ANNEX 3

EQUATIONS AND METHODS FOR COMPUTING THE REFERENCE ATTENUATION A_{cr}

The minimum input parameters required to compute the reference attenuation relative to free space are the radio frequency f in megahertz, the path distance d in kilometers, and antenna heights above ground h_{g1} and h_{g2} in meters. Estimates of surface refractivity N_s, terrain irregularity Δh, and the ground constants σ and ϵ may be selected as described in section 2, when measured values are not available.

When detailed profiles of individual paths are not available, equations (3) through (6), section 2, are used to estimate median values of the additional parameters $h_{e1,2}$, $d_{L1,2}$ and $\theta_{e1,2}$.

When detailed profile information is available for a specific path, the actual horizon distances d_{L1} and d_{L2}, horizon elevation angles θ_{e1} and θ_{e2}, and effective antenna heights h_{e1} and h_{e2} above the dominant reflecting plane are used in computing A_{cr}. The location of a horizon obstacle may be determined by testing all possible horizon elevations and selecting the one for which the horizon elevation angle θ_{e1} or θ_{e2} is a maximum:

$$\theta_{e1} = \frac{0.001\,(h_{L1} - h_{s1})}{d_{L1}} - \frac{d_{L1}}{2a} \text{ radians} \tag{3.1a}$$

$$\theta_{e2} = \frac{0.001\,(h_{L2} - h_{s2})}{d_{L2}} - \frac{d_{L2}}{2a} \text{ radians}, \tag{3.1b}$$

where $h_{s1,2}$ are the antenna heights above sea level in meters, a is the effective earth's radius in kilometers, $h_{L1,2}$ are the heights in meters above sea level of the horizon obstacles, and $d_{L1,2}$ are the great circle distances in kilometers from each antenna to its horizon. The prediction method is limited to values of $\theta_{e1,2} \leq 0.2$ radians. For larger

elevation angles the assumption of an effective earth's radius a, based on the surface refractivity N_s, is not applicable.

An alternative procedure is first to compute a least-squares fit of a straight line to terrain elevations above sea level. The heights $h_{s1,2}$ and $h_{L1,2}$ are then defined relative to this curve fit, rather than relative to sea level. This amounts to replacing sea level by an arc of radius "a" that is a least-squares fit to the great circle path terrain profile.

For line-of-sight paths, the effective antenna heights $h_{e1,2}$ are defined as the height of each antenna above the dominant reflecting plane between the antennas, or the structural height, whichever is greater. The effective heights may be calculated as heights above a smooth curve fitted to great circle profile terrain elevations that are intervisible to both antennas. A straight line is first fitted by least squares to equidistant heights h_i, and an amount $d_i^2/2a$ is then subtracted at each distance d_i to allow for the path curvature $1/a$. When terrain is so irregular that it cannot be reasonably well approximated by one or more such reflecting planes, the effective heights are estimated using (4a) or (4b) in the main body of this report.

The total input required to compute A_{cr} is then: f, d, h_{g1}, h_{g2}, polarization, and actual or estimated values of N_s, Δh, σ, and ϵ. When available for specific paths, the parameters $\Delta h(d)$, d_{L1}, d_{L2}, θ_{e1}, θ_{e2}, h_{e1}, and h_{e2} are also included as input.

3-1. Two-Ray Optics Formulas for Computing A_o and A_1

At distances d_o and d_1, which are well within radio line of sight, but are so chosen that the difference between the direct and ground-reflected rays never exceeds one fourth of the wavelength, the following formula is used to compute the attenuation relative to free space:

$$A = -10 \log_{10} \left[1 + R_e^2 - 2 R_e \cos\left(\frac{2\pi\Delta r}{\lambda} - c\right)\right] + G_p - 10 \log_{10} (g_{o1}\ g_{o2})\ \text{dB}. \tag{3.2}$$

Here g_{o1} and g_{o2} represent the directive gain for each antenna in the direction of the other, while $2\pi\Delta r/\lambda$ is the path length difference between direct and ground-reflected rays, expressed in electrical radians and in degrees as

$$\frac{2\pi\Delta r}{\lambda} = 4.1917 \times 10^{-5}\ f\ h_{e1}\ h_{e2}/d \ \text{radians}, \tag{3.3a}$$

$$= 2.4017 \times 10^{-3}\ f\ h_{e1}\ h_{e2}/d \ \text{degrees}, \tag{3.3b}$$

with f in MHz, $h_{e1,2}$ in meters, and d in kilometers. R_e is the magnitude of an effective reflection coefficient and c is its phase relative to π radians. Assuming matched polarizations, the median path antenna gain may be approximated as

$$G_p = 10 \log_{10} (g_{o1}\ g_{o2})\ \text{dB}, \tag{3.4}$$

and these terms in (3.2) then cancel each other.

