4. CHARACTERISTICS OF AN AGGREGATE OF ULTRAWIDEBAND SIGNALS

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4.1 Introduction

The proliferation of UWB devices throughout the United States has been predicted by many industry sources. Hence, it is important that the effects of an aggregate of such devices on RF spectrum users be well understood by regulators, spectrum users, and UWB system designers. This section describes models that can be used to predict interference effects of many UWB devices on traditional narrowband RF receivers.

This model assumes that the *victim receiver* is *narrowband* and hence, it is sufficient to evaluate UWB parameters such as effective isotropically radiated average power (EIRP) and antenna gains at the center frequency of the receiver. The calculation of the power at the victim receiver requires an estimation of the basic transmission loss over the propagation path from the transmitters to the receiver. Single frequency propagation models used in traditional radio link calculations will be utilized in conjunction with the models described in this section. In the analysis which follows, it was convenient to use the basic transmission gain (denoted below as g_b) instead of loss. The basic transmission gain and loss are reciprocals and have the same absolute value but opposite signs when given in decibels.

In the first part of this section, the aggregate effects of a few similar devices in the immediate vicinity of a victim receiver are discussed. This is followed by the development of a statistical model that can be used to calculate the average received power from many UWB devices randomly distributed over the surface of the Earth. This model can be used to predict interference power for both terrestrial and airborne receivers.

4.2 Deterministic Interference Model for UWB Devices in the Vicinity of a Victim Receiver

The mean power in the receiver bandwidth due to UWB devices is simply the sum of the power received from each source or

$$w_r = \sum_{n=1}^{N} w_{t_n} g_{t_n} g_{b_n} g_{r_n} \quad , \tag{4.1}$$

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where w_r is the received power, w_t is the emitted EIRP in the receiver bandwidth, g_t is the transmitter gain, and g_r is the receiver gain in the direction of the nth transmitting device. When the locations of the devices are known and N is small, computing the received power is a relatively straightforward matter.

More realistically, one may have only a rough estimate of the ostensible number of such devices deployed in a particular geographic area (e.g., an average areal density) surrounding a particular RF receiver. In such cases, Equation 4.1 is not very useful since the parameters (perhaps the most important one being g_{b_n}) are not known. Hence statistical models and estimates are required to make any progress in predicting the potential for interference. The development of such a model is given in the next section.

Equation 4.1 is valid for commonly encountered random RF signals because the variance of the sum of zero mean random variables is the sum of the individual variances. When the received signals are normally distributed, the mean power is all that is needed to describe the statistics of the resulting interference. If there are many such devices with the same statistical properties (not necessarily normally distributed) then the statistics of the sum will approach a normal distribution [1]. In such cases, the models that predict the mean interference power provide the only statistic necessary to describe the process.

This leads directly to the question of how many signals must be added before the aggregate signal realistically appears to be normally distributed. Perhaps some insight can be gained by examining the results for a band limited fixed time-base dithered UWB signal as described in Section 3.3. In this example, the signal statistics are approximately normal for bandwidths well below the PRR. As the bandwidth increases, the absolute value of the excess increases and the statistics are no longer normal. The excess for an aggregate of such devices can be calculated as described below.

The aggregate excess for the sum of several random variables is related to the excess of each random variable γ_{2_n} as follows

$$\gamma_2 = \frac{\sum_n \gamma_{2_n} [\mu_{2_n}]^2}{\mu_2^2} \quad , \tag{4.2}$$

where μ_{2_n} is the second central moment of each variable and μ_2 is the sum of the moments. Since the processes are zero mean, the second central moment is just the signal power given in Equation 4.1.

The aggregate excess for band limited signals (e.g., as given in Section 3.3) is therefore

$$\gamma_2 = \frac{\sum_n \gamma_{2_n} [w_{t_n} g_{t_n} g_{r_n} g_{b_n}]^2}{[\sum_n w_{t_n} g_{t_n} g_{r_n} g_{b_n}]^2} \quad . \tag{4.3}$$

When the individual excesses, EIRP, and gains are the same, Equation 4.3 reduces to the *well known* result

$$\gamma_2 = \frac{\gamma_{2_n}}{N} \quad , \tag{4.4}$$

which indicates that the excess can decrease fairly rapidly as additional devices are added.

