APPENDIX C. CONVERSION OF POWER MEASURED IN A CIRCUIT TO INCIDENT FIELD STRENGTH AND INCIDENT POWER DENSITY, AND CORRECTIONS TO MEASURED EMISSION SPECTRA FOR NON-CONSTANT EFFECTIVE APERTURE MEASUREMENT ANTENNAS

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This appendix derives conversions between power measured in a circuit and incident field strength and effective radiated power from a transmitter.[2] Necessary corrections to measured emission spectra for non-constant effective aperture measurement antennas are also derived and explained.

C.1 Directivity, Gain, Effective Antenna Aperture, and Antenna Correction Factor

The starting point is *directivity* and *gain*, which are both measures of how well energy is concentrated in a given direction. *Directivity*, *d*, is the ratio of power density, p_{den} in that direction, to the power density that would be produced if the power were radiated isotropically, $p_{den\text{-}iso}$:

$$d = \frac{p_{den}}{p_{den-iso}} \quad . \tag{C.1}$$

The reference can be linearly or circularly polarized. By geometry, directivity is:

$$d = \frac{4p\, p_{den}}{\frac{\sqrt{E}}{m}\int\int E^2 d\Omega} \tag{C.2}$$

where E is the field strength [1]. Directivity makes reference only to power in space around the antenna; it is unrelated to power at the antenna terminals. There is loss between the terminals and free space. *Gain*, *g*, includes these antenna losses; gain is the field intensity produced in the given direction by a fixed input power, p_{in}, to the antenna. Gain and directivity are related by efficiency, ,:

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[2] In this appendix, quantities in linear units (e.g., milliwatts) are written in lower case; decibel quantities [10 log (linear power ratio)] are written in upper case.

$$g = d\epsilon \tag{C.3}$$

or,

$$g = \frac{(4\pi\ p_{den})}{p_{in}} \quad . \tag{C.4}$$

Through reciprocity, directivity is independent of transmission or reception, as is gain. *Effective antenna aperture*, a_e is unrelated to the physical aperture of an antenna. a_e is defined as:

$$a_e = \frac{\lambda^2 g}{4\pi} \tag{C.5}$$

where 8 is the free-space wavelength. Note that a_e has the units of area. For an antenna matched to a load, the power in the load, p_{load}, is related to the free-space power density by a_e:

$$p_{load} = p_{den} \cdot a_e \quad . \tag{C.6}$$

If p_{load} is in a 50-ohm circuit and p_{den} is in a free-space impedance of 377 ohms, then the following relations apply:

$$p_{load} = \frac{V_{load}^2}{50} \tag{C.7a}$$

and

$$p_{den} = \frac{V_{space}^2}{377} \quad . \tag{C.7b}$$

Rewriting Eq. C.6 gives:

$$\frac{V_{load}^2}{50} = a_e \cdot \left(\frac{V_{space}^2}{377} \right) \tag{C.8}$$

Note that the voltage in the circuit, V^2_{load}, is in units of volts, but that the free-space field strength, V^2_{space} is in units of volts/m. The effective aperture, in units of m^2, converts the free-space power density on the right to the power in a circuit on the left.

At this point, we introduce the *antenna correction factor, acf,* which is defined as follows:

$$acf \;=\; \frac{V_{space}^2}{V_{load}^2} \quad . \tag{C.9}$$

Note that acf has the units of m^{-2}. Rewriting Eq. C.8 with this substitution for acf gives:

$$acf \;=\; \frac{377}{50} \cdot \left(\frac{1}{a_e} \right) \quad . \tag{C.10}$$

Because a_e is dependent upon both gain and frequency, so is acf. Substituting Eq. C.5 into Eq. C.6 gives:

$$acf \;=\; \left(\frac{377}{50} \right) \left(\frac{4\pi}{\lambda^2 g} \right) \quad . \tag{C.11}$$

