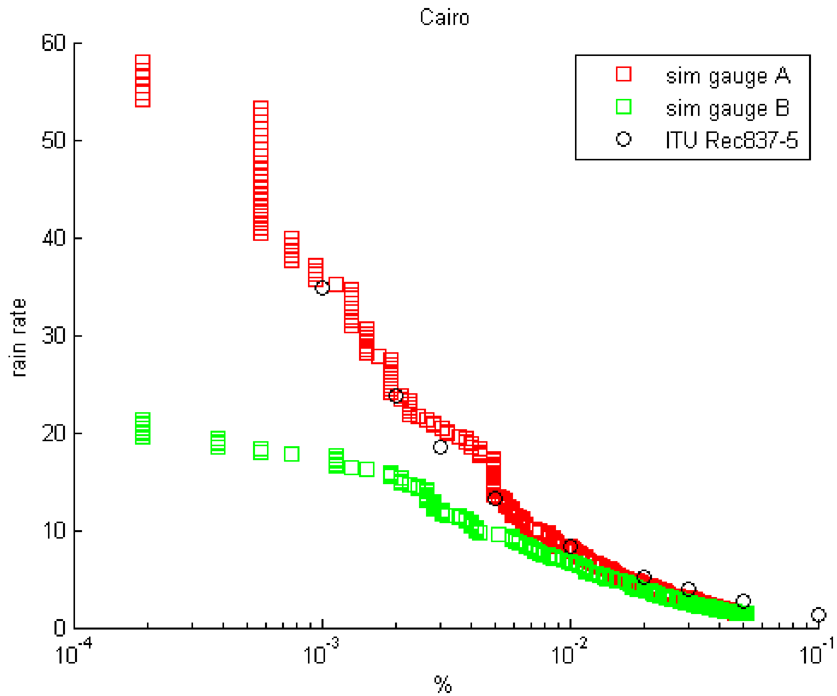


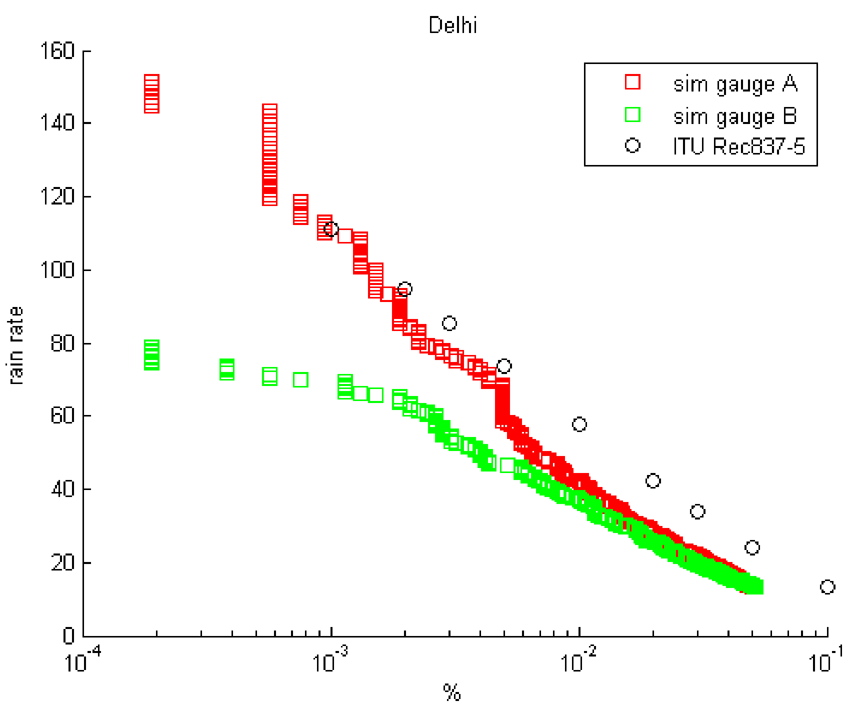
Figure shows the ITU Rec 837 curve for percentage times less than 0.1%, in comparison with the measured Chilbolton and Sparsholt annual curves (Jan 05-Dec 05) and the cdfs for simulated rain gauges A and B (7.5km apart).

