FMCW Radar Waveforms in the HF Band

Mike D.E. Turley

Defence Science and Technology Organisation, Edinburgh, SA, 5111, Australia.
Email: mike.turley@dsto.defence.gov.au

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***Abstract*— The HF band is used for various communications and radar requirements. Australian users of the band are required to adhere to spectrum management recommendations set by the Australian ACMA based on regulations set by the International Telecommunications Union (ITU). Here we examine the applicability of the ITU recommendations on Linear Frequency Modulated Continuous Waveforms (LFMCW) and amplitude modulated LFMCW waveforms. We find the ITU-R 40 dB bandwidth formula difficult to meet, even for excellent out-of-band spectral performance, due to the poor carrier frequency relation. Even though simulations and state-of-art waveform generators indicate excellent potential performance, the linearity issue of high power amplifiers still causes recommendation for no change to the 20 dB/decade roll-off rate.**

I. INTRODUCTION

Radio waves in the HF band (3-30MHz) are capable of spanning large distances on the Earth's surface due to the refractive properties of the ionospheric region. The ionospheric medium typically supports reflection of these radio waves at heights of 90-350km above the Earth's surface. HF radar defence systems have existed since the British Chain Home line-of-sight air defence system of 1937. Skywave radar operating in the HF band presents a means to survey large areas of land and sea for air and maritime targets [1], [2]. Surface wave radar also operates in the HF band and makes use of electromagnetic coupling of the HF waves to the sea surface. This coupling provides a means to detect targets over-the-horizon beyond the line-of-sight limit experienced by microwave radar systems. Other forms of HF radar also exist including hybrid Skywave-Line-Of-Sight systems (SkyLOS). Targets of interest include aircraft, ships, missiles, commercial activities, and illicit incursions and trafficking.

Conventionally these HF radar systems [1] use a form of LFMCW waveform [3]. The waveform parameter operational envelope will depend on the particular application. For monostatic configurations the waveform may also include amplitude pulse modulation, while in bistatic configurations the transmission may be continuous. Radar waveforms are generally designed to achieve specific functional requirements including good range and Doppler ambiguity profiles. The LFMCW provides excellent range resolution for a given transmission bandwidth, and allows the transmitter power amplifiers to output at a constant power. Other waveforms are designed to meet other requirements, such as the ITU-R Out-of-Band (OoB) emissions [4], for example OoB emission reduction via time and frequency weightings [5]. Recently greater interest in multiple-input multiple-output (MIMO) systems where the design objectives are transmitter element waveform orthogonality using either temporal, spectral diversity . The implementation of such designs is of course limited by the performance of the transmission system.

In this paper we examine several waveforms applicable to HF radar and compare their waveform spectra to the ITU-R emission masks [4]. Some of these radar waveforms may fall into the ITU-R taxonomy of either LFMCW or pulsed FMCW waveforms. Section II provides a theoretical foundation for analysis of the LFMCW waveform. Time and frequency techniques for sidelobe suppression in the OoB domain are discussed in Section III. The ITU-R OoB emission mask is given in Section IV. Actual data measured using a Waveform Generator Test Bed is analysed in Section V, and spectral measurement of these waveforms is considered in Section VI. Finally Conclusions and Recommendations are made.

II. LINEAR FMCW WAVEFORM ANALYSIS

In this section we examine the theoretical spectrum produced by the Linear FMCW waveform.

A. LFMCW Spectrum

Let us define the instantaneous frequency f_I of the LFMCW waveform, for a single repetition as:

$$f_I(t) = \frac{B}{T}t + f_c \qquad |t| \leq T/2 \tag{1}$$

with repetitions obeying:

$$f_I(t) = f_I(t - T) \qquad \forall t$$

where f_c is the waveform centre frequency, B is the waveform bandwidth, T is the waveform repetition interval, and t is the time variable. This waveform is shown in Figure 1 for five waveform repetition intervals.

The phase of the signal with instantaneous frequency (1) can be calculated as:

$$\begin{aligned}\phi(t) &= 2\pi \int_0^t f_I(t')\mathrm{d}t' \qquad |t| \leq T/2 \\ &= \frac{\pi B}{T}t^2 + 2\pi f_c t \qquad |t| \leq T/2 \end{aligned} \tag{2}$$

where the signal has constant amplitude and using the convenience of complex notation becomes:

$$v(t) = e^{j\phi(t)} \tag{3}$$

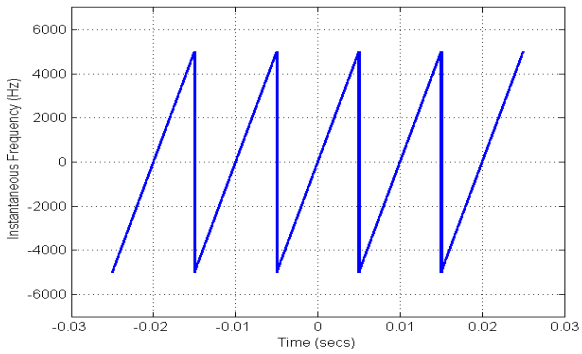

Fig. 1. Example LFMCW waveform for five repetitions.