If the frequency, f, is in megahertz, then the substitution [8= c/(fA10^6) = 3A10^8/(fA10^6)] gives:

$$acf \;=\; \left(\frac{377}{50} \right) (4\pi) \left[\left(\frac{10^{12}}{(3 \cdot 10^8)^2} \right) \;\cdot\; (f,\ MHz)^2 \cdot \left(\frac{1}{g} \right) \right] \tag{C.12}$$

which, calculating the constant term values, gives:

$$acf \;=\; \left(1.03 \cdot 10^{-3}\right) \;\cdot\; (f,\ MHz)^2 \;\cdot\; \left(\frac{1}{g} \right) \quad . \tag{C.13}$$

Taking 10log(*acf*) gives *ACF* in dB:

$$(ACF,\ dB) \;=\; (-29.78\ dB) \;+\; 20\log(f,\ MHz) \;-\; 10\log(g) \quad . \tag{C.14}$$

C.2 Free Space Field Strength Conversion

Signals are commonly measured in circuits as either voltages or log-detected voltages proportional to power. In either case, the signal measurement within a circuit is usually converted to equivalent power in the circuit impedance. This conversion is usually accomplished automatically within the measurement device (e.g., a spectrum analyzer). This measured power within a circuit, p_{load}, is converted to incident field strength using either the antenna correction factor or antenna gain relative to isotropic, as follows. Writing Eq. C.6 with a substitution for a_e from Eq. C.5 gives:

$$p_{load} = \left(\frac{\lambda^2 \cdot g \cdot \left(V_{space}\right)^2}{4\pi \cdot 377} \right) \quad , \tag{C.15}$$

and substituting for (8= c/f), $f = 10^6$Af(MHz), and c = 3A10^8 m/s gives:

$$p_{load} = \left[\frac{\left(3 \cdot 10^8\right)^2 \cdot g \cdot \left(V_{space}\right)^2}{(f,\ MHz)^2 \cdot \left(10^6\right)^2 \cdot 4\pi \cdot 377} \right] \quad . \tag{C.16}$$

For power in milliwatts and field strength in microvolts/meter, the conversions (power, mW) = 1000A(power, W) and (field strength, v/m) = 10^{-6}A(field strength, µV/m) are used:

$$\left(p_{load},\ mW\right) = \left[\frac{1000 \cdot \left(3 \cdot 10^8\right)^2 \cdot g \cdot \left(10^{-6}\right)^2 \cdot \left(V_{space},\ \mu V/m\right)^2}{(f,\ MHz)^2 \cdot \left(10^6\right)^2 \cdot 4\pi \cdot 377} \right] \tag{C.17}$$

which, upon computing the constant terms, becomes:

$$\left(p_{load},\ mW\right) = \left[\frac{1.90 \cdot 10^{-8} \cdot g \cdot \left(V_{space},\ \mu V/m\right)^2}{(f,\ MHz)^2} \right] \quad . \tag{C.18}$$

Taking 10log of both sides gives:

$$10\log\left(p_{load},\ mW\right) = (-77.2\ dB) + 10\log(g) + 20\log\left(V_{space},\ \mu V/m\right) - 20\log(f,\ MHz) \quad . \tag{C.19}$$

Rearrangement of terms yields:

$$\left(field\ strength,\ dB\mu V/m\right) = \left(P_{load},\ dBm\right) + (77.2\ dB) + 20\log(f,\ MHz) - G \quad . \tag{C.20}$$

Note that P_{load} is related to the power measured within a circuit (e.g., a spectrum analyzer) by the correction for path gain between the antenna and the analyzer: $P_{load} = P_{meas}$ - (path gain to antenna). This changes Eq. C.20 to:

$$\left(fieldstrength,\ dB\mu V/m\right) = \left(P_{meas},\ dBm\right) - (path\ gain) + (77.2\ dB) + 20\log(f,\ MHz) - G. \tag{C.21}$$

Eq. C.21 is key for the conversion of measured power in a circuit into incident field strength in dBµV/m. For example, suppose that an antenna has gain G = 17 dB, at a frequency of 2300

MHz, with 28 dB path gain between the antenna and the spectrum analyzer. The measured power is -12 dBm on the spectrum analyzer display. Then field strength is +87 dBμV/m.