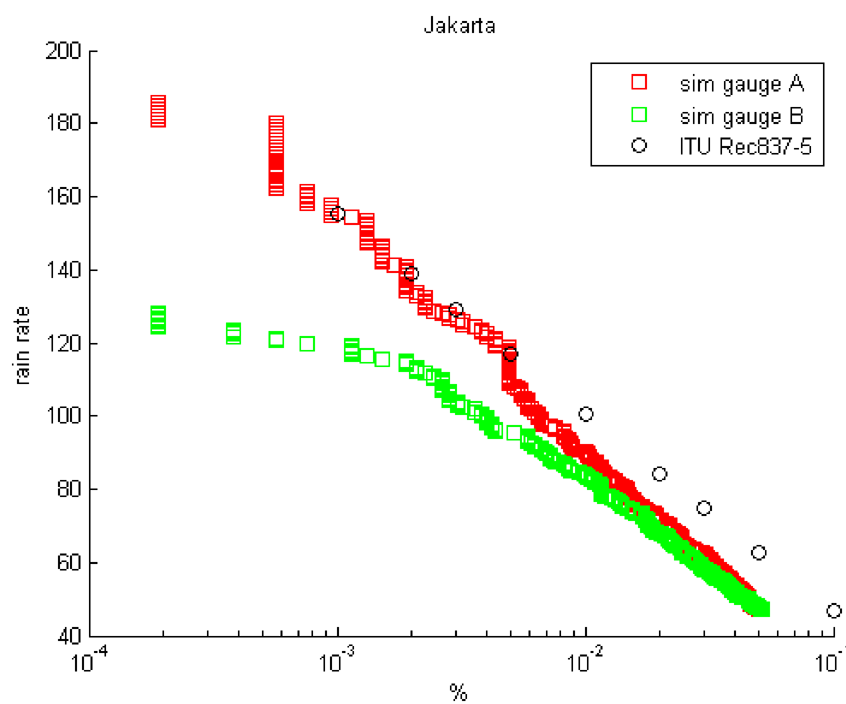
The cdf of A has been scaled using parameters a and b to fit it to the ITU curve. The cdf of B uses the same a and b, but isn't as closely tied to the ITU curve.

There is good agreement of the simulated curves with the ITU curve for % values less than 0.05%

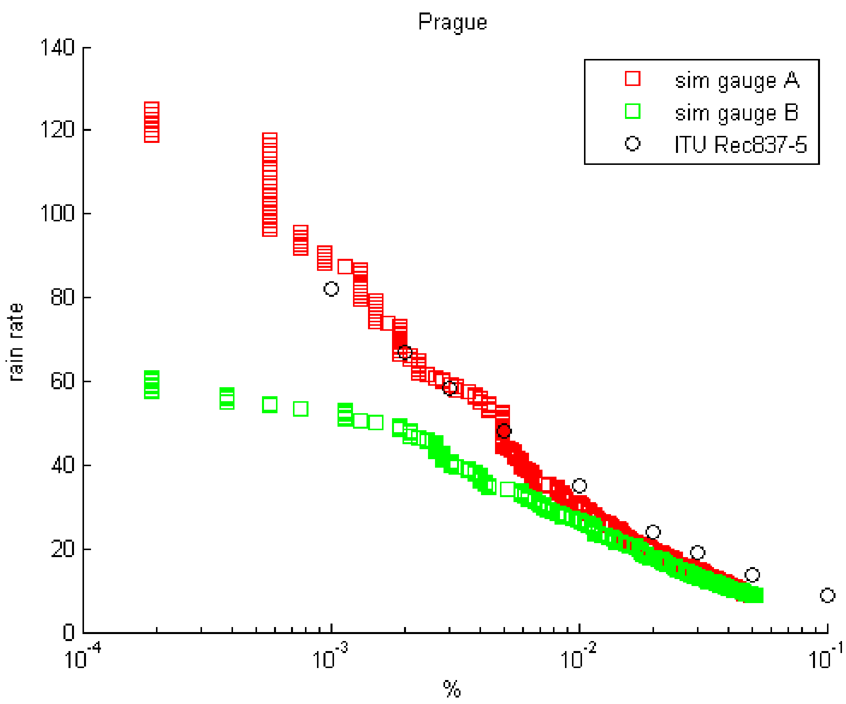
Simulating the tail of the annual distribution – results for different locations (1)

