$$g(0.9, f) = 0.18 \sin \left[5.22 \log_{10} (f/200)\right] + 1.23. \qquad (1.11b)$$

For $f > 1600$:

$$g(0.1, f) = g(0.9, f) = 1.05. \qquad (1.11c)$$

Equations (1.11a-c) represent approximations to the empirical estimates of g(0.1) and g(0.9) plotted in figure 1.5 for all year.

1-2. Variability with Location

There is a path-to-path variation in the available wanted signal power that may be referred to as location variability. Such random variations from location to location for any given q_T may be assumed to be normally distributed with a standard deviation σ_{La} dB. Then $A(q_L)$ is the attenuation below free space not exceeded for at least a fraction q_L of all randomly chosen paths for which other parameters, such as frequency, antenna heights, and path length, are fixed. A value $q_L = 0.5$ would represent median conditions, while $q_L = 0.1$ would mean that the antennas are assumed to be located at sites selected to be among the best 10 percent of all possible locations within a given area. For low antennas, over irregular terrain, the best estimate from presently available data indicates a value of $\sigma_{La} \simeq 10$ dB, as shown in annex 2. The location variability $Y(q_L)$ for any fraction q_L may be expressed in terms of the standard deviation σ_{La} and a standard normal deviate $z_o(q_L)$,

$$Y_L \equiv Y(q_L) = -\sigma_{La}\, z_o(q_L) \text{ dB}, \qquad (1.12a)$$

where

$$z_o(q_L) = \sqrt{2}\, \text{erf}^{-1} (2q_L - 1). \qquad (1.12b)$$

1-3. Prediction Error and Service Probability

A communication link is assumed to provide satisfactory service of a given grade g_r if the available signal-to-noise ratio R exceeds the required protection ratio $R_r(g_r)$ for at least a fraction q_T of the time and a fraction q_L of all similar links. The noise-limited service requirement may be written as

$$R(q_T, q_L, Q) = R_o + Y_T + Y_L + Y_c \quad \mathrm{dB}, \qquad (1.13a)$$

where R_o is the median available signal-to-noise ratio and the symbols Y_T, Y_L, and Y_c represent biases or safety factors required to achieve protection for at least the desired fraction q_T of the time, the desired fraction q_L of locations, and with a probability Q. These biases are considered in the order given, allowing first for time availability q_T, then for location variability q_L, with q_T fixed, and finally for service probability Q, with both q_T and q_L fixed. That is, we may write (1.13a) as

$$R(q_T, q_L, Q) = R_o + Y_T(q_T) + Y_L(q_T, q_L) + Y_c(q_T, q_L, Q) \quad \mathrm{dB}. \qquad (1.13b)$$

In this discussion we consider interference due to noise at the terminals of the receiving antenna, including both internal system noise and externally generated unwanted signals whose effect can be represented by an equivalent noise power whenever the probability of power-independent distortion is negligible. Further discussion of interference from unwanted signals may be found in annex V of the report by Rice et al. (1967).

The biases Y_T, Y_L, and Y_c expressed in decibels are assumed to be normally distributed in time and with location, with errors of prediction also normally distributed. The biases may then be expressed in terms of a standard normal deviate $z_o(q)$, and the variance of the available signal-to-noise ratio in time, σ_T^2, with location, σ_L^2, and with prediction error, σ_c^2. The variance σ_T^2 is defined in terms of variances in time σ_{Ta}^2 and σ_{Tn}^2 associated with the available wanted signal power W_a and the noise power W_n, respectively, and of ρ_T, the coefficient of correlation between them:

$$\sigma_T^2 = \sigma_{Ta}^2 + \sigma_{Tn}^2 - 2\rho_T \sigma_{Ta} \sigma_{Tn} \quad \text{dB}^2, \tag{1.14a}$$

where σ_{Ta} is defined by (1.9) and (1.10).

