A Study on Formulating the Statistical Characteristics of the Rain Rate Duration

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Background

The rain rate duration is a fundamental parameter for evaluating the dynamic effects of rain attenuation.

Studies on the rain dynamics have been continuing in WP3J for a new recommendation since 2003.

Modeling the rain dynamics can contribute to other studies.
For examples;
the synthetic rain-rate time-series generation
the time variation of the two-dimensional simulated rain field.
etc.

Parameters for Rain Dynamics

(1) The number of rain rate events exceeding rain rate R and duration D

$$N_R(D/R)$$ ⇒ Empirical formula

(2) The number of rain fade events exceeding rain fade L and duration D

$$N_L(D/L)$$

(3) The probability distribution on number of duration D at rain rate R or at rain attenuation L

$$P_n(D/R) \quad \text{or} \quad P_n(D/L)$$

(4) The probability distribution on time of duration D at rain rate R or at rain attenuation L

$$P_t(D/R) \quad \text{or} \quad P_t(D/L)$$

Duration of Rainfall Rate Events

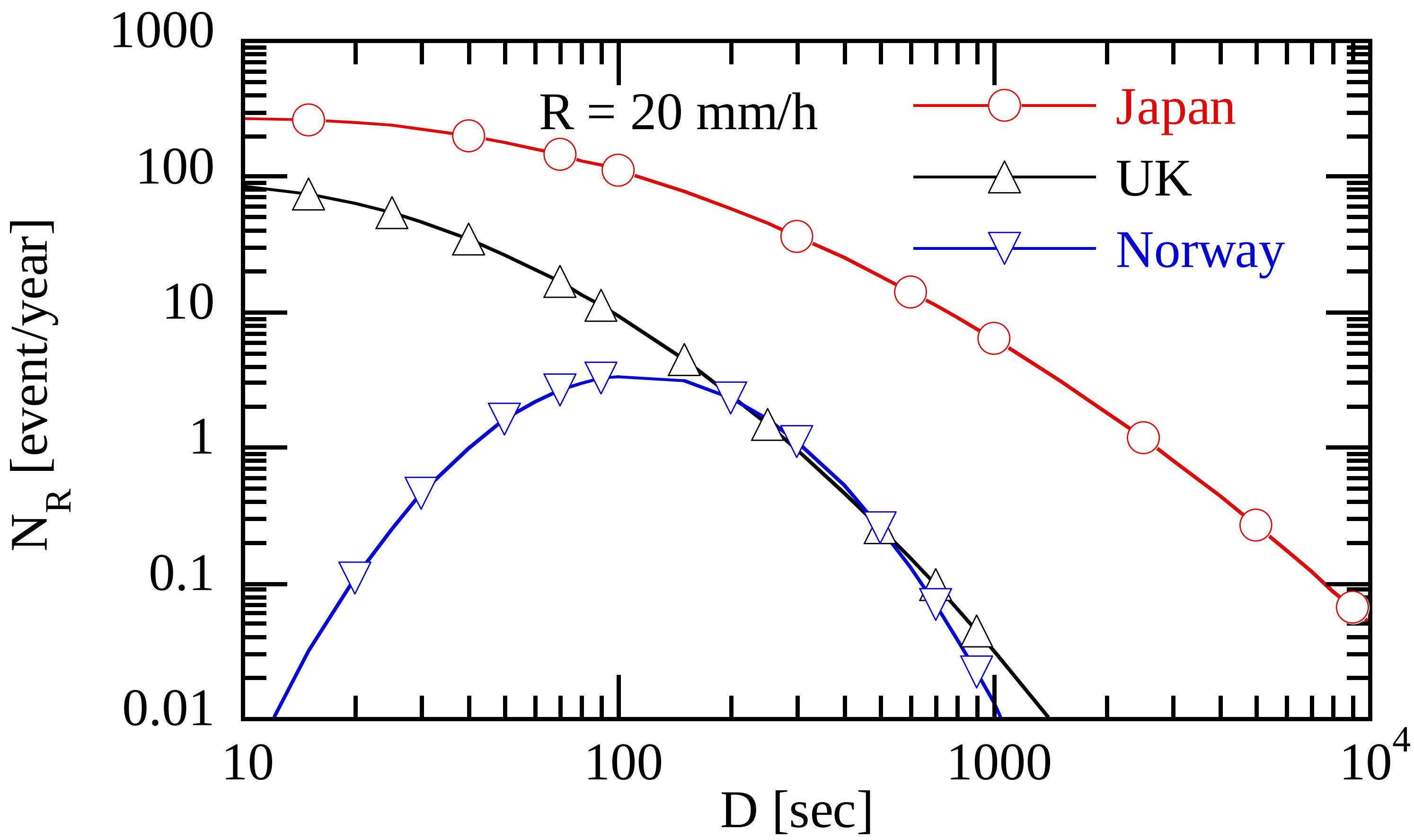
$N_R(D/R)$: The annual number of rain rate events that exceed rain rate R and duration D.