If an equation is required to relate field strength in dBμV/m to the *ACF*, then Eq. C.8 is used to relate acf to voltage in a circuit and free-space field strength:

$$\frac{V_{load}^2}{50} = \left(p_{load}\right) = \frac{V_{space}^2}{\left(50 \cdot acf\right)} \quad . \tag{C.22}$$

Converting power in watts into power in milliwatts, and converting voltage in volts/meter into microvolts/meter, gives

$$\left(p_{load,}\ mW\right) = \left[\frac{1000 \cdot \left(10^{-6}\right)^2 \cdot \left(V_{space},\ \mu V/m\right)^2}{50acf}\right] \tag{C.23}$$

which, upon computing the group of constant terms, gives:

$$\left(p_{load},\ mW\right) = \frac{\left(2 \cdot 10^{-11}\right) \cdot \left(V_{space},\ \mu V/m\right)^2}{acf} \tag{C.24}$$

and which, taking 10log of both sides, means that

$$\left(P_{load},\ dBm\right) = \left(-107\ dB\right) + \left(V_{space},\ dB\mu V/m\right) - ACF \quad . \tag{C.25}$$

Rearranging terms and substituting [P_{meas} - (path gain)] for P_{load}:

$$V\left(dB\mu V/m\right) = P_{meas} - \left(path\ gain\right) + \left(107\ dB\right) + ACF \quad . \tag{C.26}$$

For example, suppose a measurement of -12 dBm is taken on a spectrum analyzer, with 28 dB net gain in the path between the antenna and the analyzer, and measurement antenna acf of 111 (corresponding to G = 17 dBi at 2300 MHz, as in the example above). Then *ACF* = 20.5 dB, and the corresponding free-space field strength is computed using Eq. C.26 to be +87 dBμV/m.

C.3 Effective Isotropic Radiated Power Conversion

It may be necessary to know the effective isotropic radiated power (EIRP) that a device transmits. The conversion from measured power in a circuit to EIRP is described in this section.

Free space loss must be determined: For transmit power (watts) p_t; transmit antenna gain relative to isotropic g_t; receive antenna gain relative to isotropic g_r; receive power (watts) p_r; and receive antenna effective aperture a_e; the effective isotropic radiated power is

$$(eirp) = (p_t \cdot g_t) \tag{C.27}$$

and

$$p_r = a_e \cdot \left(\frac{eirp}{4\pi r^2} \right) \tag{C.28}$$

with r being the distance between the transmit and receive antennas.

The effective aperture of the antenna is the effective aperture of an isotropic antenna multiplied by the antenna's gain over isotropic, or

$$a_e = \left(\frac{\lambda^2}{4\pi} \right) \cdot g_r \quad . \tag{C.29}$$

A change to decibel units makes Eq. C.28:

$$P_r = EIRP + G_r + 20\log(\lambda) - 20\log(4\pi) - 20\log(r) \quad . \tag{C.30}$$

Substituting c/f for 8,

$$P_r = EIRP + G_r + 20\log(c) - 20\log(f) - 20\log(4\pi) - 20\log(r) \quad . \tag{C.31}$$

If frequency is in megahertz and distance is in meters, then Eq. C.31 becomes

$$P_r = EIRP + G_r + 20\log(c) - 20\log(f, MHz \cdot 10^6) - 20\log(4\pi) - 20\log(r) \quad , \tag{C.32}$$

which yields

$$P_r = EIRP + G_r + 169.5 - 20\log(f, MHz) - 120 - 22 - 20\log(r, meters) \tag{C.33}$$

or

$$P_r = EIRP + G_r + 27.5 - 20\log(f, MHz) - 20\log(r, meters) \quad , \tag{C.34}$$

Similarly, for r in kilometers, Eq. C.31 becomes

$$P_r = EIRP + G_r - 32.5 - 20\log(f,\ MHz) - 20\log(r,\ km) \qquad \text{(C.35)}$$

and Eq. C.31 for r in miles is

$$P_r = EIRP + G_r - 36.5 - 20\log(f,\ MHz) - 20\log(r,\ miles) \quad . \qquad \text{(C.36)}$$