Similarly the path-to-path variance σ_L^2 of the available signal-to-noise ratio may be expressed as:

$$\sigma_L^2 = \sigma_{La}^2 + \sigma_{Ln}^2 - 2\rho_L \sigma_{La} \sigma_{Ln} \quad \text{dB}^2, \tag{1.14b}$$

where σ_{La} and σ_{Ln} are the location-to-location standard deviations of the available wanted signal power W_a and the noise power W_n, respectively, and ρ_L is the normalized coefficient of correlation between them. For service limited by a background of man-made noise, a study of available data indicates that $\sigma_{Tn} \simeq 4$ dB and $\sigma_{Ln} \simeq 4$ dB, while $\sigma_{La} \simeq$ 10 dB, as previously stated. The correlation coefficients ρ_T and ρ_L are usually assumed to be positive because when the received signal is high as a result of good propagation conditions the noise level is likely to be high also.

The estimated standard error of prediction σ_c may be defined in terms of the variance σ_{ca}^2 associated with the received power exceeded at least a fraction q_T of the time at a fraction q_L of the locations, the variance σ_{cn}^2 associated with the corresponding noise power, and the correlation ρ_c between them, with additional allowances for time and location variability:

$$\sigma_c^2 = \sigma_{ca}^2 + \sigma_{cn}^2 - 2\rho_c \, \sigma_{ca} \, \sigma_{cn} + 0.12 \, \sigma_T^2 \, z_o^2(q_T) + 4 \, z_o^2(q_L) + \sigma_x^2 \quad dB^2, \tag{1.15}$$

$$\text{where } \sigma_{ca} = 5 \, [1 + 0.6 \exp(- d_e/100)] \quad dB. \tag{1.16}$$

The estimated prediction error σ_c includes an allowance σ_x for errors in predicting the required signal-to-noise ratio R_r (g_r). The coefficients 0.12 and 4 in (1.15) and 5 and 0.6 in (1.16) represent the best empirical estimates presently available.

Estimates of $\sigma_{cn} \simeq 4$ dB, $\sigma_x \simeq 5$ dB are probably adequate, and the standard deviation σ_{ca} is computed using (1.16) as a function of the effective distance d_e, defined by (1.3). Little information is available regarding the correlation ρ_c between prediction errors for wanted signals and noise. The correlation is expected to be positive and less than unity.

The safety factors or biases shown in (1.13) may be allowed for separately in the order given. The bias Y_T, assuming $q_T = 0.99$, is the difference between an available signal-to-noise ratio $R(0.99, q_L)$ dB exceeded at least 99 percent of the time at any location and the median-time value $R(0.5, q_L)$. Then the bias Y_L, assuming $q_T = 0.99$ and $q_L = 0.1$, is the difference between the ratio $R(0.99, 0.1)$ exceeded at least 99 percent of the time at 10 percent of the locations and the median-time, median-location value $R(0.5, 0.5)$:

$$Y_T = - \sigma_T(\rho_T) \, z_o(q_T) \quad dB, \tag{1.17a}$$

$$Y_L = - \sigma_L(\rho_L) \, z_o(q_L) \quad dB, \tag{1.17b}$$

where ρ_T is the time correlation and ρ_L is the path-to-path correlation between wanted signals and noise.

The bias or safety margin,

$$Y_c = - \sigma_c(\rho_c) \, z_o(Q) \quad dB, \tag{1.17c}$$

required to achieve a given signal-to-noise ratio with a service probability Q depends in turn on the correlation ρ_c assumed between prediction errors for wanted signals and noise in (1.15).