$$N_R(D/R) = N_0 \exp\left\{ -\frac{(\ln D_R - \mu)^2}{2\sigma^2} \right\}$$

Expression of Parameters Related to Number of Rain Events with Duration

District	**Country**	N_0	**μ**	**σ**
Tokyo	Japan	$2.32\ 10^4 R^{-1.49}$	$3.17R^{-0.09}$	$2.42R^{-0.13}$
Chilbolton	United Kingdom	$1.7\ 10^4 R^{-1.76}$	2	$(1.93-0.02045R)^{0.5}$
Lillehammer Oslo Alesund	Norway	$2.78\ 10^2 R^{-1.47}$	$6.76R^{-0.12}$	$1.16R^{-0.19}$
Santiago de Compostela	Spain	150*exp(–0.24*R)	1.61	$1.44{*}R^{0.22}$
Huelva	Spain	150*exp(–0.18*R)	1.61	$1.55{*}R^{0.06}$

Inconvenience of the existing method



The curve of Norway has a peak value because the curve was fit using data in a range that exceeds 120 seconds.

Solutions to avoid the inconvenience

(1) Limit the range

District	Country	N_0	μ	σ
Tokyo	Japan	$2.32\ 10^4 R^{-1.49}$	$3.17R^{-0.09}$	$2.42R^{-0.13}$
Chilbolton	United Kingdom	$1.7\ 10^4 R^{-1.76}$	2	$(1.93\text{-}0.02045R)^{0.5}$
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$D_{minimum}$

60sec 60 120 …. 60 …

* $D_{maximum}$ may be needed.

(2) Reconsider the formulation

N_R, the annual number of events that exceed a time duration is a cumulative distribution. It is better to express the distribution using a monotone function.
The applicability of Weibull distribution is indicated based on the study in Brazil.

Weibull distribution

PDF

$$f(x) = \frac{\alpha}{\beta^{\alpha}} \cdot x^{\alpha-1} \cdot \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

CDF

$$F(x) = \int_{x}^{\infty} f(t)dt = \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

Mean

$$\overline{x} = \beta \cdot \Gamma\left(1 + \frac{1}{\alpha}\right)$$

S.t.d

$$\sigma = \sqrt{\beta^{2} \cdot \left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \left\{\Gamma\left(1 + \frac{1}{\alpha}\right)\right\}^{2}\right]}$$

Relation between α and β

$$\beta = \sqrt{\frac{\sigma^{2} + \left(\overline{x}\right)^{2}}{\Gamma\left(1 + 2/\alpha\right)}}$$