The spectrum of this signal can be obtained from its Fourier transform:

$$v(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t} \mathrm{d}t \tag{4}$$

Upon substitution of (3) and letting $\alpha = \frac{\pi B}{T}$ and $\beta = 2\pi(f_c - f)$:

$$v(f) = \int_{-\infty}^{\infty} e^{j(\alpha t^2 + 2\beta t)} \mathrm{d}t \tag{5}$$

Using the integral relationships [6]:

$$\int \cos(\alpha t^2 + 2\beta t)\mathrm{d}t = \sqrt{\frac{\pi}{2\alpha}} \left\{ \cos\left(\frac{\beta^2}{\alpha}\right) \mathrm{C}(x) + \sin\left(\frac{\beta^2}{\alpha}\right) \mathrm{S}(x) \right\}$$

and

$$\int \sin(\alpha t^2 + 2\beta t)\mathrm{d}t = \sqrt{\frac{\pi}{2\alpha}} \left\{ \cos\left(\frac{\beta^2}{\alpha}\right) \mathrm{S}(x) - \sin\left(\frac{\beta^2}{\alpha}\right) \mathrm{C}(x) \right\}$$

where

$$x = \sqrt{\frac{2}{\pi\alpha}} (\alpha t + \beta)$$

and $\mathrm{C}(x)$ and $\mathrm{S}(x)$ are known as Fresnel Integrals defined as (shown in Figure 2):

$$\mathrm{C}(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) \mathrm{d}t$$

and

$$\mathrm{S}(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) \mathrm{d}t$$

The spectrum (5) of a single waveform repetition is then derived as:

$$v(f) = \sqrt{\frac{\pi}{2\alpha}} e^{-j\frac{\beta^2}{\alpha}} \left[\mathrm{C}(x) + j\mathrm{S}(x)\right]_{x(-\frac{T}{2})}^{x(\frac{T}{2})} \tag{6}$$

and hence the squared magnitude of the spectrum is:

$$|v(f)|^2 = \frac{T}{2B} \left\{ \left[\mathrm{C}(x_2) - \mathrm{C}(x_1)\right]^2 + \left[\mathrm{S}(x_2) - \mathrm{S}(x_1)\right]^2 \right\} \tag{7}$$

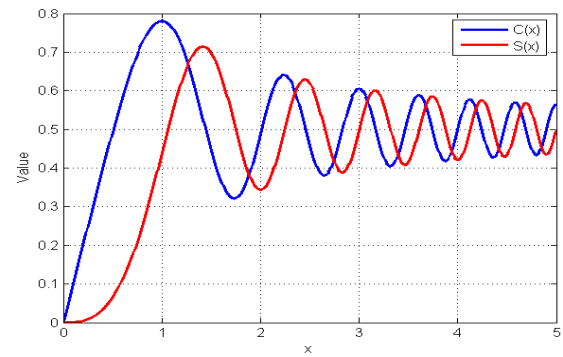

Fig. 2. Fresnel Integrals.

An example of the LFMCW spectrum magnitude is shown in Figure 3 for the case of $B = 10$ kHz, $T = 1/250$ secs and a single waveform repetition. Without loss of generality, the radar frequency f_c has been set to zero, for all subsequent analysis. The spectral oscillations are known as Fresnel ripples.

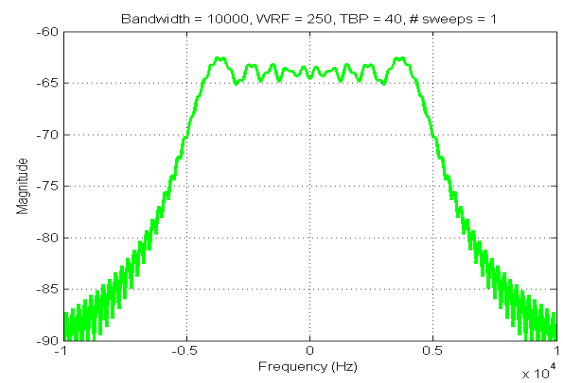

Fig. 3. Example LFMCW spectrum for a single repetition.

B. Numerical Analysis

For waveforms more general than the rectangular LFMCW numerical analysis may be performed. Figure 4 shows the half spectrum for a single sweep rectangular LFMCW waveform. Overlayed is the detected envelope spectrum. The waveform signal was also synthesised using (3) by discrete sampling at $f_s = 100 \times B$ with $f_c = 0$. For the significant portion of the plot the spectrum for the numerical technique is identical and obscured by the overplotted theoretical spectrum. At high frequency the numerical spectrum sidelobes degrade as a result of aliasing caused by sampling below the true Nyquist frequency, which could be regarded as infinite.

C. Out-of-Band Roll-Off Rate

From (7) the spectrum sidelobe roll-off rate may be approximated in the sidelobe region using the large $x > 5$ Fresnel Integral approximations, based on [6]:

$$\mathrm{C}(x) \approx \frac{1}{2} + \frac{1}{\pi x} \sin\left(\frac{\pi x^2}{2}\right) - \frac{1}{\pi^2 x^3} \cos\left(\frac{\pi x^2}{2}\right)$$

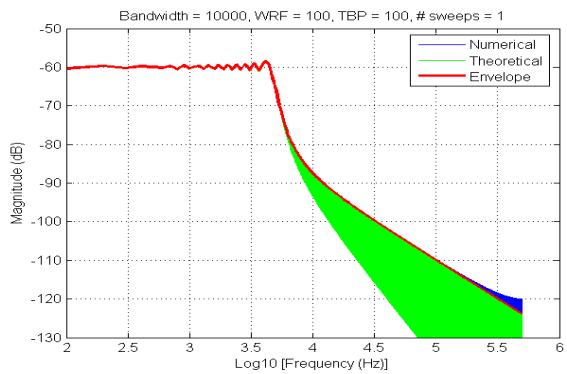

Fig. 4. Spectrum for a single repetition rectangular LFMCW waveform.

and

$$\mathrm{S}(x) \approx \frac{1}{2} - \frac{1}{\pi x}\cos\left(\frac{\pi x^2}{2}\right) - \frac{1}{\pi^2 x^3}\sin\left(\frac{\pi x^2}{2}\right)$$

It may be seen that the dominant frequency relationship is an inverse x^2 term, since $x \propto f$ through β, thus the roll-off rate, δ, becomes:

$$\begin{aligned}\delta &= 10\ \log_{10}\left(\frac{f^2}{(10f)^2}\right)\\ &= -20 \text{ dB/decade}\end{aligned} \tag{8}$$

This value becomes significant when specifying OoB emission mask formula for LFMCW or similar discontinuous waveforms.