The available wanted signal power is

$$W_a = W_t + G_p - L_b \quad \text{dBW}, \tag{1.18}$$

where W_t is the total radiated power in dBW, G_p is the path antenna gain in dB, and L_b is the basic transmission loss in dB. The path antenna gain may be expressed as

$$G_p = G_1 + G_2 - L_{gp} \quad \text{dB}, \tag{1.19}$$

where G_1 and G_2 are the free space antenna gains in decibels relative to an isotropic radiator. The loss in path antenna gain, L_{gp}, may be approximated as follows:

$$\text{For } G_1 + G_2 \leq 50 \text{ dB} \qquad L_{gp} \leq 1 \text{ dB}, \tag{1.20a}$$

For $50 \leq G_1 + G_2 \leq 100$ dB

$$L_{gp} \simeq 0.07 \exp[0.055\,(G_1 + G_2)] \quad \text{dB}. \tag{1.20b}$$

In most situations we assume that free-space gains are realized. Where the loss in gain may be appreciable, other methods for estimating G_p are described in the report by Rice et al. (1967). The approximation in (1.20b) tends to give substantially larger values of L_{gp} for large $G_1 + G_2$ than the methods reported by Rice et al. Experience with actual communication links at 4500 MHz using 10 m parabolic reflectors suggests that (1.20b) may provide more realistic estimates of link performance. However, more theoretical and experimental studies are needed to resolve this problem.

The available signal to noise ratio R is

$$R = W_a - W_n \quad \text{dB}, \tag{1.21}$$

where W_n is the total equivalent r-f noise power in dBW. It is often convenient to express W_a and W_n as power spectral densities, in dB(W/kHz), rather than expressing the total power in the r-f passband of the receiver in dBW.

The powers, or power spectral densities, W_a and W_n and the basic transmission loss L_b are assumed to be normally distributed in time and from path to path. The median values of W_a, W_n, and L_b may be denoted as W_o, W_{no}, and L_{bo}. Then the median available signal-to-noise ratio R_o is

$$R_o = W_o - W_{no} = W_t + G_p - L_{bo} - W_{no} \quad \text{dB}, \tag{1.22}$$

and the signal-to-noise ratio R may be expressed in terms of its median value R_o and the biases Y_T, Y_L and Y_c:

$$R = R_o + Y_T + Y_L + Y_c \quad \text{dB}, \tag{1.23}$$

where

$$Y_c = - \sigma_c \, z_o(Q) \quad \text{dB} .$$

The grade of service at any receiving location is satisfactory if the available wanted signal-to-noise ratio R exceeds the ratio R_r required for satisfactory service in the presence of fine-grained time and space variations of signals and noise, that is, if $R - R_r > 0$, where

$$R - R_r = R_o + Y_T + Y_L - R_r - \sigma_c \, z_o(Q) \quad \text{dB} . \tag{1.24}$$

Substituting (1.22) in (1.24) and defining

$$S_o = W_t + G_p - R_r - W_{no} + Y_T + Y_L \quad \mathrm{dB}, \qquad (1.25)$$

we can write

$$R - R_r = S_o - L_{bo} - \sigma_c \, z_o(Q) \, . \qquad (1.26)$$

The service probability Q is calculated by setting $R - R_r$ equal to zero and solving for Q in terms of the error function:

$$\sigma_c \, z_o \, (Q) = S_o - L_{bo} \quad \mathrm{dB}, \qquad (1.27a)$$

$$Q = 0.5 + 0.5 \, \mathrm{erf}[(S_o - L_{bo})/(\sigma_c \sqrt{2})] \, . \qquad (1.27b)$$

Or, for a given value of Q, such as $Q_o = 0.95$, (1.27a) may be solved for the value of S_o required to achieve this service probability.

1-4 List of Symbols and Abbreviations

In the following list the English alphabet precedes the Greek alphabet and lower case letters precede upper case letters. In general, upper case letters are used for quantities expressed in decibels.

a an effective earth's radius that allows for average refraction of radio rays near the surface of the earth, (1.2a).

A_{cr} a predicted reference value of attenuation below free space, expressed in decibels, (1.1).

$A(q_L)$ attenuation below free space not exceeded for at least a fraction q_L of all randomly chosen paths for which other parameters, such as frequency, antenna heights, and path length, are fixed.

$A(0.5)$ long-term median value of attenuation below free space for a specified period of time, climatic region, etc. (1.1).