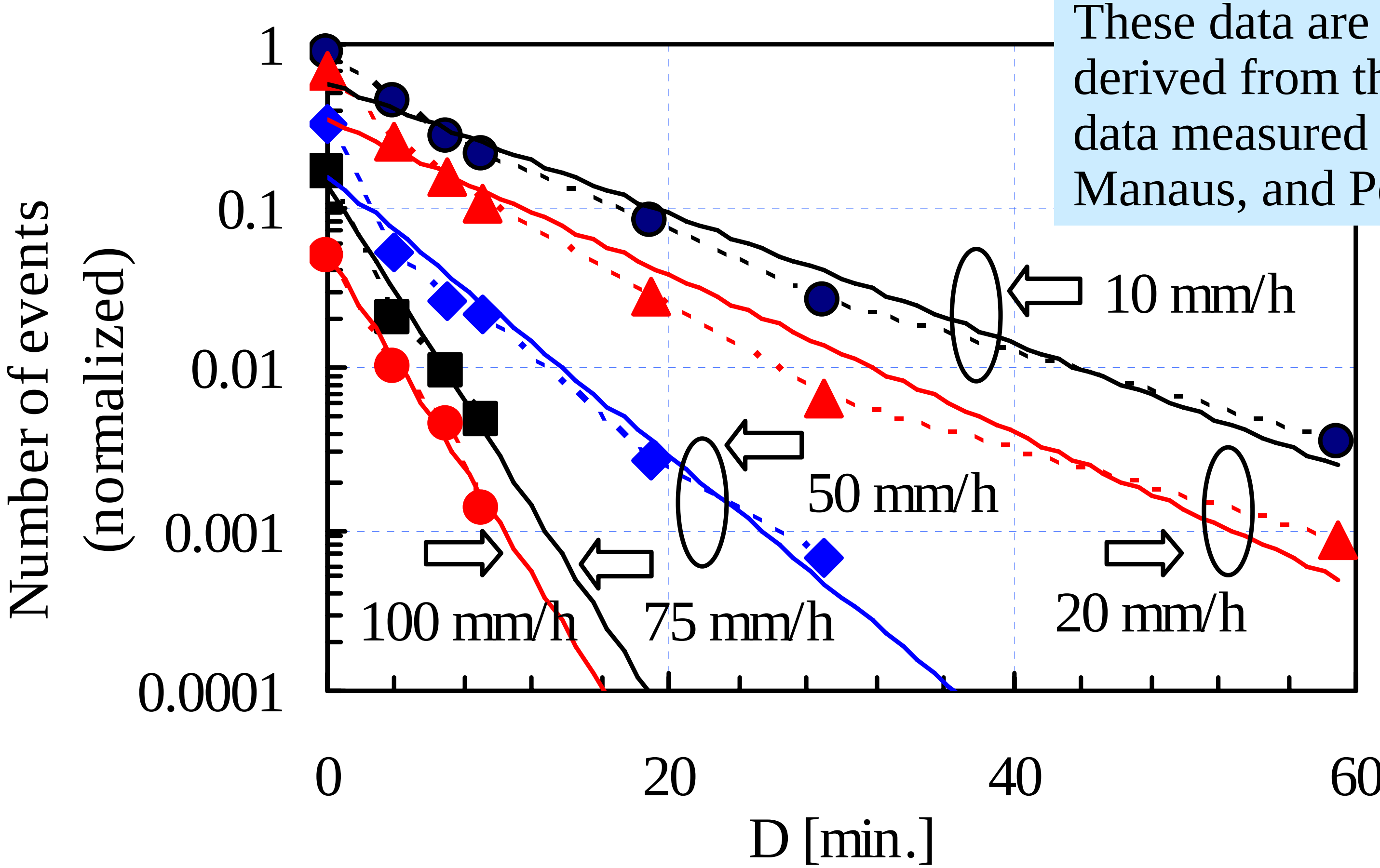
where

α equals 1; Exponential distribution

α equals 2; Rayleigh distribution

Γ( ); Gamma function

Probability distribution function of number of events in Brazil



These data are mean values derived from three sets of the data measured at Belem, Manaus, and Ponta das Lages.

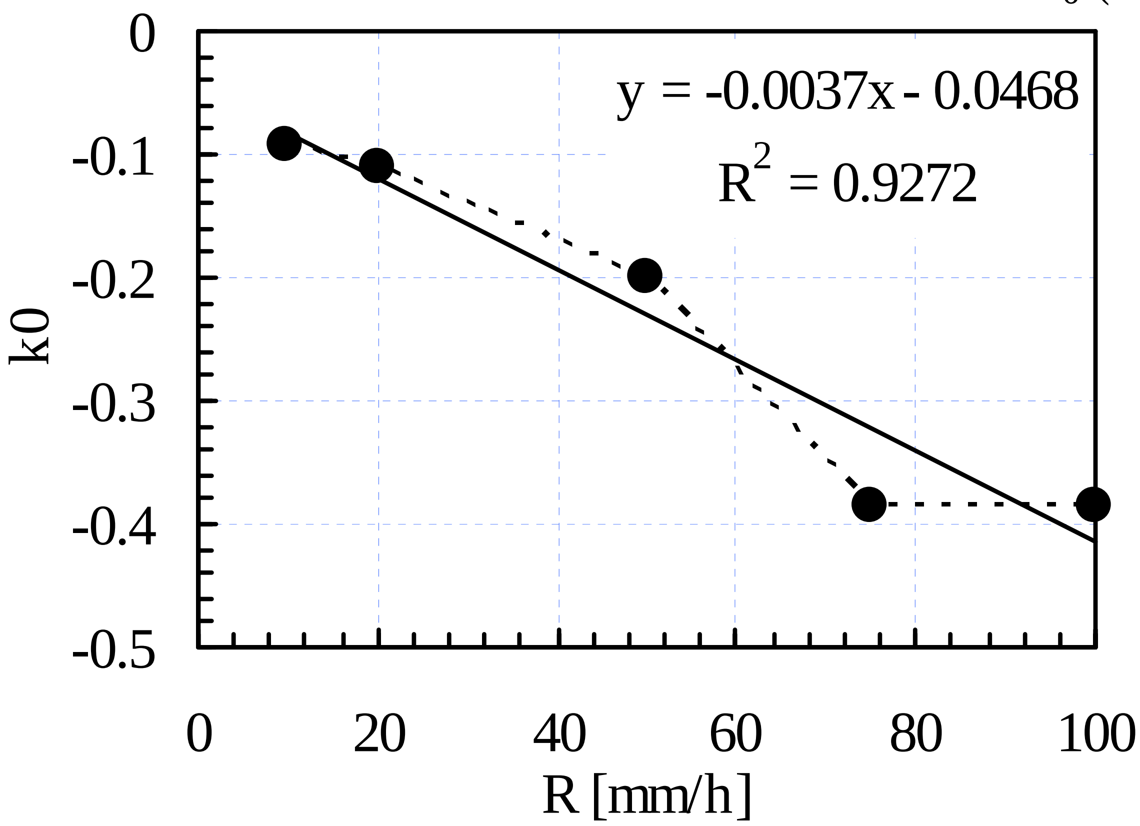
75 mm/h and 100 mm/h, the exponential formula matches well.
50 mm/h to 10 mm/h, some differences are observed but not so large