No divergence factor is included in the definition of R_e since its use will not add significantly to the accuracy of the method described for irregular terrain. (See the smooth-earth formulas for **D**, h_{te}, h_{re} in Rice et al. (1967).

Let g_{r1} and g_{r2} represent the directive antenna gains in the direction of a point of ground reflection. Then,

$$\hat{R}_e = R_{h,v} \left(\frac{g_{r1}\ g_{r2}}{g_{o1}\ g_{o2}}\right)^{1/2} \exp\left(-\frac{2\pi\sigma_h \sin\psi}{\lambda}\right). \tag{3.5}$$

Usually, $g_{r1} = g_{o1}$ and $g_{r2} = g_{o2}$, unless beams are very narrow or are directed away from the earth's surface to minimize reflection from the surface. $R_{h,v}$ is the magnitude of the theoretical plane earth reflection coefficient, the subscripts h and v referring to horizontal and vertical polarization respectively, and the factor σ_h in the exponent is the rms deviation of terrain and terrain clutter within the limits of the first Fresnel zone in the dominant reflecting plane. For this report the factor σ_h and the grazing angle ψ are defined as follows:

$$\sigma_h(d) = 0.78\,\Delta h(d)\,\exp\left\{-0.5\left[\Delta h(d)\right]^{1/4}\right\}\text{ m, for } \Delta h(d) > 4\text{ m}, \quad (3.6a)$$

$$\sigma_h(d) = 0.39\,\Delta h(d), \text{ for } \Delta h(d) \le 4\text{ m}, \quad (3.6b)$$

$$\psi = \tan^{-1}\left[(h_{e1} + h_{e2})/(1000\,d)\right]. \quad (3.7)$$

If $$R_{h,v}\exp\left[-(2\pi\,\sigma_h\sin\psi)/\lambda\right] > 0.5 \text{ and } > \sqrt{\sin\psi}, \quad (3.8a)$$

$$R_e = \hat{R}_e.$$

Otherwise, $$R_e = \left[\frac{g_{r1}\,g_{r2}}{g_{o1}\,g_{o2}}\sin\psi\right]^{1/2}. \quad (3.8b)$$

The theoretical plane earth reflection coefficients R_h, R_v and the phase angle c are functions of the radio frequency f, grazing angle ψ, and the ground constants σ and ϵ. Their magnitudes may be read from figures III. 1 through III. 8, volume 2 of the report by Rice et al. (1967), or computed as follows:

$$x = 18000\,\sigma/f, \quad q = x/(2p) \quad (3.9a)$$

$$2\,p^2 = \left[(\epsilon - \cos^2\psi)^2 + x^2\right]^{1/2} + (\epsilon - \cos^2\psi) \quad (3.9b)$$

$$b_v = \frac{\epsilon^2 + x^2}{p^2 + q^2}, \quad b_h = \frac{1}{p^2 + q^2} \text{ radians} \tag{3.10}$$

$$m_v = \frac{2(p\epsilon + qx)}{p^2 + q^2}, \quad m_h = \frac{2p}{p^2 + q^2}. \tag{3.11}$$

Then

$$R_v^2 = [1 + b_v \sin^2\psi - m_v \sin\psi] / [1 + b_v \sin^2\psi + m_v \sin\psi] \tag{3.12a}$$

$$R_h^2 = [1 + b_h \sin^2\psi - m_h \sin\psi] / [1 + b_h \sin^2\psi + m_h \sin\psi]. \tag{3.12b}$$