$A(0.1)$, $A(0.9)$ attenuation below free space not exceeded for 10 percent and 90 percent of the time, respectively, (1.4).

d great circle distance in kilometers

dB abbreviation for decibel.

d_e an effective distance in kilometers, defined by (1.3).

d_{Lo} the sum of the smooth-earth horizon distances for an effective earth's radius $a = 9000$ km, (1.2).

d_{s1} the distance between horizons for which diffraction and forward scatter losses are approximately equal over a smooth earth, (1.2a).

erf x the error function of x, (1.6), defined as $H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha$.

$\text{erf}^{-1} x$ the inverse error function of x, (1.6).

f radio wave frequency, expressed in megahertz, (1.2).

$g(q, f)$ frequency factors used to adjust the predicted variability in time at 100 MHz for use at other frequencies, (1.5) and figure 1.4.

$g(0.1, f)$, $g(0.9, f)$ frequency factors used to adjust $Y_o(0.1)$ and $Y_o(0.9)$, respectively, (1.5) and figure 1.4.

G_p path antenna gain in decibels above the unit gain of an isotropic radiator, (1.19).

G_1, G_2 free-space antenna gains, in decibels relative to an isotropic radiator, of the transmitting and receiving antennas, respectively, (1.19).

h_{e1}, h_{e2} effective heights in meters of the transmitting and receiving antennas, respectively, (1.2b).

L_b basic transmission loss in decibels, (1.18).

L_{bo}, $L_b(0.5)$ long-term median value of basic transmission loss in decibels, (1.4) and (1.22).

$L_b(0.1)$, $L_b(0.9)$ basic transmission loss not exceeded for fractions 0.1 and 0.9 of hourly medians, (1.4).

L_{gp} loss in path antenna gain, (1.19) and (1.20).

N_s atmospheric refractivity at the surface of the earth.

q a symbol that represents any fraction between 0 and 1, (1.6).

q_L any desired fraction of all randomly chosen paths for which such parameters as frequency, antenna heights, and path length are fixed, (1.12).

q_T any desired fraction of time, (1.7).

Q a symbol that represents service probability, (1.17) and (1.27).

R the available signal-to-noise ratio expressed in decibels, (1.21).

R_r the signal-to-noise ratio required to provide satisfactory service in the presence of fine-grained time and space variations of signals and noise, (1.24).

R_o median value of the available signal-to-noise ratio expressed in decibels, (1.22).

$R(q_T, q_L, Q)$ the signal-to-noise ratio available for at least a desired fraction q_T of the time at a desired fraction q_L of locations, with a probability Q, (1.13).

$R(0.5, q_L)$ the median time value of the available signal-to-noise ratio at any location.

$R(0.99, q_L)$ the available signal-to-noise ratio exceeded at least 99 percent of the time at any location.

$R(0.99, 0.1)$ the available signal-to-noise ratio exceeded at least 99 percent of the time at 10 percent of the locations.

S_o a term defined by (1.25).

$V(0.5)$ the difference in decibels between a computed reference value A_{cr} and the median attenuation $A(0.5)$ expected for a specified climate, season, time of day, or desired group of paths, (1.1).

$V(0.5, d_e)$ the adjustment factor $V(0.5)$ as a function of the effective distance d_e, figure 1.1.

W_a radio frequency signal power or power spectral density that would be available from an equivalent loss-free receiving antenna, (1.18).

W_n the total equivalent r-f noise power or power spectral density at the terminals of a loss-free receiving antenna, including both internal system noise referred to these terminals and externally generated unwanted signals whose effect can be represented by an equivalent noise power, (1.21).

W_t the total power radiated from a transmitting antenna in a given band of radio frequencies, (1.18).

W_{no} the median value of W_n, (1.22).

W_o the median value of W_a, (1.22).

x a parameter defined as $x = d_e/100$ used in (1.9) and (1.10).

Y_c an allowance in decibels for prediction error, (1.13) and (1.17).