4.3 Statistical Aggregate Model

In this subsection, we develop a statistical model that can be used to estimate received interference power from many devices randomly distributed over the area surrounding a victim receiver. It is assumed that the devices are uniformly distributed over the surface of the Earth. The model requires an estimate of the path gain over the geographical area surrounding the receiver, the average receiver and transmitter antenna gains, and the average areal density of transmitters. The areal path gain can be calculated from traditional propagation models such as the Irregular Terrain Model [2]. A simple methodology that can be used to estimate average transmitter antenna gain is given in this subsection. Example calculations are given using simple receiving and transmitting antennas.

Let a UWB device with EIRP w_t and gain g_t be located at a point in space denoted by λ_i. The gain due to free space and terrestrial propagation from the point λ_i to the victim receiver, g_b, is a random variable that depends on location, terrain, climate, and other factors. Assuming a receiver gain g_r, the received power is

$$w_r = g_r w_t g_t g_b(\lambda_i, \omega_i) \quad , \tag{4.5}$$

where λ_i represents the dependence on the spatial location and ω_i are points in some probability space that characterizes, for example, random variations in devices, how, when and where they are deployed, propagation paths, etc. The average power at the victim receiver due to many UWB devices is obtained by taking the expected value of the sum of the contribution from each device

$$
\begin{aligned}
\bar{w}_r &= \mathscr{E}\Big\{\sum_i g_r w_t g_t g_b(\lambda_i, \omega_i)\Big\} \\
&= \mathscr{E}\Big\{\sum_{\Delta\lambda} n(\Delta\lambda) g_r w_t g_t g_b(\Delta\lambda, \omega_i)\Big\}
\end{aligned}
\quad , \tag{4.6}
$$

where $\Delta\lambda$ is an area increment at the point λ_i and $n(\Delta\lambda)$ is the number of devices in $\Delta\lambda$.

In this model, it will be assumed that the devices are randomly distributed in space according to *Poisson Postulates*. Essentially this means that the number of devices in non-overlapping regions of space are *independent*, the probability structure is both space and time invariant, and the probability of exactly one device being in a small increment of space $\Delta\lambda$ is approximately proportional to the increment

$$
p(\Delta\lambda) = \rho\Delta\lambda + o(\Delta\lambda); \quad \Delta\lambda \rightarrow 0 \quad , \tag{4.7}
$$

where ρ is the average density. The probability of more than one device being in a small interval is smaller than the order of magnitude of $\Delta\lambda$(i.e., $o(\Delta\lambda)$). The average received power is then

$$
\bar{w}_r = \rho \sum_{\Delta\lambda} \mathscr{E}\{w_t g_t g_r g_b(\lambda_i, \omega_i)\}\,\Delta\lambda \quad . \tag{4.8}
$$

The expected values of the transmitted power $\bar{w}_t$ and gain $\bar{g}_t$ will depend, for example, on the range of possible devices and the antenna orientations with respect to the victim receiver. The mean path gain $\bar{g}_b$ is a function of the space coordinates. The mean receiver gain $\bar{g}_r$ will also in general be a function of the space coordinates. Assuming a distribution in 2-space corresponding to the surface of the earth, for small increments, the received power can be calculated via integration. Using polar coordinates with the victim receiver located at the origin, the average power (assuming $\bar{g}_r$ and $\bar{g}_b$ are independent) is

$$
\bar{w}_r = \bar{w}_t \bar{g}_t \rho \int_0^{2\pi} \bar{g}_r(\phi)\, d\phi \int_0^{\infty} \bar{g}_b(r)\, r\, dr \quad . \tag{4.9}
$$

In this expression, the basic path gain is the average over all possible radial paths and may be calculated, for example, by using the Irregular Terrain Model in the *area prediction mode*. The integral over ϕ includes the directive gain of a typical receiver. In this model, the parameter ρ is constant and is equal to the average number of devices per unit area.