The phase angle c in (3.2) is defined below for both horizontal and vertical polarization, c_h and c_v. The angle c_h defined as

$$c_h = \tan^{-1}\left(\frac{q}{p + \sin\psi}\right) - \tan^{-1}\left(\frac{q}{p - \sin\psi}\right) \text{ radians} \tag{3.13}$$

is always negative and ranges in value from $0 \geq c_h \leq -0.1$ radians. The angle c_v changes suddenly from near zero to $\pi/2$ at the pseudo-Brewster angle, $\sin^{-1}\sqrt{1/b_v}$. To define c_v, let

$$y_1 = (x \sin\psi + q)/(\epsilon \sin\psi + p), \quad y_2 = (x \sin\psi - q)/(\epsilon \sin\psi - p). \tag{3.14}$$

If $\epsilon \sin\psi \geq p$:

$$c_v = \tan^{-1} y_1 - \tan^{-1} y_2 + \pi \text{ radians}. \tag{3.15a}$$

If $\epsilon \sin\psi < p$ and $p \sin\psi > 0.5$:

$$c_v = \tan^{-1} y_1 + \tan^{-1} y_2 \text{ radians}. \tag{3.15b}$$

If $\epsilon \sin\psi \leq p$ and $p \sin\psi \leq 0.5$:

$$c_v = \tan^{-1} y_1 - \tan^{-1} y_2 \text{ radians}. \tag{3.15c}$$

In the above formulas, $\tan^{-1} y$ is in the first quadrant if y is positive and in the fourth quadrant if y is negative.

The two-ray optics formulas (3.2) to (3.15) are used to compute values of attenuation A_{ot} and A_{1t} at distances d_o and d_1, respectively.

For $A_{ed} \geq 0$, define

$$d_o = 4 \times 10^{-5} h_{e1} h_{e2} f \text{ km, or } 0.5\, d_L \text{, whichever is smaller,} \quad (3.16a)$$

For $A_{ed} < 0$, define

$$d_{o1} = -A_{ed}/m_d \text{ km, or } (d_L - 2) \text{ km, whichever is smaller,} \quad (3.16b)$$

$$d_o = \begin{cases} d_{o1} & \text{for } d_{o1} \geq 0.5\, d_L \\ 0.5\, d_L & \text{otherwise} \end{cases} \quad (3.16c)$$

$$d_1 = d_o + 0.25\,(d_L - d_o) \text{ km.} \quad (3.16d)$$

In (3.16) the radio frequency f is in MHz, the effective antenna heights $h_{e1,2}$ are in meters, d_L is the sum of the horizon distances in kilometers and the attenuation A_{ed} and slope m_d are defined in the next subsection (3.38).

In addition to the two-ray-theory estimates A_{ot} and A_{1t} of attenuation at the distances d_o and d_1, estimates of diffraction attenuation A_{od}, A_{1d}, and A_{Ls} are also computed at d_o, d_1, and d_{Ls}:

$$A_{od} = A_{ed} + m_d d_o \quad (3.17a)$$

$$A_{1d} = A_{ed} + m_d d_1 \quad (3.17b)$$

$$A_{Ls} = A_{ed} + m_d d_{Ls}, \quad (3.17c)$$

where A_{ed} and m_d are defined in the next subsection by (3.38).

The estimates of attenuation A_o and A_1 at the distances d_o and d_1 are then computed as weighted averages of the two-ray theory and the diffraction estimates

$$A_o = w_o A_{ot} + (1 - w_o) A_{od} \quad \text{or} \quad A_{od}, \text{ whichever is smaller,} \tag{3.18a}$$

$$A_1 = w_o A_{1t} + (1 - w_o) A_{1d} \quad \text{or} \quad A_{1d}, \text{ whichever is smaller,} \tag{3.18b}$$

$$w_o = (1 + f \Delta h \, 10^{-4})^{-1} . \tag{3.18c}$$

For distances less than the smooth-earth horizon distance d_{Ls}, the calculated reference value A_{cr} is defined by a smooth curve fitted to the three values of attenuation below free space, A_o, A_1, and A_{Ls}, at the distances d_o, d_1, and d_{Ls}.