D. Time-Bandwidth Product

The LFMCW waveform spectrum may also be analysed by examining the spectral envelope. Maxima of the $|v(f)|^2$ function may be calculated [6] or alternatively a simple algorithm may be developed to select the spectral maxima. We chose the second option. In Figure 5(a) the single-sided (half spectrum) envelope spectrum is plotted in log-log scale for a waveform bandwidth of B =10 kHz and various time-bandwidth products (TBP). The diagram shows the in-band dependance on TBP, and conversely independence in the OoB zone. The sidelobe roll-off rate asymptotes to -20 dB/decade at high frequencies. Figure 5(b) shows the derivative of the envelope functions for each TBP.

The spectral shape dependence on TBP value and its relation to the OoB sidelobe roll-off zone should clearly be factors that influence specification of OoB emission mask formula for LFMCW waveforms.

In Figure 6 the bandwidth parameter has been reduced to $B = 1$ kHz with the same TBP values. As expected the spectral envelopes show reduced bandwidth, and scaling governed by the $T/(2B)$ relation as defined in (7). The sidelobe roll-off rate remains identical.

Figures 5 & 6 show that the spectral shape is a function of the TBP only, the bandwidth parameter scales with frequency, and magnitude scales as $\sqrt{T/(2B)}$.

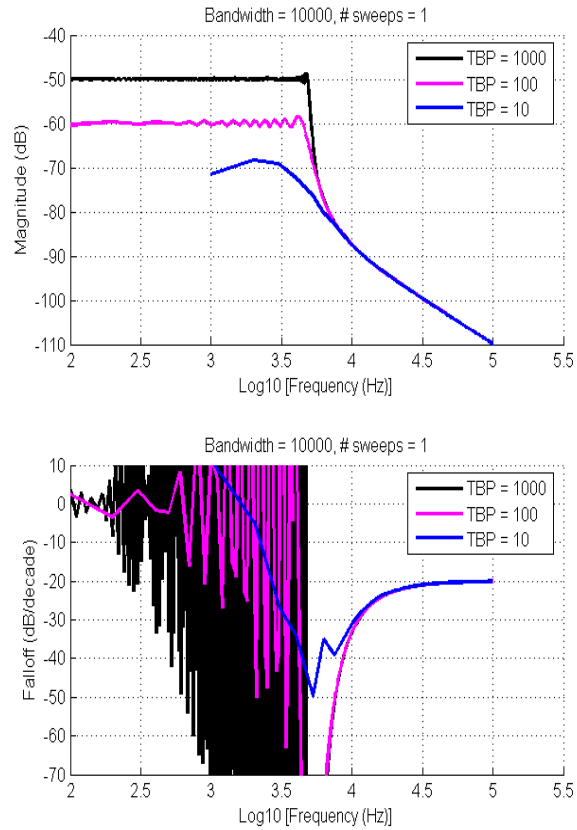

Fig. 5. Spectral envelope and sidelobe roll-off of LFMCW waveforms with bandwidth of 10 kHz as a function of TBP.

E. Observation Time / # chirps

The preceding analysis has been performed by considering only a single waveform repetition (or sweep). The actual spectrum observed will depend on the measurement duration. Firstly let us consider the spectrum of a rectangular LFMCW waveform for an integer number of repetitions. The repetitive form of the waveform (1) means the multiple sweep spectrum (4) may be calculated as the sum of single sweep spectra:

$$v(f) = \sum_{s=1}^{N} e^{j2\pi f\left(s-\frac{N+1}{2}\right)T} \cdot \int_{-\frac{T}{2}}^{\frac{T}{2}} v(t)e^{-j2\pi ft}\mathrm{d}t \tag{9}$$

where s & N denote sweep index & number, and we have made use of the Fourier transform time shift property. An example spectrum for an eight sweep waveform is displayed in Figure 7. The spectrum appears as the sum of harmonic lines separated by the waveform repetition frequency (WRF $= 1/T = 100$ Hz). The envelope spectrum was obtained by searching for the maximum value over each WRF interval. The envelope spectrum is of similar (but not identical) appearance to the single sweep spectrum, and a coherent gain at the harmonic line frequencies is observed.

In Figure 8 the envelope spectra for various number of waveform sweeps is shown, with each spectrum normalised

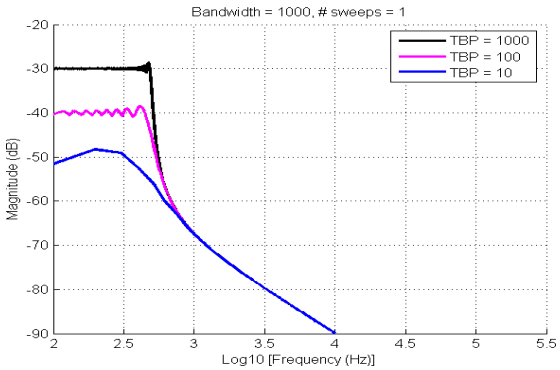

Fig. 6. Spectral envelope and sidelobe roll-off of LFMCW waveforms with bandwidth of 1 kHz as a function of TBP.