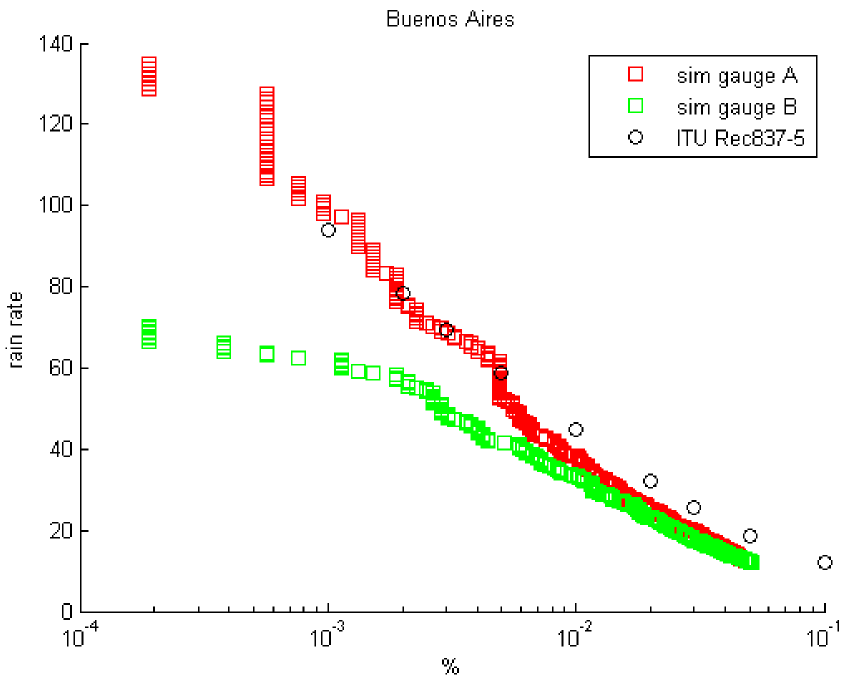






Simulating the tail of the annual distribution – results for different locations (2)





Spatial correlation of scaled simulated rain fields

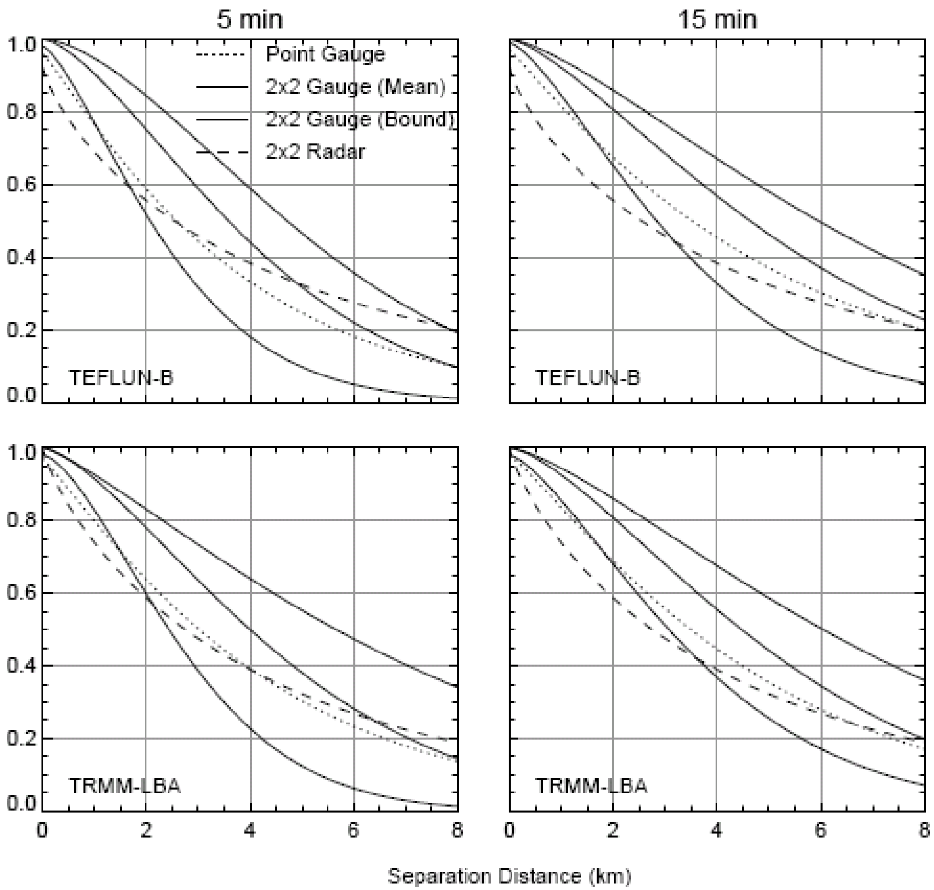
- Double checking the correlation factor of simulated fields (scaled by a and b to fit the tail of the ITU distribution) reveals that it falls off slower with distance than the measured data (on a pixel by pixel comparison)