For $0 < d \leq d_{Ls}$:

$$A_{cr} = A_o + k_1 (d - d_o) + k_2 \log_{10} (d/d_o) \quad \text{dB} . \tag{3.19}$$

The constants k_1 and k_2 in (3.19) are evaluated as follows. First estimates $\hat{k}_1$, $\hat{k}_2$ of the slopes k_1, k_2 in (3.19) are computed as

$$\hat{k}_2 = \frac{(A_{Ls} - A_o)(d_1 - d_o) - (A_1 - A_o)(d_{Ls} - d_o)}{(d_1 - d_o) \log_{10} (d_{Ls}/d_o) - (d_{Ls} - d_o) \log_{10} (d_1/d_o)} \quad \text{dB,}$$

or 0, whichever is larger algebraically, (3.20)

$$\hat{k}_1 = [(A_{Ls} - A_o) - \hat{k}_2 \log_{10} (d_{Ls}/d_o)]/(d_{Ls} - d_o) \quad \text{dB/km} . \tag{3.21}$$

If $\hat{k}_1 < 0$ set $k_1 = 0$ and

$$k_2 = (A_{Ls} - A_o)/\log_{10}(d_{Ls}/d_o). \tag{3.22}$$

If the reference attenuation A_{cr} computed from (3.19) is less than zero at any distance $0 \le d \le d_{Ls}$, let $A_{cr} = 0$ for that distance.

3-2. Formulas for Computing Diffraction Attenuation A_d

In the far diffraction region, the attenuation A_d is computed as a weighted average of two estimates, A_r for smooth terrain and A_k for highly irregular terrain. In general, A_d is defined by (13) as

$$A_d = (1 - w)\, A_k + w\, A_r \quad \text{dB},$$

where the empirically determined weighting factor w is defined as

$$w = \left\{1 + 0.1\left[\frac{\Delta h(d)}{\lambda}\left(\sqrt{\frac{h_{e1}\, h_{e2} + C}{h_{g1}\, h_{g2} + C}} + \frac{a\,\theta_e + d_L}{d}\right)\right]^{\frac{1}{2}}\right\}^{-1}, \tag{3.23}$$

with $\frac{\Delta h(d)}{\lambda} \le 1000$. In the accompanying computer program and output $C = 0$. For low antennas with known path parameters $C \simeq 10$.

In (3.23) the radio wavelength λ, terrain irregularity $\Delta h(d)$, and effective and structural antenna heights $h_{e1,2}$, $h_{g1,2}$ are in meters; the effective earth's radius a, the horizon distance d_L, and the distance d, at which A_k and A_r are computed are in kilometers; and the sum of the elevation angles θ_e is in radians. For very smooth terrain, the weight $w \simeq 1$ and $A_d \simeq A_r$, and for highly irregular terrain, the

weight $w \simeq 0$ and $A_d \simeq A_k$. The prediction approaches A_k when either the frequency or the terrain irregularity are very large; therefore, a limit is placed on this ratio.

The diffraction attenuation is computed at distances d_3 and d_4, chosen well beyond the horizon:

$$d_3 = d_L + 0.5\,(a^2/f)^{\frac{1}{3}} \text{ km}, \quad d_4 = d_3 + (a^2/f)^{\frac{1}{3}} \text{ km}. \tag{3.24}$$

If $d_3 < d_{Ls}$ set $d_3 = d_{Ls}$.

At these distances, d_3 and d_4, the attenuations A_3 and A_4 are computed using the following formulas, substituting d_3 and d_4 for d in (3.23) to obtain w_3 and w_4:

$$A_3 = (1 - w_3)\,A_{k3} + w_3\,A_{r3} \tag{3.25a}$$

$$A_4 = (1 - w_4)\,A_{k4} + w_3\,A_{r4} \tag{3.25b}$$

$$\theta_3 = \theta_e + d_3/a, \quad \theta_4 = \theta_e + d_4/a. \tag{3.25c}$$

The estimates A_{k3} and A_{k4} for highly irregular terrain are computed as though the horizon obstacles were sharp ridges or hills, and the attenuation is computed for a double knife-edge path.

$$v_{1.3} = 1.2915\,\theta_3 \sqrt{f\,d_{L1}\,(d_3 - d_L)/(d_3 - d_{L2})} \tag{3.26a}$$

$$v_{2.3} = 1.2915\,\theta_3 \sqrt{f\,d_{L2}\,(d_3 - d_L)/(d_3 - d_{L1})} \tag{3.26b}$$

$$v_{1.4} = 1.2915\,\theta_4 \sqrt{f\,d_{L1}\,(d_4 - d_L)/(d_4 - d_{L2})} \tag{3.26c}$$

$$v_{2.4} = 1.2915\,\theta_4 \sqrt{f\,d_{L2}\,(d_4 - d_L)/(d_4 - d_{L1})} \tag{3.26d}$$

$$\begin{cases} A(v) = 6.02 + 9.11\,v - 1.27\,v^2 & \text{for} \quad 0 \le v \le 2.4 \qquad (3.27a) \\ A(v) = 12.953 + 20\log_{10} v & \text{for} \quad v > 2.4 \qquad (3.27b) \end{cases}$$

$$A_{k3} = A(v_{1.3}) + A(v_{2.3}), \quad A_{k4} = A(v_{1.4}) + A(v_{2.4}). \qquad (3.27c)$$