Linear fit of k_0

Exponential model $$S(D) = \exp\{k_0(R)\cdot(D-1)\}$$

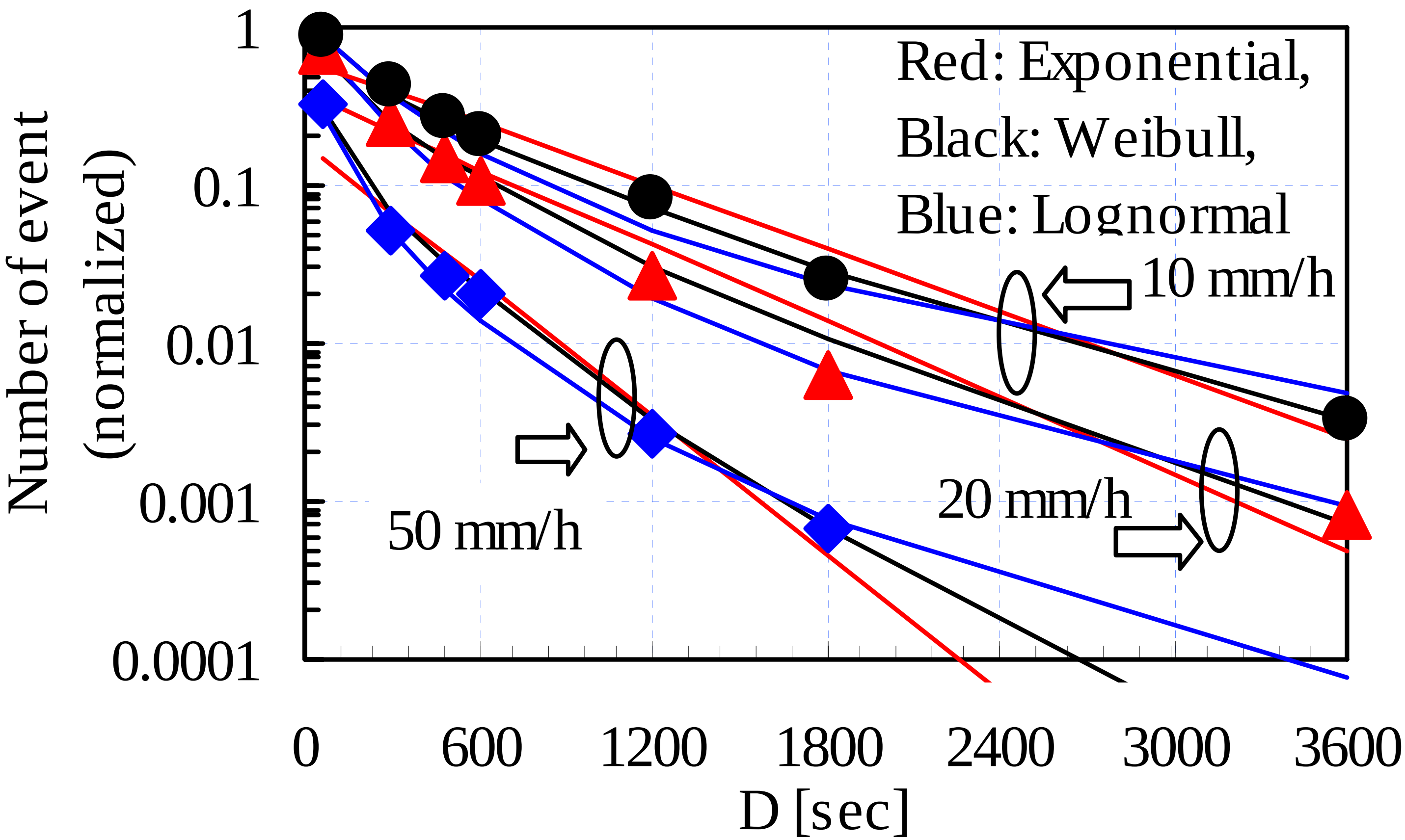
The k_0 coefficient expresses the inclination of the curve according to duration D as a function of rain rate R.