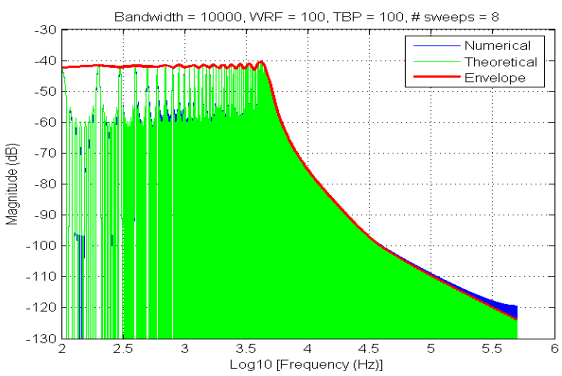

Fig. 7. Spectrum morphology for a repetitive rectangular LFMCW waveform.

to its peak value. For longer observation of the waveform, better peak to sidelobe performance is achieved.

F. In-band Performance

For radar applications the acceptance of alternative waveforms typically critically depends on at least range discrimination performance. The waveform ambiguity function may be calculated as:

$$|\chi(\tau, f_d)|^2 = \left| \int v^*(f) v(f - f_d) e^{-j2\pi f \tau} \mathrm{d}f \right|^2 \tag{10}$$

The zero Doppler ambiguity function (or range pattern) may thus be calculated discretely as:

$$\chi(k, 0) = F^{-1} \left\{ w_R(f) |v(f)|^2 \right\} \tag{11}$$

where F^{-1} represents the inverse discrete Fourier transform, k is the range bin number, and a range processing taper function w_R is introduced to reduce range sidelobe performance. Figure 9 displays the range pattern for the various waveform modulation methods (described later), incorporating a rectangular taper $w_R(f) = 1$. The best far-range sidelobe performance is achieved by the amplitude modulation technique, which will be described in the next section.

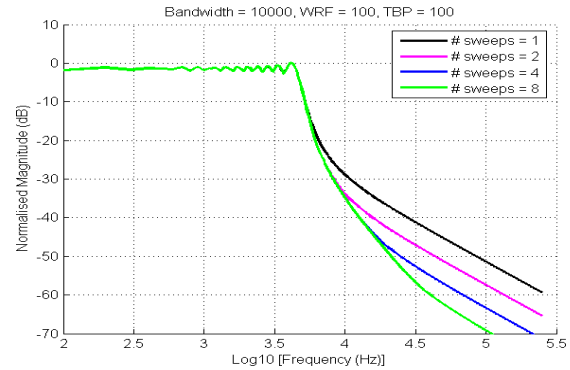

Fig. 8. Normalised Spectra as a function of waveform repetitions.

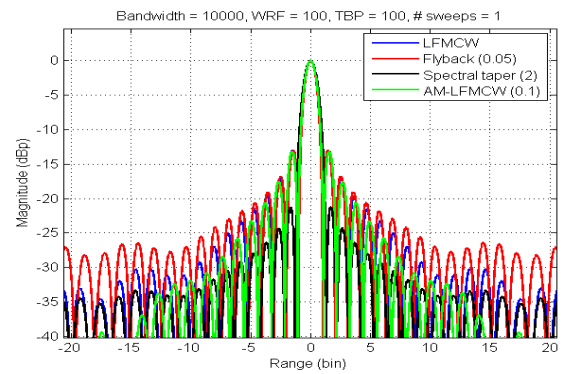

Fig. 9. Zero-Doppler waveform ambiguity function

III. OUT-OF-BAND EMISSION SUPPRESSION

Alternative waveforms (to LFMCW) designed with objectives to meet both the ITU-R recommendations and radar system requirements are of interest. The following subsections discuss frequency and amplitude modulation modifications to the rectangular LFMCW waveform. This type of reduction of sidelobes through time and frequency weighting is discussed by Kowatsch *et al.* [5]. The principal involved is to improve the OoB emission spectrum whilst maintaining in-band performance. The obvious modification is to smooth the frequency transition region at the ends of each chirp, by smoothing the signal in either the instantaneous frequency or amplitude domains.

Three modifications to the basic rectangular LFMCW waveform are considered. All methods are somewhat adhoc, as they are not directly designed with the objective to minimise OoB emissions.

A. Spectral Taper Method

The waveform may be designed in the spectral domain. An adhoc means is to start with the LFMCW single sweep spectrum (6), modulated by a taper function $w_{ST}(f)$ [7], then inverse Fourier transformed to derive the signal:

$$v_{fm}(t) = \int_{-\infty}^{\infty} w_{ST}(f) \cdot v(f) e^{j2\pi f t} \mathrm{d}t \tag{12}$$

Figure 10 shows an example spectral envelope calculated from the sampled signal using the Hann taper function, see black curve. Some gain in OoB emission performance is achieved, though the roll-off rate is identical to the blue curved rectangular LFMCW technique. This modification results in slight changes to the signal phase, amplitude and instantaneous frequency, as displayed in Figure 11, 12 & 13.

