alent rate of B cps per recirculation period, the minimum time to accumulate and analyze MT_s seconds of data is

$$T_{AA} \geq M\left(T_s + \frac{N}{bp_0}\right). \qquad (4)$$

If we substitute the typical values used above, put $b=0.01$, and allow a modest time for system logic and control functions, we see that the values of T_{AA} for an over-all statistical error of 5 per cent are actually longer than the time for conventional swept analysis, *for all* vibration signals with frequency contents greater than 2 cps.

The speed-up device does not seem too effective for the usual types of random vibration and acoustic data. (Nonrandom data is not under discussion.) It would seem to offer some speed advantage, only if the random signals of interest had extremely low frequencies, or if the analyzing bandwidth B was required to be exceptionally narrow ($B \ll 1$ cps). Even so, a considerable amount of auxiliary equipment is needed (*e.g.*, a digital computer and a tape-shuttle unit, the latter to avoid gaps in the data of duration $(+N/bp_0)$ occurring during analysis intervals) and the method loses appeal.

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Determination of Error Rates for Narrow-Band Communication of Binary-Coded Messages in Atmospheric Radio Noise*

Montgomery [1] has investigated the effects of Gaussian noise on binary-coded messages for amplitude and angle modulation narrow-band communication systems. His analysis is based on the result that for Gaussian noise the envelope of the noise at the detector output is Rayleigh distributed so that the probability distribution function for each component of the noise (in-phase and quadrature) is given by [2]

$$P(N_i \geq E) = \operatorname{erfc}\left(\frac{E}{N_0}\right),$$

where p is the probability that the rms amplitude of the component N_i exceeds the rms amplitude E, and N_0 is the long-term rms amplitude of the total noise envelope.

Systems which operate at less than 20 Mc are subjected to atmospheric radio noise which is not Rayleigh distributed, except at its lowest levels.

The distribution function of atmospheric noise has been investigated [3]–[5], and it has been found that the high noise levels are distributed during a short period of time (a few minutes to an hour) as follows

$$P(N \geq E) = e^{-(E/k)^\alpha},$$

where k and α are constants easily determined by the parameters defining the shape of the complete distribution function [3], [5] and N has been normalized to the rms amplitude of the total noise so that N can also be thought of as the SNR. The Rayleigh distribution is a special case of the above ($k=1$, $\alpha=2$). Due to the exponent α, expressions of the above form are sometimes called power Rayleigh distributions.

Letting the entire amplitude distribution for atmospheric noise be given by $P(N \geq E) = F(E)$, the density function is $p(E) = -F'(E)$, and, if the reasonable assumptions are made that the phase of the noise is uniformly distributed and that the amplitude E and phase θ are uncorrelated, their joint distribution is

$$p(E, \theta) = \frac{-F'(E)}{2\pi}.$$

Making the change of variable

$$E_q = E \sin\theta \quad \text{and} \quad E_p = E\cos\theta$$

where E_q and E_p are the quadrature and in-phase components of $\vec{E}$,

$$p(E_q, E_p) = \frac{p(E,\theta)}{E}.$$

The distribution function for the rms amplitude of either of the components is obtained by letting $N_i = |E_q|$, and making use of the fact that, if $N_i = |E_q|$,

$$P(N_i \leq E) = p(-E \leq E_q \leq E) = \int_{-E}^{E} p(E_q)dE_q.$$

Therefore,

$$P(N_i \leq E) = \frac{1}{2\pi}\int_{-E}^{E}\int_{-\infty}^{\infty} \frac{-F'(\sqrt{E_q^2 + E_p^2})}{\sqrt{E_q^2+E_p^2}}\, dE_q dE_p.$$

By changing to polar co-ordinates, $P(N_i \leq E)$ can be put in the form

$$P(N_i \leq E) = \frac{1}{2\pi}\left[\int_0^{2\pi}\int_0^{E} -F'(\rho)d\rho d\phi + 4\int_{\mathrm{Cos}^{-1} E/\rho}^{\pi/2}\int_E^{\infty} -F'(\rho)d\rho d\phi\right],$$

which reduces to

$$P(N_i \geq E) = \frac{2}{\pi}\int_E^{\infty} -F'(\rho)\,\mathrm{Cos}^{-1}\left(\frac{E}{\rho}\right) d\rho,$$

where

$$\rho^2 = E_p^2 + E_q^2, \quad \text{and} \quad \phi = \tan^{-1}\frac{E_q}{E_p}.$$