- Comparison with small scale measurements shows that if we assume that a simulated pixel is 100m*100m, then the spatial autocorrelation in the simulated data falls off too slowly. If we assume that 1 simulated pixel is equivalent to 50m*50m, this corresponds better with the measured data.

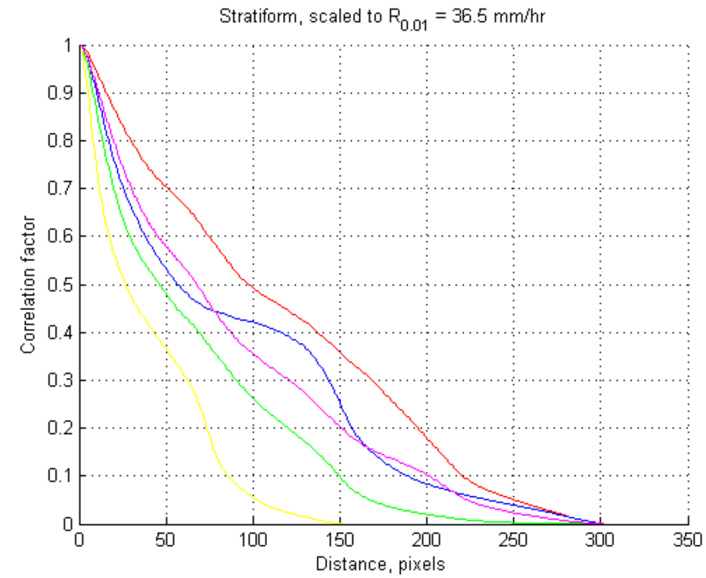
- Overall, when the arrays are scaled to fit the tail of the ITU distribution, the simulated fields become more correlated. This makes sense, as we're altering the standard deviation of the simulated fields, pushing the spread of simulated values closer together.

- Tweaking the pixel size will make the spatial correlation more consistent with reality, but will reduce the total area covered.

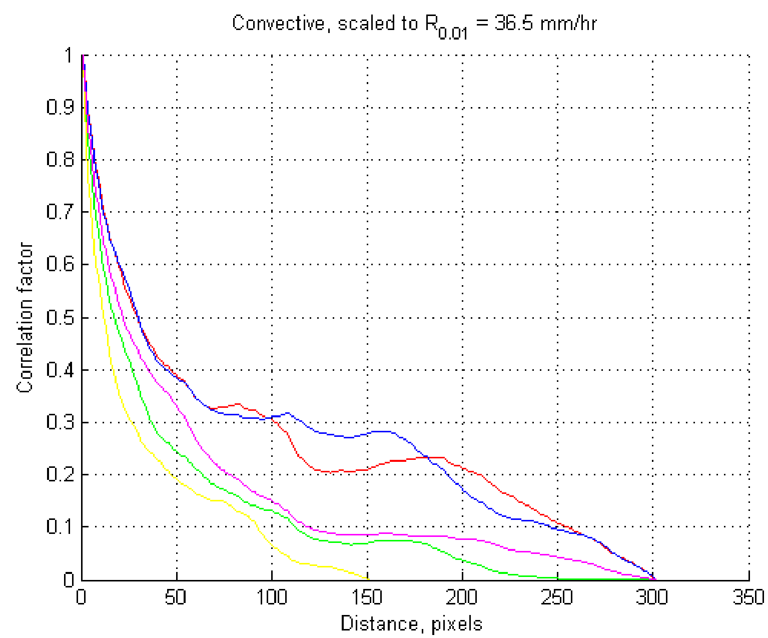
2D autocorrelation comparisons: $R_{0.01}$