$$k_0(R) = -0.0037\cdot R - 0.0468$$



N_R in Brazil can be formulated by the exponential distribution

Trial results of curve fitting using three different formulas



Exponential distribution has only one degree of freedom
Weibull and lognormal models have two degrees of freedom

Residuals of three types of approximation

		Residuals		
		Exponential	Weibull	Lognormal
R [mm/h]	10	0.895	0.996	0.993
	20	0.773	0.999	0.996
	50	0.726	0.995	0.984
	75	0.971	0.997	0.983
	100	0.999	0.995	0.982

The Weibull model yields the best match to the measured data in the middle range of the rain rate.

Applicability of the Weibull distribution to a temperate climate zone

First, the applicability of the Weibull distribution as a probability distribution function is considered. It is treated as a formal statistical distribution and not as an approximation model.

Statistical parameters such as the mean value and standard deviation are required.

We can use the number of events per year, N_0, the mean duration, x(R), and the standard deviation, σ (R), as measured parameters in the Tokyo area.

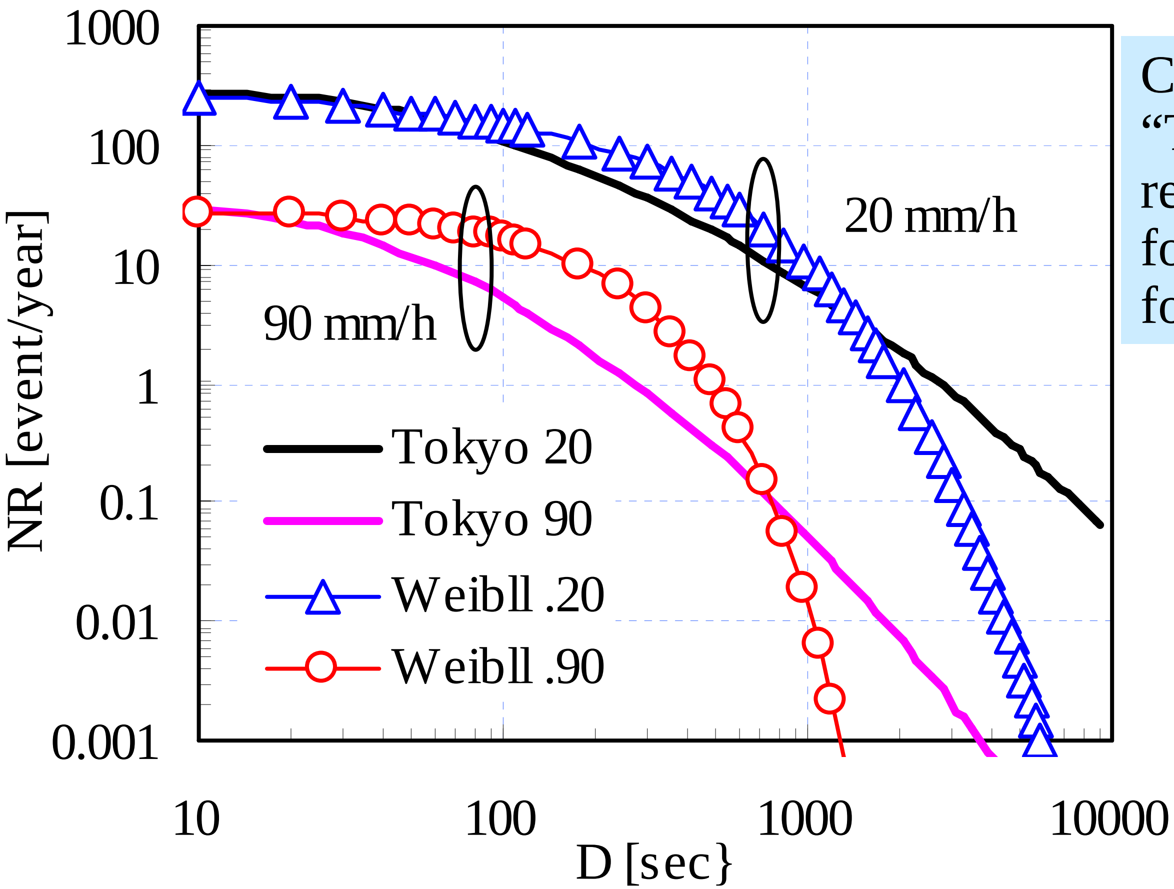
$$N_0 = 2.32 \cdot 10^4 \cdot R^{-1.49}$$