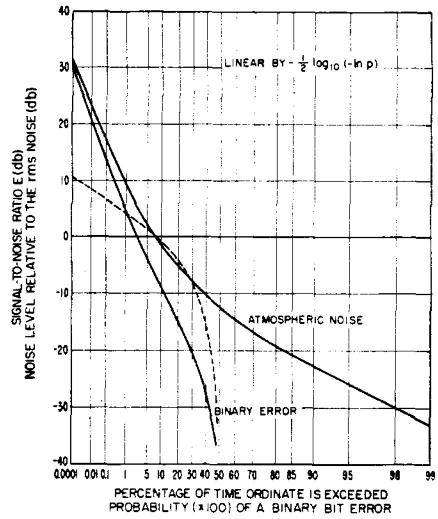

Fig. 1—An amplitude-probability distribution function for atmospheric radio noise and the probability of binary bit error vs SNR.

If the above integral instead of the probability integral is used in Montgomery's analysis, valid results are obtained for systems operating under atmospheric radio noise. In general, the $F'(\rho)$ in the above must be obtained numerically from the $F(E)$ given in the references, although this is somewhat cumbersome.

The SNR's usually of interest, however, occur on the portion of the distribution defined by the power Rayleigh. In this case then,

$$P(N_i \geq E) = \frac{2\alpha}{\pi k}\int_E^{\infty}\left(\frac{\rho}{k}\right)^{\alpha-1} e^{-(\rho/k)^\alpha}\,\mathrm{Cos}^{-1}\left(\frac{E}{\rho}\right)d\rho,$$

which, when placed in the form

$$P(N_i \geq E) = \frac{2e^{-(E/k)^\alpha}}{\pi}\int_0^{\infty} \cdot\,\mathrm{Cos}^{-1}\left\{\frac{E}{k\left[u + \left(\frac{E}{k}\right)^\alpha\right]^{1/\alpha}}\right\} e^{-u}du,$$

is very easily evaluated by Gauss-Laguerre quadratures.

As an example, Montgomery shows that $\pi/2$ coherent phase modulation results in smaller error rates than the other systems considered (noncoherent carriers keying, coherent carrier keying, and noncoherent frequency shift keying). The probability of a binary bit error for the $\pi/2$ coherent biphase system is equal to one half the probability that the quadrature component of the noise envelope will exceed the signal, that is

$$p(\text{binary bit error}) = (1/2)P(N_i > E).$$

Fig. 1 shows an example of the amplitude distribution function for atmospheric noise taken from [4] ($\alpha=0.424$, and $k=0.0915$ for the power Rayleigh portion) along with the probability of a binary error vs average carrier-to-noise ratio E(in db). The dashed curve gives the probability of a binary error for Gaussian noise. The figure is plotted on Rayleigh co-ordinates, so the power Ray-

* Received September 19, 1963.

leigh distributions are straight lines. The Rayleigh distribution has a slope of $-\frac{1}{2}$, and the noise is seen to be Rayleigh distributed at the lowest levels. The curved section of the distribution is the arc of a circle tangent to the Rayleigh and power Rayleigh lines. The errors for SNR that are below the power Rayleigh section were computed by numerical integration, using methods based on the $P(N_i \geq E)$ integral involving $F'(\rho)$.

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Design Considerations for Ferrite Junction Circulators*

Skomal[1] has considered circulation action in junction circulators to be caused by bound surface waves on a ferrite disc; however, the authors have considered another approach to the problem. We have assumed that the ferrite element acts as a dielectric-ferrimagnetic dimensional resonator which stores and reradiates the energy.[2,3] For a cylindrical ferrite element, the diameter necessary for dimensional resonance from Soohoo[4] for a lossless medium is

$$d = \frac{b\lambda_0}{2\sqrt{\mu_e \epsilon}}, \tag{1}$$

where b is an arbitrary constant, ϵ the relative permittivity, μ_e the effective permeability and λ_0 the free space wavelength. For a transversely magnetized ferrite

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu}, \tag{2}$$

where μ and κ are the usual components of the susceptibility tensor.