The rounded earth attenuations A_{r3} and A_{r4} are defined as

$$A_{r3,4} = G(x_{3,4}) - F(x_1) - F(x_2) - 20 \quad \text{dB}, \qquad (3.28)$$

where the functions $F(x_{1,2})$ and $G(x_{3,4})$ depend on the radio frequency, polarization, and ground constants σ and ϵ, the distances $d_{L1,2}$, $d_{3,4}$, and the effective earth's radii $a_{1,2}$ for the terrain between the antennas and their horizons and $a_{3,4}$ for the terrain between horizons. The latter are defined as

$$a_1 = d_{L1}^2/(0.002\,h_{e1}) \ \text{km}, \quad a_2 = d_{L2}^2/(0.002\,h_{e2}) \ \text{km} \qquad (3.29a)$$

$$a_3 = (d_3 - d_L)/\theta_3 \ \text{km}, \quad a_4 = (d_4 - d_L)/\theta_4 \ \text{km}. \qquad (3.29b)$$

Then the distances $x_{1,2,3,4}$ are defined as

$$x_1 = B_1\,a_1^{-\frac{2}{3}}\,d_{L1} \ \text{km}, \quad x_2 = B_2\,a_2^{-\frac{2}{3}}\,d_{L2} \ \text{km} \qquad (3.30)$$

$$x_3 = B_3\,a_3^{-\frac{2}{3}}\,(d_3 - d_L) + x_1 + x_2 \ \text{km} \qquad (3.31a)$$

$$x_4 = B_4\,a_4^{-\frac{2}{3}}\,(d_4 - d_L) + x_1 + x_2 \ \text{km}, \qquad (3.31b)$$

where the parameter $B_{1,2,3,4}$ is defined for both vertical and horizontal polarization as

$$B_{1,2,3,4} = 416.4\, f^{\frac{1}{3}} \left[1.607 - K_{h,v}(a_{1,2,3,4})\right]. \tag{3.32}$$

The parameters $K_h(a)$ for horizontal and $K_v(a)$ for vertical polarization are defined as

$$K_h(a) = 0.36278\,(a\,f)^{-\frac{1}{3}}\left[(\epsilon - 1)^2 + x^2\right]^{-\frac{1}{4}} \tag{3.33a}$$

$$K_v(a) = K_h(a)\,\left[\epsilon^2 + x^2\right]^{\frac{1}{2}}, \tag{3.33b}$$

where x is defined by (3.9a) as $x = 18000\,\sigma/f$, and the ground constants σ and ϵ are included in the input.

The functions $F(x_1)$ and $F(x_2)$ may be read from figures 8.5 or 8.6 of the report by Rice et al. (1967) or may be computed using the following formulas.

1. For $0 < x_{1,2} \le 200$ <u>and</u> $0 \le K_{h,v}(a_{1,2}) \le 10^{-5}$:

$$F(x_{1,2}) = 40 \log_{10} x_{1,2} - 117, \quad \underline{\text{or}} \tag{3.34a}$$

$$F(x_{1,2}) = -117 \text{ dB}, \tag{3.34b}$$

whichever yields the smaller absolute value.

2. For $0 < x_{1,2} \le 200$ <u>and</u> $10^{-5} \le K_{h,v}(a_{1,2}) < 1$,

and $x \ge -450/\left\{\log_{10}\left[K_{h,v}(a_{1,2})\right]\right\}^3$,

$F(x_{1,2})$ is calculated using (3.34a). Otherwise,

$$F(x_{1,2}) = 20 \log_{10} K_{h,v}(a_{1,2}) + 2.5 \times 10^{-5}\, x_{1,2}^2 / K_{h,v}(a_{1,2}) - 15 \text{ dB}. \tag{3.34c}$$

Note that when $K_{h,v}(a_{1,2}) > 0.1$ no test on x is required and (3.34c) is always used.