$$\overline{x(R)} = 535.1 \cdot R^{-0.2651}$$

$$\sigma(R) = 1768.1 \cdot R^{-0.5637}$$

M. Ishida, O. Sasaki, T. Taga, and S. Ichitubo, “A study on rain fade duration distribution characteristics on millimeter wave radio link,” Proc. of Climdiff-2003, Clim52 (2003).

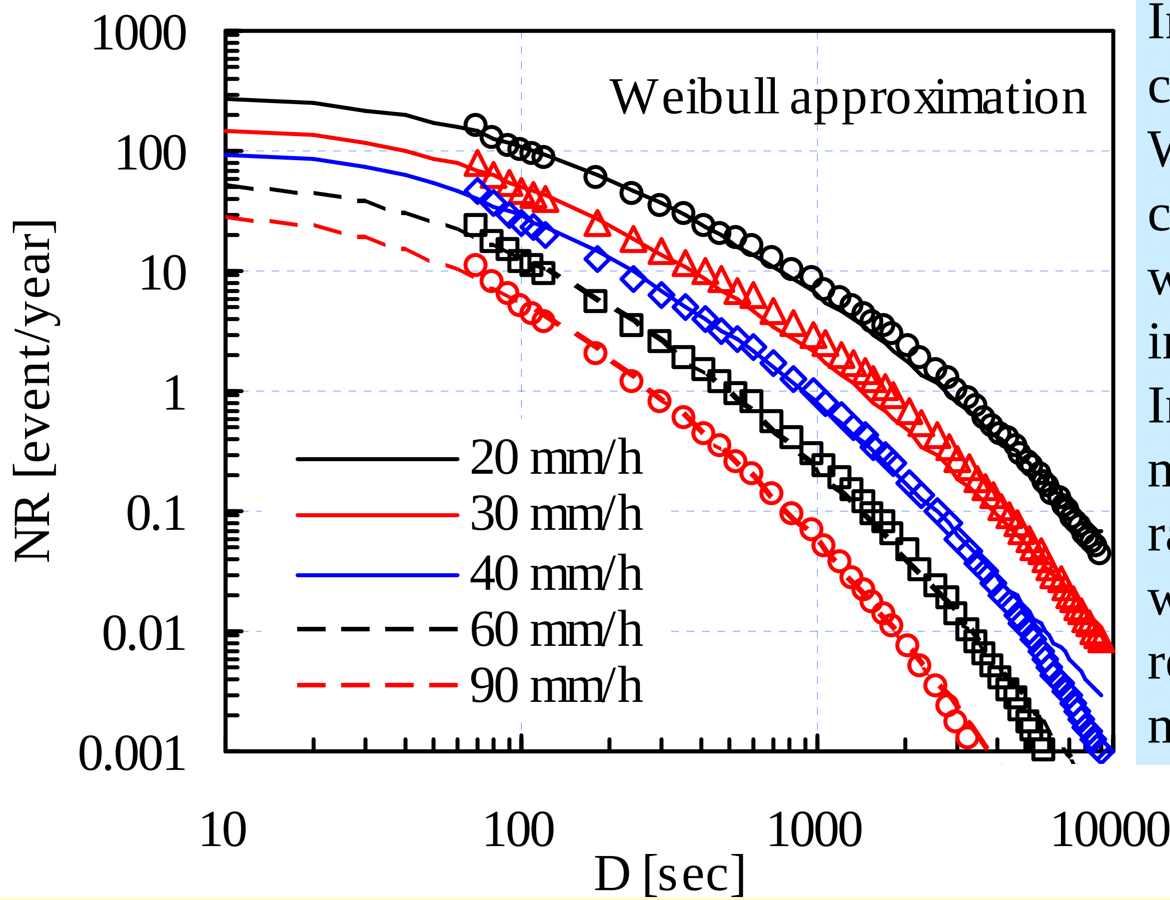
Comparison of the log-normal model to the Weibull distribution



Curves for “Tokyo 20” and “Tokyo 90” are the calculated results from lognormal formula and they are targets for the estimation of N_R.