Y_L an allowance in decibels for random variations in transmission loss from location to location, (1.13) and (1.17).

Y_T an allowance in decibels for long-term variability in time, (1.13) and (1.17).

$Y(q_L)$ the location variability for any fraction q_L of all randomly chosen paths for which other parameters, such as frequency, path length and antenna heights, are fixed, (1.12a).

$Y(q_T)$ the variability in time about the long-term median exceeded at least a fraction q_T of the time, (1.7).

Y(0.1), Y(0.01) the difference between the long-term median at-
Y(0.9), Y(0.99) tenuation and that not exceeded for fractions 0.1, 0.01, 0.9, and 0.99 of the time, respectively, (1.5) and following discussion.

$Y_o(0.1)$, $Y_o(0.9)$ values of Y(0.1) and Y(0.9), respectively, expected at 100 MHz in a continental temperate climate, plotted versus an effective distance d_e, figure 1.4 and (1.4).

z_o a standard normal deviate, (1.6).

$z_o(q)$ a standard normal deviate where the symbol q represents any fraction between zero and unity, (1.6).

$z_o(q_L)$ a standard normal deviate where the symbol q_L represents any desired fraction of all randomly chosen paths for which such parameters as frequency, distance, and antenna heights are fixed, (1.12) and (1.17).

$z_o(q_T)$ a standard normal deviate where the symbol q_T represents any desired fraction of time, (1.7) and (1.17).

$z_o(Q)$ a standard normal deviate where the symbol Q represents the desired service probability, (1.17).

θ_{s1} the angular distance at which scatter and diffraction transmission losses are approximately equal over a smooth earth of effective radius $a = 9000$ km, (1.2).

ρ_c the cross-correlation coefficient between the variations of the received signal power exceeded at least a fraction q_T of the time at a fraction q_L of locations and variations of the corresponding noise power, (1.15).

ρ_L the coefficient of correlation between the location-to-location variations of the available wanted signal power and the noise power, (1.14b).

ρ_T the coefficient of correlation in time between the available wanted signal power and the noise power, (1.14a).

σ_c estimated prediction error, defined by (1.15).

σ_{ca}, σ_{cn} the standard deviations of the received wanted signal power and noise power, respectively, exceeded at least a fraction q_T of the time at a fraction q_L of the locations, (1.15); σ_{ca} is defined by (1.16), $\sigma_{cn} \simeq 4$ dB.

$\sigma_c(\rho_c)$ a value of σ_c computed using a specified value of ρ_c, (1.17c).

σ_L, σ_L^2 the standard deviation and variance, respectively, from location to location of the available signal-to-noise ratio, (1.14b).

σ_{La}, σ_{Ln} the location-to-location standard deviations of the available wanted signal power and noise power respectively, (1.14b); available data indicate $\sigma_{La} \simeq 10$ dB and $\sigma_{Ln} \simeq$ 4 dB.

$\sigma_L(\rho_L)$ the location-to-location standard deviation of the available signal-to-noise ratio assuming a specified value of ρ_L, (1.17b).

σ_T, σ_T^2 the standard deviation and variance, respectively, of the available wanted signal-to-noise ratio in time, (1.14a).

σ_{Ta}, σ_{Tn} the standard deviation in time of the available wanted signal power and noise power, respectively, (1.14a); σ_{Ta} is computed using (1.9) and (1.10); available data indicate $\sigma_{Tn} \simeq 4$ dB.

$\sigma_{Ta}(0.1), \sigma_{Ta}(0.9)$ standard deviations representing the bi-normal time distribution of attenuation relative to free space, illustrated in figure 1.6, (1.9), and (1.10).

$\sigma_T(\rho_T)$ the standard deviation in time of the available signal-to-noise ratio assuming a specified value of ρ_T, (1.17a).

σ_x a term used in estimating prediction error that allows for errors in predicting the required signal-to-noise ratio, $\sigma_x \simeq$ 5 dB, (1.15).