* Received September 23, 1963.
[1] E. N. Skomal, "Theory of operation of a 3-port Y-junction ferrite circulator," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-11, pp. 117–122; March, 1963.
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Substituting (2) into (1) it follows that

$$d = \frac{b\lambda_0}{2\epsilon^{1/2}}\left[\frac{\gamma^2 H_i^2 - f^2 + 4\pi M_s \gamma^2 H_i}{(\gamma^2 H_i^2 - f^2 + 4\pi M_s \gamma^2 H_i) - (4\pi M_s \gamma f)}\right]^{1/2}, \tag{3}$$

where f is the frequency in megacycles, H_i is the internal field from Kittel's equation and the other terms are standard terminology. By substituting Kittel's equation for internal field into (3), a relation for diameter can be obtained which equates dimensional resonance to $4\pi M_s$, the applied dc magnetic field and the demagnetizing factors.

Fig. 1 shows a comparison between calculated diameters and experimental data for $\kappa\mu$ band. For this example, $H_a = 500$, $4\pi M_s = 2400$ and $\epsilon = 11.5$. The constant b was chosen to be 1.5 which is the ratio of λ_g/λ_0 at midband. Note the agreement at these frequencies. Fig. 2 shows some computed data for S band which agree with the normal range of diameters reported for gallium substituted yttrium ferrites. However, notice the erratic behavior of the diameter which results

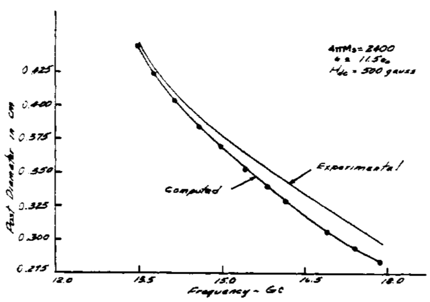

Fig. 1—Comparison of experimental and calculated diameters.

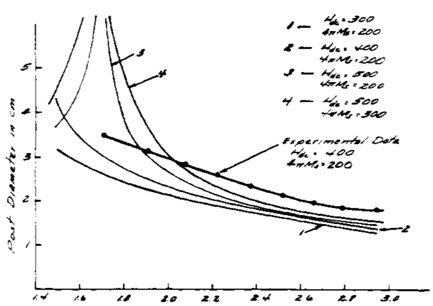

Fig. 2—Variation of diameter with $4\pi M_s$.

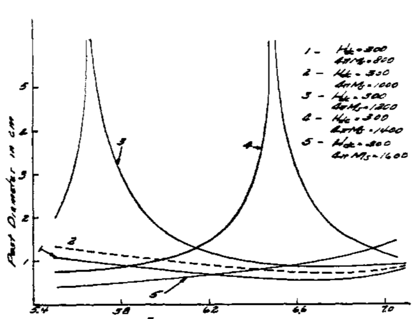

Fig. 3—Calculated diameters for varying $4\pi M_s$; $H_{dc} = 300$.

from the effective permeability approaching zero. At the higher frequencies, X band and above, this problem is avoided, as the point of zero permeability is displaced due to higher values of $4\pi M_s$ and higher operating frequencies. Fig. 3 shows a similar curve for C band and the same effect, operation near zero permeability, occurs.

These curves are interesting in that their implied temperature stability is dependent upon the choice of operating parameters. At C band this is more graphically illustrated than at S band, but, experimentally, both frequency regions show this effect. In Fig. 3 it is apparent that a change in M_s, due to variations in temperature, will cause a change in the dimension necessary for dimensional resonance, and as a result the circulation frequency changes. This means that the ferrite designer must use judicious care in choosing the ferrite, and the dc bias to obtain μ_e which is insensitive to temperature. One other effect we have noticed in our calculation is that in operation above ferrimagnetic resonance diameter, variation with $4\pi M_s$ and H_{dc} is minimized. This implies that temperature sensitivity is reduced for operation above ferrimagnetic resonance.

In conclusion, we have investigated a simple model which relates the diameter of ferrite elements in junction circulators to dimensional resonance. By no means do we claim that the model is complete; except for the limited applications within, and it appeared to satisfy some questions on temperature problems in junction circulators. Viewed in this light the model appears to be valid.

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"A Reciprocity Theorem for the Interaction of Electromagnetic Plane Waves with a One-Dimensional Inhomogeneous Slab"*

In the above correspondence,[1] Swift has gone through a rather elaborate analysis to show that the (horizontally and vertically

* Received October 21, 1963.
[1] C. T. Swift, PROC. IEEE (*Correspondence*), vol. 51, p. 1268; September, 1963.