The statistical parameters derived from equations in the previous slide do not work well to estimate the cumulative distribution by substituting directly into the Weibull distribution

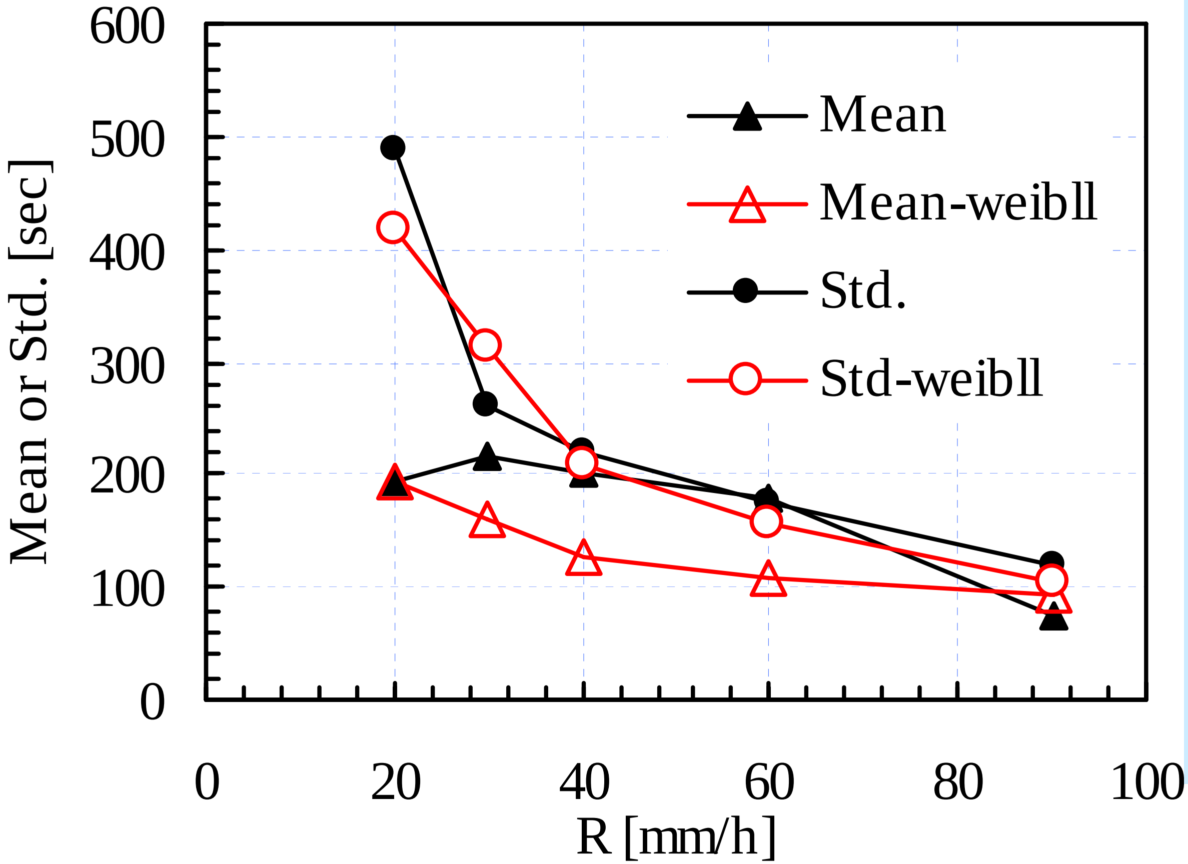
Comparison of the log-normal model to modified Weibull distribution



In the general form of cumulative distribution of the Weibull distribution, the x corresponds to duration D, which ranges from zero to infinity.
In this case, zero as the minimum of the integration range of D is replaced by 60, which is the minimum time resolution of the duration measurement.

After the replacement, the curve fitting was accomplished by searching for a suitable set of α and β. This figure shows that the calculated results and target curves agree well. But we need $D_{minimum}$.