3. For $200 < x_{1,2} \leq 2000$, define

$$w_{1,2} = 0.0134\, x_{1,2} \exp(-0.005\, x_{1,2}). \tag{3.35a}$$

Then

$$F(x_{1,2}) = w_{1,2}(40 \log_{10} x_{1,2} - 117) + (1 - w_{1,2})(0.05751\, x_{1,2} - 10 \log_{10} x_{1,2}) \text{ dB}. \tag{3.35b}$$

4. For $x_{1,2} > 2000$,

$$F(x_{1,2}) = 0.05751\, x_{1,2} - 10 \log_{10} x_{1,2} \text{ dB}. \tag{3.36}$$

The parameter $G(x_{3,4})$ is defined as

$$G(x_{3,4}) = 0.05751\, x_{3,4} - 10 \log_{10} x_{3,4} \text{ dB}. \tag{3.37}$$

Values of $A_{k3,4}$ as given by (3.27) and of $A_{r3,4}$ as given by (3.28) are substituted in (3.24a, b) to obtain A_3 and A_4. These computed values of A_3 at d_3, and A_4 at d_4 are used to compute the slope m_d and intercept A_{ed} that define a straight line. The reference attenuation A_{cr} at any distance $d_{Ls} \leq d \leq d_x$ is then

$$A_{cr} = A_d = A_{ed} + m_d\, d \text{ dB}, \tag{3.38a}$$

$$A_{ed} = A_{fo} + A_4 - m_d\, d_4, \text{ and } m_d = (A_4 - A_3)/(d_4 - d_3), \tag{3.38b}$$

where A_{fo} is a "clutter factor", defined as

$$A_{fo} = 5 \log_{10} \left[1 + h_{g1}\, h_{g2}\, f \sigma_h (d_{Ls})\, 10^{-5}\right] \text{ dB},$$

$$\text{or 15 dB, whichever is smaller,} \tag{3.38c}$$

and the terrain roughness term $\sigma_h (d_{Ls})$ is obtained by substituting d_{Ls} for d in (3.6) and (3).

3-3. Formulas for Computing Scatter Attenuation A_s

At distances d_5 and d_6, defined below, the following formulas are used to obtain initial estimates $\hat{A}_5$ and $\hat{A}_6$ of forward scatter attenuation relative to free space:

$$d_5 = d_L + 200 \text{ km}, \quad d_6 = d_L + 400 \text{ km} \tag{3.39}$$

$$\theta_5 = \theta_e + d_5/a, \quad \theta_6 = \theta_e + d_6/a \text{ radians} \tag{3.40}$$

$$\left.\begin{aligned} H_{5,6} &= \left(\frac{1}{h_{e1}} + \frac{1}{h_{e2}}\right) / (\theta_{5,6}\, f \left| 0.007 - 0.058\, \theta_{5,6} \right|) \text{ dB} \\ &\underline{\text{or}} \text{ 15 dB, whichever is smaller.} \end{aligned}\right\} \tag{3.41}$$

$$S_5 = H_5 + 10 \log_{10} (f\, \theta_5^4) - 0.1\, (N_s - 301) \exp(-\theta_5\, d_5/40) \quad \text{dB} \tag{3.42a}$$

$$S_6 = H_6 + 10 \log_{10} (f\, \theta_6^4) - 0.1\, (N_s - 301) \exp(-\theta_6\, d_6/40) \quad \text{dB}. \tag{3.42b}$$

Substitute d_5, θ_5, S_5, and d_6, θ_6, S_6 in the following expressions to obtain $\hat{A}_5 = \hat{A}_s$ at d_5, and $\hat{A}_6 = \hat{A}_s$ at d_6.