For measurements of Part 15, Part 18, and ultrawideband transmitters, Eq. C.34 is convenient. If the gain of the receive antenna is known and the received power has been measured at a known distance from the emitter, then Eq. C.34 can be rearranged to yield *EIRP:*

$$(EIRP,\ dBW) = (P_r,\ dBW) - G_r - 27.5 + 20\log(f,\ MHz) + 20\log(r,\ meters) \quad . \qquad \text{(C.37)}$$

If the received power is measured in dBm rather than dBW, Eq. C.37 becomes

$$(EIRP,\ dBW) = (P_r,\ dBm) - G_r - 57.5 + 20\log(f,\ MHz) + 20\log(r,\ meters) \quad . \qquad \text{(C.38)}$$

If EIRP in decibels relative to a picowatt (dBpW) is required, then Eq. C.38 becomes:

$$(EIRP,\ dBpW) = (P_r,\ dBm) - G_r + 62.5 + 20\log(f,\ MHz) + 20\log(r, meters) \quad . \qquad \text{(C.39)}$$

For example, if a value of -10 dBm is measured at a frequency of 2450 MHz, with an antenna gain +16.9 dBi, at a distance of 3 meters, then the EIRP value is +113 dBpW.

Effective radiated power relative to a dipole (ERP_{dipole}) is sometimes required. EIRP is 2.1 dB higher than ERP_{dipole}.

Finally, the conversion to incident power density is considered. The incident power density, P_{den}, in W/m^2, is equal to the incident field strength squared (units of $(V/m)^2$), divided by the ohmic impedance of free space:

$$(P_{den},\ W/m^2) = \left(\frac{(field\ strength,\ V/m)^2}{377} \right) \quad . \qquad \text{(C.40)}$$

Using more common units for field strength of (dBµ*V/m*), and more common units for incident power of (µW/cm^2):

$$(1\ W/m^2) = \frac{100\ \mu W}{cm^2} = \frac{10^{12}}{377}(\mu V/m)^2 \qquad \text{(C.41)}$$

gives

$$1\ \left(\mu W/cm^{2}\right) = \frac{10^{10}\ \left(\mu V/m\right)^{2}}{377} \quad . \tag{C.42}$$

The incident power density is thus

$$\left(P_{\rho},\ \mu W/cm^{2}\right) = \frac{377}{10^{10}} \cdot \left(field\ strength,\ \mu V/m\right)^{2} \tag{C.43}$$

or

$$\left(P_{\rho},\ \mu W/cm^{2}\right) = 3.77 \cdot 10^{-8} \cdot \left(field\ strength,\ \mu V/m\right)^{2} \quad . \tag{C.44}$$

C.4 Correction of Measured Emission Spectra for Non-Constant Effective Aperture Measurement Antennas

With reference to Eq. C.5, the effective aperture of an isotropic antenna as a function of frequency, f, is:

$$a_{e} = \frac{g}{4\pi f^{2}} \tag{C.45}$$

or

$$A_{e} = G - 10\log\left(4\pi\right) - 20\log\left(f\right) \quad . \tag{C.46}$$

If gain relative to isotropic, g, is arbitrarily normalized to 4B, then the functional dependence of effective aperture on frequency is clear:

$$A_{e} \propto -20\log\left(f\right) \quad . \tag{C.47}$$

The effective aperture of a constant-gain isotropic antenna decreases as the square (i.e., 20 log) of the frequency. If an isotropic antenna were realized physically, and were then used to measure an emission spectrum, it would be necessary to correct the measured spectrum for this drop in aperture with increasing frequency.

For an isotropic antenna, the measured spectrum amplitudes would have to be increased as (20log(frequency)) to represent the energy that would be coupled into a constant effective aperture. Parabolic reflector antennas in principle have constant apertures; their gain nominally increases at the rate of (20 log(frequency)). Consequently emission spectra measured with high-performance parabolic antennas do not require antenna aperture corrections.