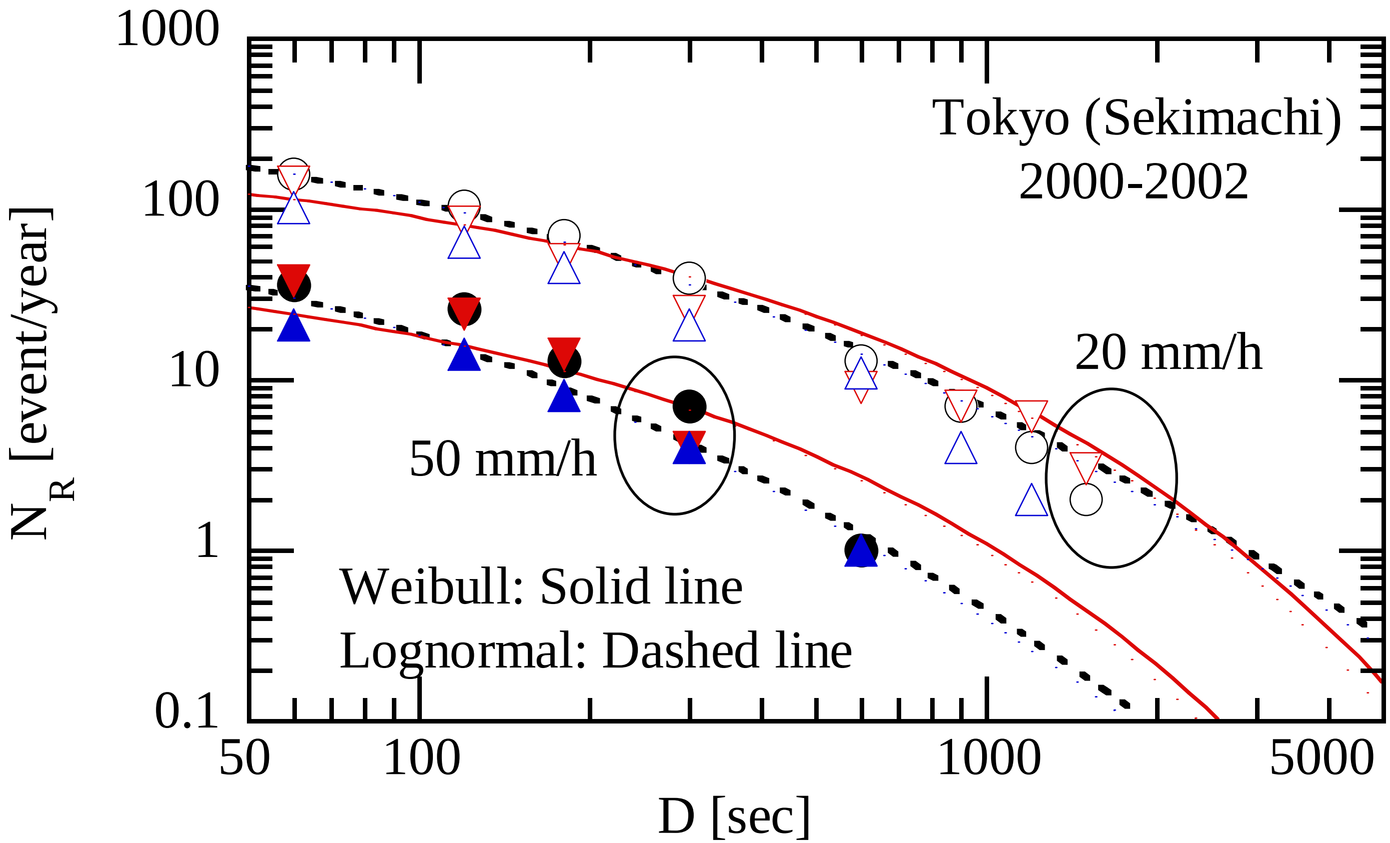
Rain rate dependencies of mean values and standard deviations



From the set of α and β, both the mean values and standard deviations can be calculated.
The N_R can be estimated using the exponential model in the case of strong rain. Below the medium rain rate, the differences between the mean and the standard deviation become remarkable.

The applicability of the exponential model with a single parameter is reduced below that for the medium rain rate and the effect of the two parameters in the Weibull distribution is notable.

Comparison of measured and estimated results



Actual data have differences such as an annual variation.

Although there are some differences between the estimated results of the lognormal formula and those from the Weibull distribution, the Weibull distribution expresses the measured results even in a temperate climate zone by optimizing the parameters.

Conclusion

Based on a report that indicates the applicability of the exponential type of formula in a tropical climatic zone, measured data were used to check the applicability for the temperate climatic zone.

(i) The exponential formula can be used in heavy rain situations such as above approximately 60 mm/h in a temperate climatic zone.
(ii) In cases of medium or light rain, the Weibull distribution can be applied by optimizing the parameters.
(iii) By using exponential types of formula the inconvenience of a decrease in the estimated number of events in a narrow range of the duration time can be avoided.