Spatial point- and area-correlation function estimated from gauge-rainfall fields, and the uncertainty bound for the area-correlation function. Also shown is the correlation function estimated from radar-rainfall fields. [Gebremichael et al, 2004]

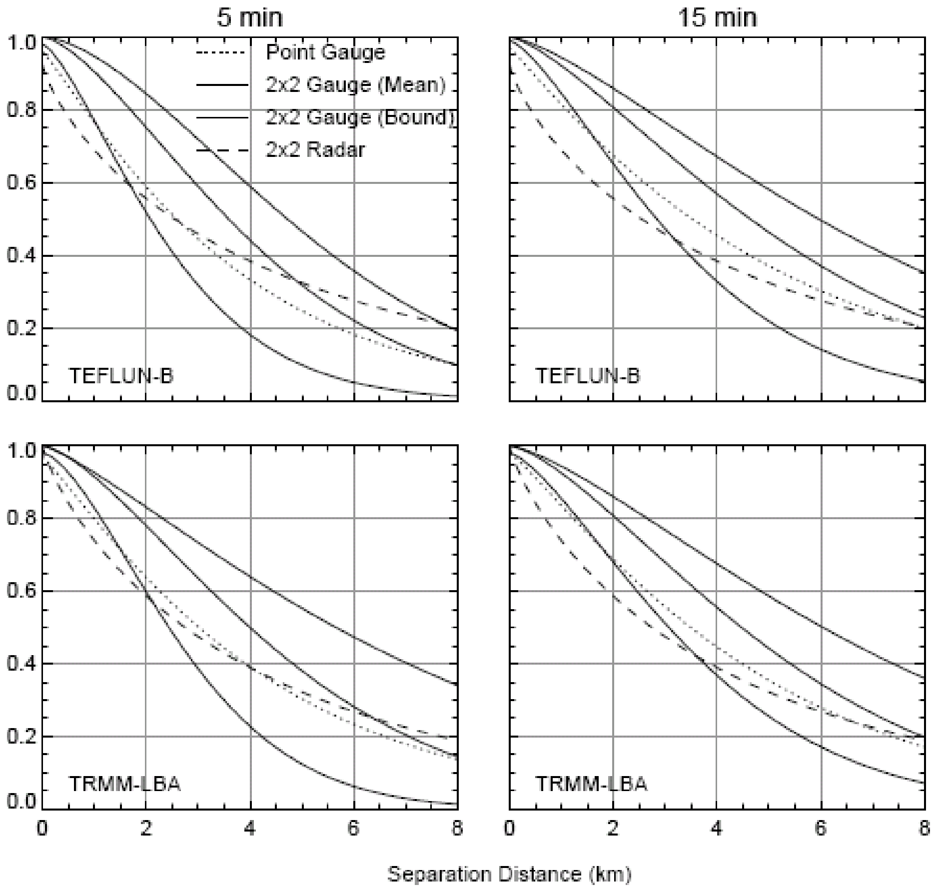


Simulated stratiform field $R_{0.01}$= 36.5 mm/hr

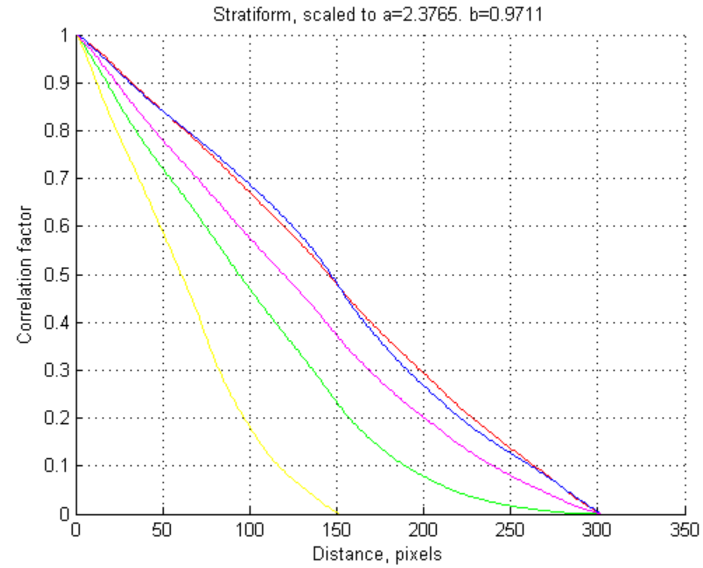


Simulated convective field $R_{0.01}$= 36.5 mm/hr

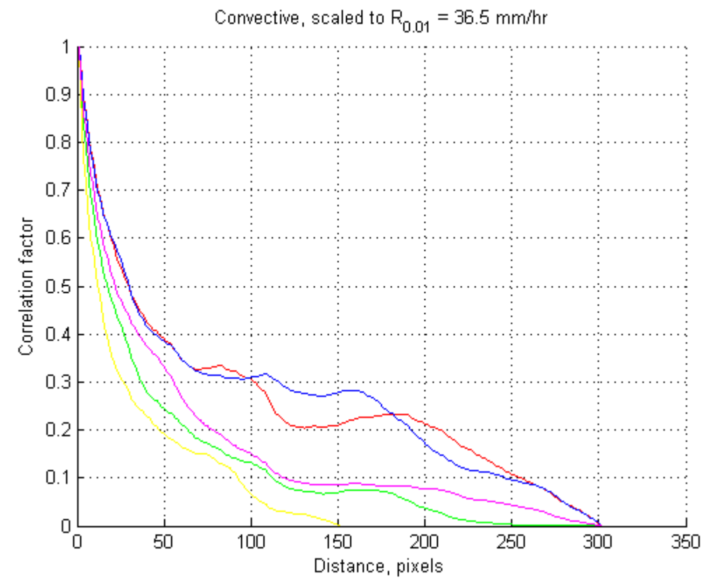
2D autocorrelation comparisons: a & b



Spatial point- and area-correlation function estimated from gauge-rainfall fields, and the uncertainty bound for the area-correlation function. Also shown is the correlation function estimated from radar-rainfall fields. [Gebremichael et al, 2004]



Simulated stratiform field a=2.3765, b=0.9711



Simulated convective field a=2.3765, b=0.9711

Conclusions

Results suggest that if we simulated an entire year’s worth of 2D rain fields in the proportions given by M_c and M_s and scaled them to $R_{0.01}$ then the resulting statistics would be consistent with the ITU model.

This is not practical, due to computer memory and processing time constraints.

The parameters a and b allow us to scale cdfs of the simulated rain fields to the ITU model for rain rate cdf, for the tail of the ITU distribution, i.e. time percentages less than 0.05%.

The simulated database required to do this has 526 arrays, in the stratiform/convective proportions given by M_c and M_s.

Comparison of the scaled simulated and ITU modeled cdfs are in good agreement for the tail of the ITU distribution (note for the single site of Chilbolton – potential future work is to test the algorithm with other locations).

Caveats

Results seem to suggest that this method of scaling works. There are a few caveats:

1. Tails of distributions are never that well defined, due to the limited numbers of data points that create them. Hence, fitting simulated data to the tail of the ITU model carries some risk, and relies on the ITU model accurately reproducing the behaviour of the measured statistics.

2. To fit the simulated data to the tail of the ITU model, we're not only adjusting the mean of the simulated data distribution, we're also changing the standard deviation. This unfortunately affects the spatial correlation, but can be mitigated by adjusting the equivalent pixel size to make the spatial correlations more consistent with reality. Doing this impacts the total area covered by the simulated field.

3. After scaling, the simulated cdf percentage values only map to the ITU curve for percentage values less than 0.05%

Future work

Expand the model to take into account the vertical variation of rain fields, allowing it to be applied more accurately to satellite system planning.

Introduce an accurate method of simulating the temporal variation of the field (evolution and advection of the rain cells).

Further investigations with measured rain field data into the spatial autocorrelation of rain fields on different scales, in different locations and using different spatial resolutions should be carried out.