Wideband horn antennas represent an intermediate case between the (-20 log) decrease in effective aperture of theoretical isotropic antennas and the constant-aperture condition of nominal parabolic reflector antennas. For example, the gain curve of a widely used double-ridged waveguide horn (Figure C.1) increases with frequency, but at the rate of approximately (6.7 log(frequency)). Since this is [(20-6.7)log] = 13.3 log below a constant-aperture condition, the spectra measured with such an antenna must be corrected at the rate of (13.3 log(frequency)).

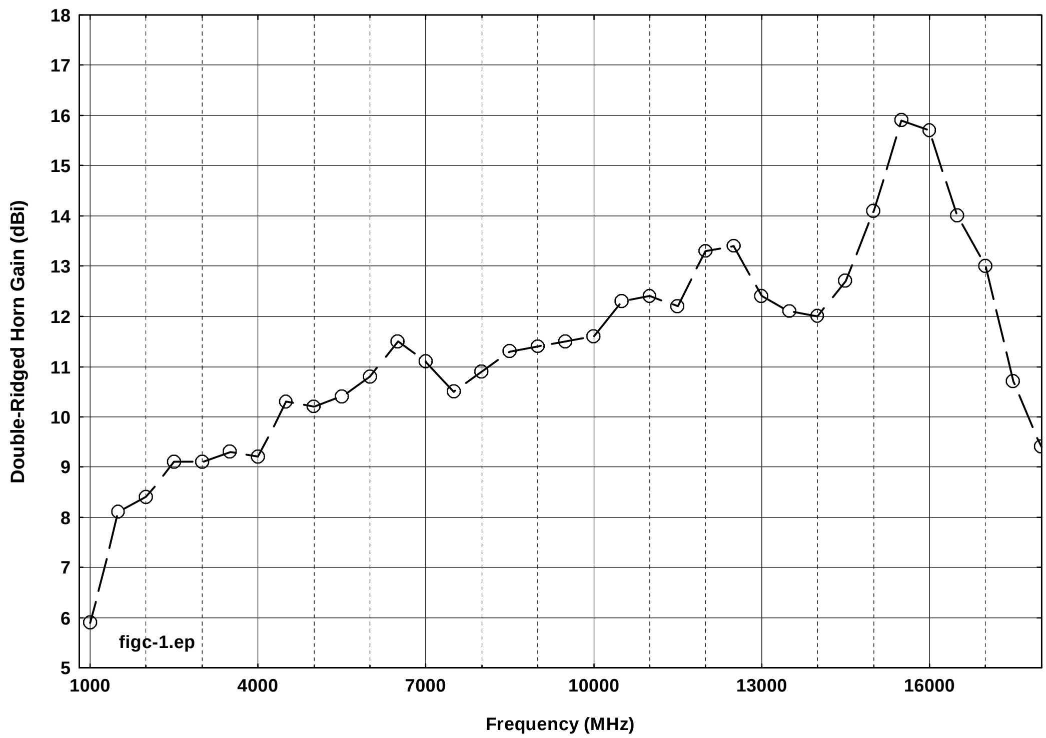

Figure C.1. Broadband double-ridged waveguide antenna gain as a function of frequency. The effective aperture would be constant if the gain curve varied as 20log frequency).

Operationally, this correction is performed by ITS engineers as follows: The spectrum is measured across a given frequency range and the uncorrected curve is stored. During the data analysis phase, the original spectrum is corrected relative to an arbitrarily chosen frequency, according to the following equation:

$$\left(P_{corrected}\right) = \left(P_{measured}\right) + 13.3\left[\log\left(\frac{f_{measured}}{f_{reference}}\right)\right] \quad . \tag{C.48}$$

For example, if a spectrum is measured between 1 GHz and 5 GHz, a convenient reference frequency might be 1 GHz, since the corrected spectrum will then have a zero correction at the left-hand side of the graph. The correction will increase from zero at 1 GHz to a maximum of 9.3 dB at 5 GHz on the right-hand side of the graph, as in Figure C.2.