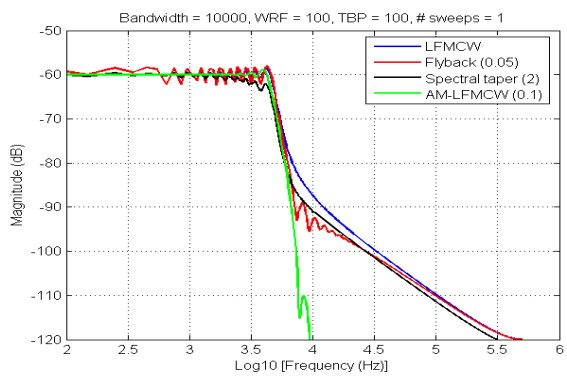

Fig. 10. Envelopes of half Spectra for various waveform modulations.

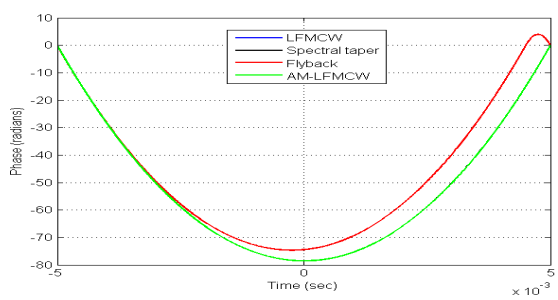

Fig. 11. Phase for various waveform modulations.

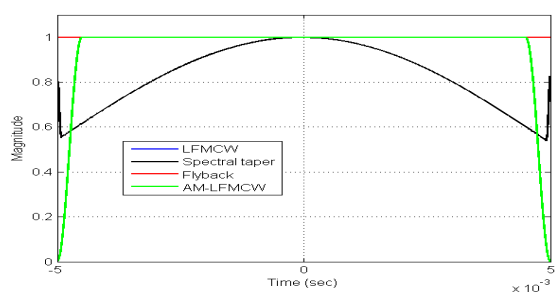

Fig. 12. Magnitude for various waveform modulations.

B. Flyback Waveforms

Examination of the rectangular LFMCW waveform indicates the instantaneous frequency is a discontinuous function. A simple method to reduce this discontinuity is to introduce

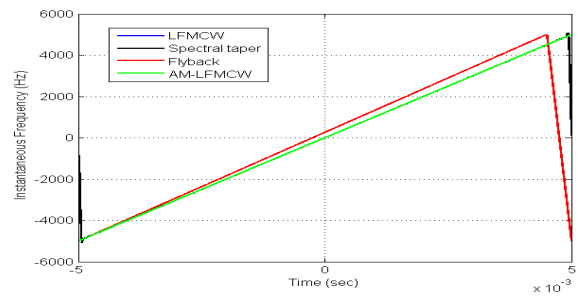

Fig. 13. Instantaneous frequency for various waveform modulations.

a counter sloping chirp at the waveform end [8]:

$$f_I(t) = \begin{cases} \frac{1}{1-\gamma}\frac{B}{T}t + f_c & -T/2 \leq t \leq (1-2\gamma)T/2 \\ -\frac{1}{\gamma}\frac{B}{T}t + f_c & \text{otherwise} \end{cases}$$

where $0 \leq \gamma \leq 1/2$ is the flyback factor. If desired higher order interpolations may be incorporated.

C. Amplitude Modulation

The conventional method for reducing roll-off rates is the application of amplitude tapering [7]. Here we apply a cosine-Tukey amplitude taper to the signal.

$$v_{am}(t) = w_{CT}(t) \cdot v(t) \tag{13}$$

As observed in Figure 10 this technique provides excellent OoB emission control, even with low percentage tapering (here 10%, Figure 12), thereby causing little associated loss in coherent gain. The method does however rely on linear high power amplifier performance (linearity).

IV. ITU-R MASK

The ITU-R spectral management document SM.1541-1 [4] provides recommendations on OoB emission advice for a taxonomy of radar waveforms including the rectangular LFMCW waveform. An emission mask is defined for three segments of the spectrum called the necessary domain, the OoB emission domain, and the spurious domain. The domain definitions are:

Necessary Bandwidth
The width of the frequency band which is just sufficient to ensure the transmission of information at the rate and with the quality required under specified conditions.

Out-of-band emission
The frequency range, immediately outside the necessary bandwidth but excluding the spurious domain, in which OoB emissions generally predominate.

Spurious
The frequency range beyond the OoB domain in which spurious emissions generally predominate.

For FMCW radar the necessary bandwidth is defined as twice the frequency deviation B_D:

$$B_N = 2B_D = B \tag{14}$$

whereas for FM Pulse waveforms:

$$B_N = \frac{1.79}{\sqrt{t_d t_r}} + 2B_C \tag{15}$$

where B_C is the frequency deviation, t_r is the pulse rise time, and t_d is the pulse duration at half amplitude.

The OoB domain emission limits for primary radar are based on a 40 dB bandwidth and a roll-off rate of 20 dB/decade. For FMCW radars the 40 dB bandwidth formula is:

$$B_{-40} = B_N + 0.0003 f_c; \tag{16}$$

whereas for FM Pulse waveforms:

$$B_{-40} = 2(B_C + \frac{A}{t_r}) + \frac{K}{\sqrt{t_d t_r}}; \tag{17}$$

where

$$A = \begin{cases} 0.105 & \text{if } K = 6.2 \text{ where } P > 100\text{kW} \\ 0.065 & \text{if } K = 7.6 \text{ where } P < 100\text{kW} \end{cases}$$

Considering the operational envelope for carrier frequency and bandwidth for radar in the HF band $4 < f_c(\text{MHz})) < 30$ and $4 < B < 50$(kHz), then the ratio of the 40 dB bandwidth formula to the bandwidth spans (16):

$$1.024 < \frac{B_{40}}{B} < 3.25$$

whereas for example the trapezoidal FM pulsed waveforms of JRG13 [9] span (17):

$$1.9 < \frac{B_{40}}{B} < 5.7$$

making the FMCW radar task comparatively difficult.