For $\theta d \leq 10$:

$$\hat{A}_s = S + 103.4 + 0.332\, \theta d - 10 \log_{10} (\theta d) \quad \text{dB}. \tag{3.43a}$$

For $10 \leq \theta d \leq 70$:

$$\hat{A}_s = S + 97.1 + 0.212\, \theta d - 2.5 \log_{10} (\theta d) \quad \text{dB}. \tag{3.43b}$$

For $\theta d \geq 70$:

$$\hat{A}_s = S + 86.8 + 0.157\,\theta d + 5 \log_{10} (\theta d) \text{ dB}. \tag{3.43c}$$

3-3.1. For $H_5 \leq 10$ dB

When the frequency gain function, H_5, computed at d_5 is less than or equal to 10 dB, formulas (3.39) through (3.43) give the actual predicted scatter loss at the distances d_5 and d_6, and

$$A_5 = \hat{A}_5 \text{ dB}, \quad \text{and} \quad A_6 = \hat{A}_6 \text{ dB}.$$

The scatter attenuation A_s, at any distance d, is then given by (17) and (18) as

$$A_s = A_{es} + m_s\, d \text{ dB},$$

where

$$A_{es} = A_5 - m_s\, d_5, \quad \text{and} \quad m_s = (A_6 - A_5)/(d_6 - d_5).$$

The distance d_x, where diffraction and scatter attenuations are equal, is

$$d_x = (A_{es} - A_{ed})/(m_d - m_s) \text{ km}, \tag{3.44a}$$

$$\underline{\text{or}}\ d_L + 0.25\,(a^2/f)^{\frac{1}{3}} \log_{10} f, \text{ whichever is greater}, \tag{3.44b}$$

where A_{ed} and m_d are defined by (15), and (3.38b). When (3.44b) is used to define d_x, redefine A_{es} as

$$A_{es} = A_{ed} + (m_d - m_s)\, d_x \tag{3.44c}$$

The reference attenuation A_{cr} for transhorizon paths is then

$$\left.\begin{array}{l} \text{for } d_{Ls} \leq d \leq d_x,\ A_{cr} = A_d = A_{ed} + m_d\, d \text{ dB} \\ \text{for } d_x \leq d \leq 1500 \text{ km},\ A_{cr} = A_s = A_{es} + m_s\, d \text{ dB} \end{array}\right\}. \tag{3.45}$$

3-3.2. For $H_5 > 10$ dB and ≤ 15 dB

When the frequency gain function H_5 computed at d_5 is greater than 10 dB, the estimates $\hat{A}_5$ and $\hat{A}_6$ are modified by comparison with the scatter loss expected over a smooth earth, $\Delta h = 0$. To determine the distance d_{xo}, where diffraction and scatter losses would be equal over a smooth earth, the diffraction loss, with $\Delta h = 0$, is also computed.

For the special case, $\Delta h = 0$, let $A_{do} = A_{ed}$, $m_{do} = m_d$, and $\hat{A}_{5o}$ be the preliminary estimate of scatter attenuation at d_5. Assume that the slope m_s is not changed. Then one estimate of d_{xo} is obtained by substituting in (3.44):

$$d_{x1} = \hat{d}_{xo} = (\hat{A}_{5o} - m_s d_5 - A_{do})/(m_{do} - m_s) \text{ km}. \tag{3.46a}$$

When H_5 is large, a good estimate of d_{xo} is

$$d_{x2} = \hat{d}_{xo} = d_L + 0.25\,(a^2/f)^{\frac{1}{3}} \log_{10} f \text{ km}. \tag{3.46b}$$

For smaller values of H_5, d_{x1} is the better estimate of d_{xo}, and for larger values d_{x2} is the better estimate. Therefore, a weighted function is used to compute d_{xo} as follows:

$$d_{xo} = d_{x1}\,(3 - 0.2\,H_5) + d_{x2}\,(0.2\,H_5 - 2) \text{ km}. \tag{3.46c}$$

For $\Delta h = 0$, scatter and diffraction losses are equal at d_{xo}. The diffraction attenuation A_{xo} at d_{xo} is

$$A_{xo} = A_{do} + m_{do}\, d_{xo} \text{ dB}. \tag{3.47}$$

It is assumed that, in general, the forward scatter attenuation A_{sx} at $d = d_{xo}$ for any value of Δh is

$$A_{sx} = A_{xo} + (\hat{A}_5 - \hat{A}_{5o}) \text{ dB}. \tag{3.48}$$

The intercept at $d = 0$ would then be

$$A_{es} = A_{sx} - m_s d_{xo} \text{ dB}. \tag{3.49}$$

Substituting this value of A_{es} in (3.44a or b) determines the distance d_x, and for any distance $d \geq d_x$,

$$A_{cr} = A_s = A_{es} + m_s d \text{ dB}.$$