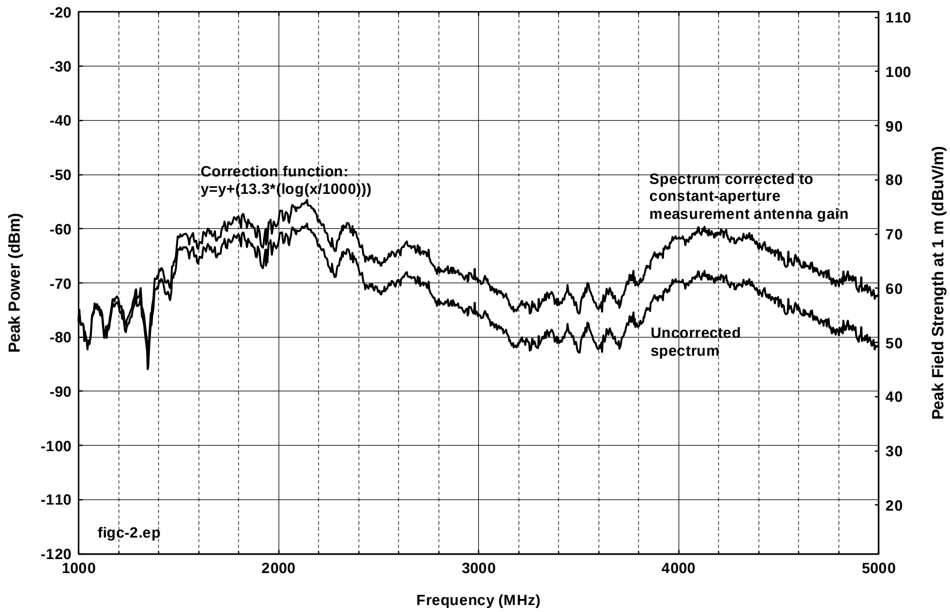

Figure C.2. Demonstration of emission spectrum measurement corrected to a constant effective aperture measurement antenna. With this correction it becomes possible to add a second axis for incident field strength.

The graph’s power-axis label needs to reflect the fact that the spectrum has been rendered as measured with a constant-effective-aperture antenna. This can be done in two ways. The first is to reference the antenna’s gain relative to isotropic at the reference frequency. If the antenna in question had 5.9 dBi gain at the reference frequency of 1 GHz, then the axis label could read: “Power measured with constant-aperture antenna, 5.9 dBi gain at 1 GHz (dBm).”

The label above might as easily refer to the antenna correction factor (acf) of the antenna at the reference frequency. The second method is to compute the effective aperture of the antenna and quote that in the label. In this example, the gain of 5.9 dBi at 1 GHz yields an effective aperture in Eq. C.45 of 3.1×10^{-7} m^2. The corresponding axis label could read:

“Power measured with antenna of constant effective aperture 3.1×10^{-7} m^2 (dBm)”

While accurate, this expression’s reference to an effective aperture is unconventional.

An advantage of correcting a measured emission spectrum to constant measurement antenna effective aperture is that a second axis can be added to the right-hand side of the graph showing field strength. With the effective aperture correction having been made, the field strength becomes an additive factor to the power measured in a circuit. In this example (5.9 dBi gain at 1 GHz, constant aperture correction made to the rest of the spectrum), Eq. C.21 is used to arrive at the following conversion to field strength:

$$\left(Field\ strength,\ dB\mu V/m\right) = P_{meas} - \left(path\ gain\right) + 77.2 + (60 - 5.9) \tag{C.49}$$

or

$$\left(Field\ strength,\ dB\mu V/m\right) = P_{meas} - \left(path\ gain\right) + \left(131\ dB\right) \quad . \tag{C.50}$$

Figure C.2 shows an example of a corrected spectrum with the field strength axis added in accordance with Eq. C.48 and Eq. C.50.

C.5 References

[1] E. C. Jordan, Ed., *Reference Data for Engineers: Radio, Electronics, Computer, and Communications*, Seventh ed., Macmillan, 1989, pg. 32-33.

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