The amplitude of the mask in the necessary region depends on a reference and measurement bandwidth as defined in ITU-R measurement document M.1177 [10]. Relative values are given in dBpp units. Here we substitute the spectral peak value as the reference value.

The regulations specify a spurious attenuation level for the spurious domain as the least stringent of 60 dB or 43 dB + $10\log(PEP)$, where PEP is the peak envelope power. Here we assume the 60 dB value.

In Figure 14 the ITU emission masks are displayed against the FMCW spectra for a single radar chirp with bandwidth of 10 kHz and various TBPs. For these cases the waveforms achieve suitable roll-off rate, but are non-compliant with the emission masks, especially at low TBP and low carrier frequency.

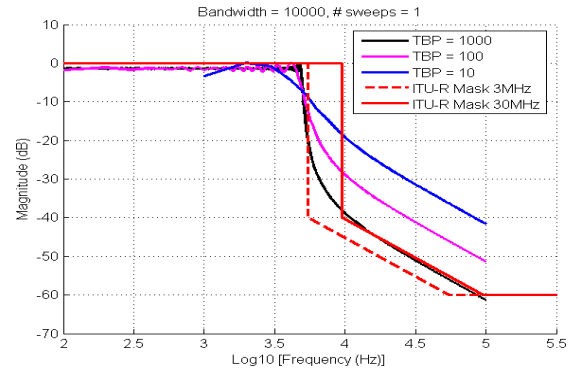

Fig. 14. ITU-R Masks for various TBPs and constant bandwidth.

V. WAVEFORM GENERATOR TEST BED

Here we provide results of measurements produced by the DSTO HF Radar Waveform Generator Test Bed. Three DSTO waveform functions were selected for input to a BAE Systems Waveform Generator. These were the LFMCW, cosine-Tukey AM-LFMCW, and a coded waveform from a MIMO orthogonal waveform eight set. The Waveform Generator was set to a centre frequency of 15.05 MHz, then a BAE Systems HF Direct Digital Receiver together with a DSTO down converter were used to acquire data at an IQ sampling frequency of 1 MHz.

Figure 15 shows the real signal component for a single chirp of an LFMCW waveform (B=10 kHz, WRF=50 Hz). The corresponding spectrum and its envelope are shown in Figure 16; this matches the expected theoretical spectrum. The AM-LFMCW waveform with 10% cosine-Tukey weighting was input to the Waveform Generator Test Bed. Figure 17 shows the real signal component for a single chirp (B=10 kHz, WRF=50 Hz), with the corresponding spectrum shown in Figure 18. In effect the excellent OoB response is achieved by trading the in-band to OoB transistion width. This suggests additional tolerance should be included in the necessary bandwidth definition (14) for the FMCW case to include a transition bandwidth B_T:

$$B_N = 2B_D + B_T$$

Comparison of the half spectrum envelopes for the LFMCW and AM-LFMCW data is shown in Figure 19. The overlayed ITU-R FMCW mask (at extreme carrier frequencies) indicates the difficulty of compliance for even the excellent spectral OoB performance achieved by the AM-FMCW waveform. For further comparison Figure 20 shows the spectral response at a higher TBP. Finally for reference the measured spectral response of the MIMO waveform is shown in Figure 21.

VI. MEASUREMENT

Under the ITU-R recommendations, techniques for the spectrum measurement are given in document ITU-R M.1177 [10]. The reference bandwidth value for FMCW radar waveforms

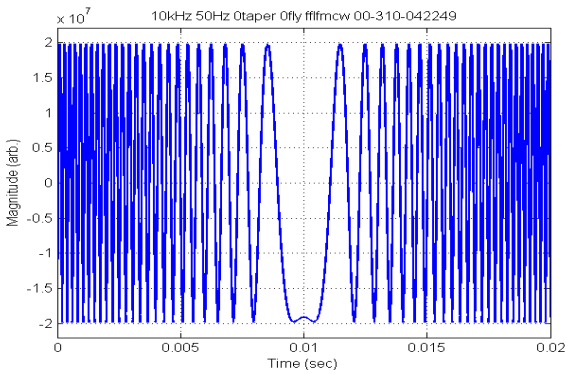

Fig. 15. Waveform Generator LFMCW, real component of signal.

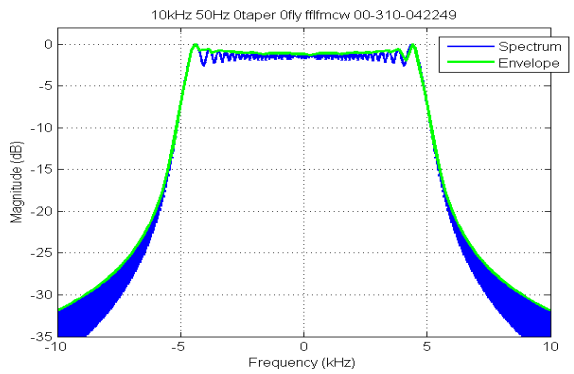

Fig. 16. Waveform Generator LFMCW spectrum.

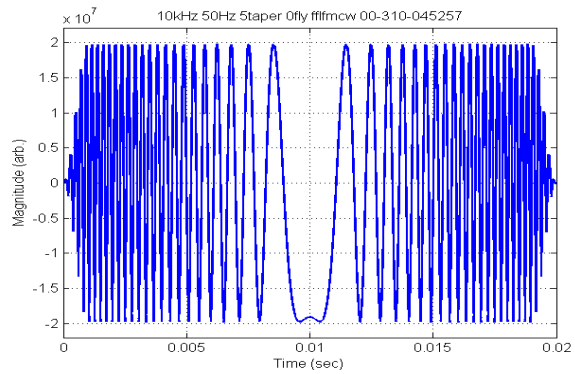

Fig. 17. Waveform Generator AM-LFMCW (10% Cosine-Tukey), real component of signal.

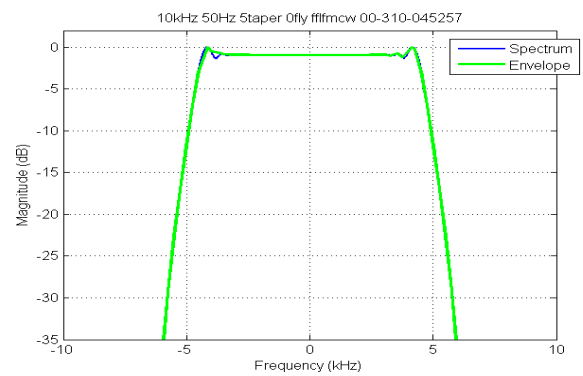

Fig. 18. Waveform Generator AM-LFMCW (10% Cosine-Tukey) spectrum.

is defined as:

$$B_{ref} = \sqrt{\frac{B}{T}} \quad \text{Hz} \tag{18}$$

The document also defines the receiver measurement bandwidth (sometimes referred to as resolution bandwidth):

$$B_m \leq \sqrt{\frac{B}{T}} \quad \text{Hz} \tag{19}$$

Given the actual spectrum depends on the observation time, the measurement duration will influence the measured spectrum and hence the peak to OoB emission spectrum.

For all of the described waveforms the actual spectrum observed depends on the observation period. The design objectives are typically based on the spectrum of a single complete waveform repetition cycle (chirp or sweep). Thus the designed spectrum will only be observed for a single complete chirp measurement, which can only be achieved if the exact waveform period and start are available. Figure 22 shows the half envelope spectrum for 100 trials of Waveform Generator Test Bed data, using a 10% Cosine-Tukey AM-LFMCW with B=10 kHz and WRF=50 Hz. In each trial the measurement duration was selected as a random value between 0.5 and 8 waveform periods, and the start time was also random. The lower blue curve corresponds to the exact single period measurement duration. All other trial spectra do not reflect the true spectrum and default to a roll-off of 20dB/decade. From these trial spectra the 40dB bandwidth was calculated and shown in Figure 23. A reasonable estimate of the true 40dB bandwidth only occurs for measurement periods greater than 5 chirps. The problem presented by this type of spectral analysis is that the measurement duration does not coincide with an integer number of waveform periods. A means to overcome this problem is to apply a taper function to the whole measurement duration. The analysis was repeated using an Hann taper with the results shown in Figures 24 & 25. The 40dB bandwidth estimate is robust for measurement durations greater than about 2 chirps. However observation of the spectra indicate some trial spectra over-achieve in the OoB domain, due to the taper influence, and that in-band spectral response is occasionally poor.

VII. CONCLUSIONS

LFMCW waveforms only achieve 20 dB/decade roll-off. At HF frequencies the ITU-R OoB emission mask is tight at low TBPs and low HF carrier frequency. The cosine-Tukey AM-LFMCW waveform presents a conceptually simple modulation with excellent range ambiguity performance. This meets the 40 dB/decade proposal for the theoretical and the demonstrated waveform generator test data. We find the ITU-R 40 dB bandwidth formula difficult to meet, even for excellent out-of-band spectral performance, due to the poor carrier

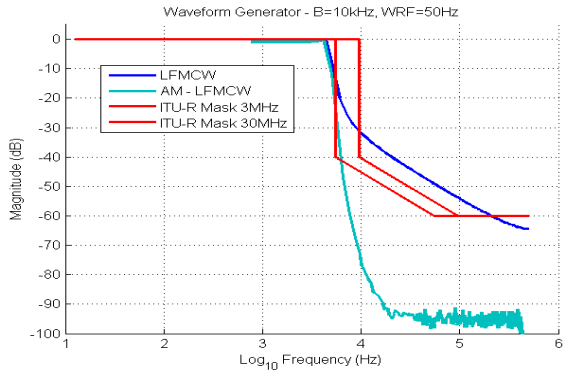

Fig. 19. Waveform Generator envelope half spectra, B=10kHz, WRF=50Hz.

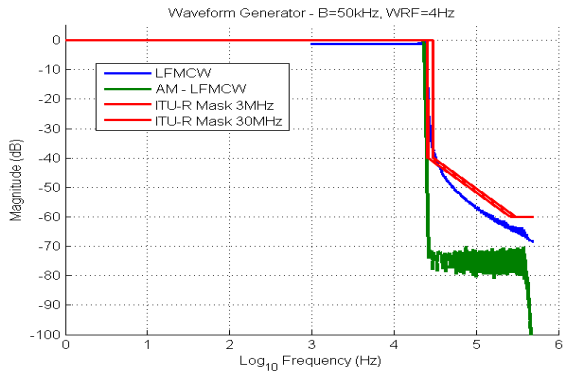

Fig. 20. Waveform Generator envelope half spectra, B=50kHz, WRF=4Hz.

frequency relation. Even though simulations and state-of-art waveform generators indicate excellent potential performance, however it has not been demonstrated that HPA systems can achieve this response. The linearity issue of high power amplifiers still causes recommendation for no change to the 20 dB/decade roll-off rate. In addition the wider in-band to OoB transition regions indicates a need for new definition of the B_{40} bandwidth. MIMO orthogonal waveforms may present a new classification of radar waveform.

VIII. RECOMMENDATIONS

- The 40 dB bandwidth of FMCW radar should be related in-line with FMCW pulsed radar.
- The output of high power HF transmission systems should be measured for comparison against the demonstrated state of art waveform generators. Hence the current 20 dB/decade roll-off rates should be maintained.
- Measurements should be performed over at least five waveform periods with use of a taper function to ensure waveform end effects are accounted.

IX. ACKNOWLEDGEMENT

The author wishes to thank Matt Young of DSTO for providing the Waveform Generator Test data.

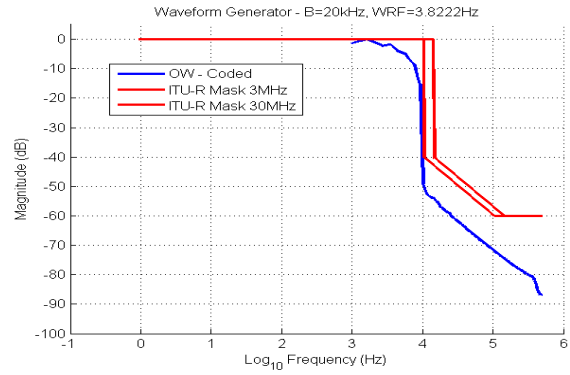

Fig. 21. Waveform Generator envelope half spectra of coded waveform, B=20kHz, WRF=3.82Hz.

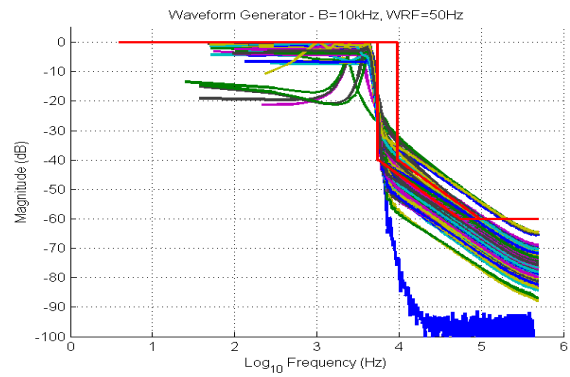

Fig. 22. Waveform Generator AM-LFMCW (10% Cosine-Tukey) half spectrum from 100 acquisitions from 0.5 to 8 sweeps.

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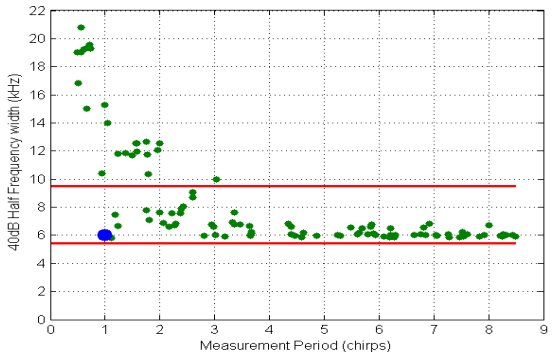

Fig. 23. Waveform Generator AM-LFMCW (10% Cosine-Tukey) half 40dB widths from 100 trials.

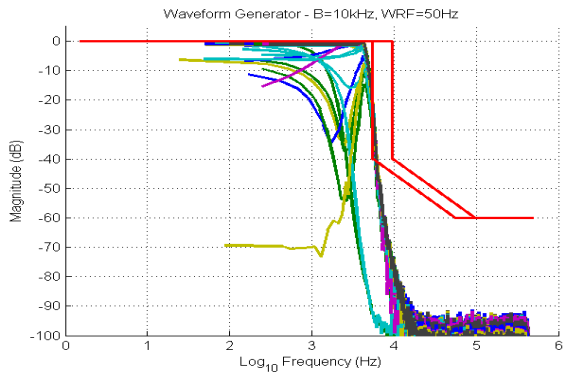

Fig. 24. Waveform Generator AM-LFMCW (10% Cosine-Tukey) half spectrum from 100 acquisitions from 0.5 to 8 sweeps.

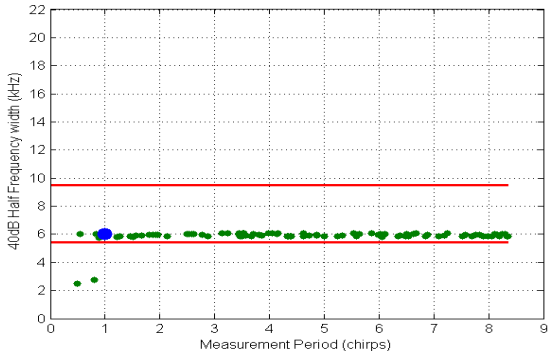

Fig. 25. Waveform Generator AM-LFMCW (10% Cosine-Tukey) half 40 dB widths from 100 trials.