4.3.1 Example Calculation Using the Irregular Terrain Model (ITM)

Converting Equation 4.9 to decibels, we have

$$\begin{aligned} \bar{W}_r &= \bar{W}_t + \bar{G}_t + \mathrm{P} + \Gamma_r + \Gamma_b \\ \Gamma_r &= 10\log_{10}\frac{1}{2\pi}\int_0^{2\pi} \bar{g}_r(\phi)\,d\phi \\ \Gamma_b &= 10\log_{10} 2\pi \int_0^{\infty} \bar{g}_b(r)\,r\,dr \quad . \end{aligned} \tag{4.10}$$

As is customary, upper case letters are used to denote decibel equivalents . The mean transmitter power can be estimated from specifications or measurements of typical UWB devices.

A Method for Estimating $\bar{G}_t$

In this model, it is assumed that the transmitting antennas are randomly oriented. The average is obtained by assuming a probability distribution for the orientations and applying a typical UWB transmitter gain function which can be defined in terms of the usual spherical coordinate system angles θ and α. In what follows, θ is the angle from the pole of the sphere located, for example, at the top of the transmitter antenna (e.g., the top of a vertical dipole) and α is the azimuth.

In the case of a victim receiver near the ground, it is reasonable to assume that the direction of propagation to the receiver is uniformly distributed over a solid angle Ω_0 defined by a *band* on the unit sphere bounded by spherical angles θ_0 and $\pi - \theta_0$ $(0 \le \alpha \le 2\pi)$. The expected value of the gain in the direction of the victim receiver in terms of the directive gain function $g_t = f(\Omega)$ is

$$\mathscr{E}\{f(\Omega)\} = \int_{\Omega_0} f(\Omega)\frac{d\Omega}{\Omega_0} \quad , \tag{4.11}$$

where

$$\Omega_0 = 2 \int_0^{2\pi} \int_{\theta_0}^{\pi/2} \sin\theta \, d\theta \, d\alpha = 4\pi\cos\theta_0 \quad . \tag{4.12}$$

The expected value of g_t is then

$$\mathcal{E}\{f(\theta,\alpha)\} = \frac{1}{4\pi\cos\theta_0} \int_0^{2\pi} \int_{\theta_0}^{\pi-\theta_0} f(\theta,\alpha) \sin\theta \, d\theta \, d\alpha \quad . \tag{4.13}$$

As an example, consider a short dipole where $f(\theta,\alpha) = 1.5\sin^2\theta$. The expected value of the gain is

$$\mathcal{E}\{g_t\} = 1.5\left(1 - \frac{\cos^2\theta_0}{3}\right) \quad . \tag{4.14}$$

When the transmitters are oriented so that $\theta_0 \approx \pi/2$, $\bar{G}_t \approx 1.76$ dB and when $\theta_0 = 0, \bar{G}_t = 0$ dB.

Calculation of Areal Gain Γ_b Using ITM

The ITM in area prediction mode was used to obtain the average path gain $\bar{g}_p$ relative to free space g_{fs} as function of distance from the victim receiver. The basic path gain $\bar{g}_b = \bar{g}_p g_{fs}$ was then integrated to obtain Γ_b. Table 4.1 gives typical ITM parameter settings used for examples given below unless otherwise specified.

Referring to Equation 4.10, the basic path gain is integrated over the interval $[0,\infty]$. The usual free space gain formula is only valid in the far field and has a singularity at $r = 0$. In the near field (less than a few wavelengths), power is transferred between the antennas via *mutual* coupling. For the purposes of this analysis, close proximity free space coupling was approximated by fitting a function to data obtained from a numerical analysis of the maximum coupling between two half-wave dipoles in the near field [3]. The resulting function used to calculate free space gain is

$$G_{fs} \approx -20\log_{10}\left(\frac{4\pi r}{\lambda} + 1.64\right) \quad , \tag{4.15}$$

which closely approximates near-field results and gives the usual far-field behavior when the antennas are separated by more than a few wavelengths. The numerical results for near-field coupling and G_{fs} (maximum coupling less the gain of the half-wave dipoles) as a function of antenna separation are shown in Figure 4.1.

For large distances, the integration is truncated well into the diffraction region (beyond the smooth earth radio horizon) where contributions are negligible. Figure 4.2 shows the basic transmission gain $\bar{G}_b$ and path gain $\bar{G}_p$, obtained from ITM for the parameters given in Table 4.1.

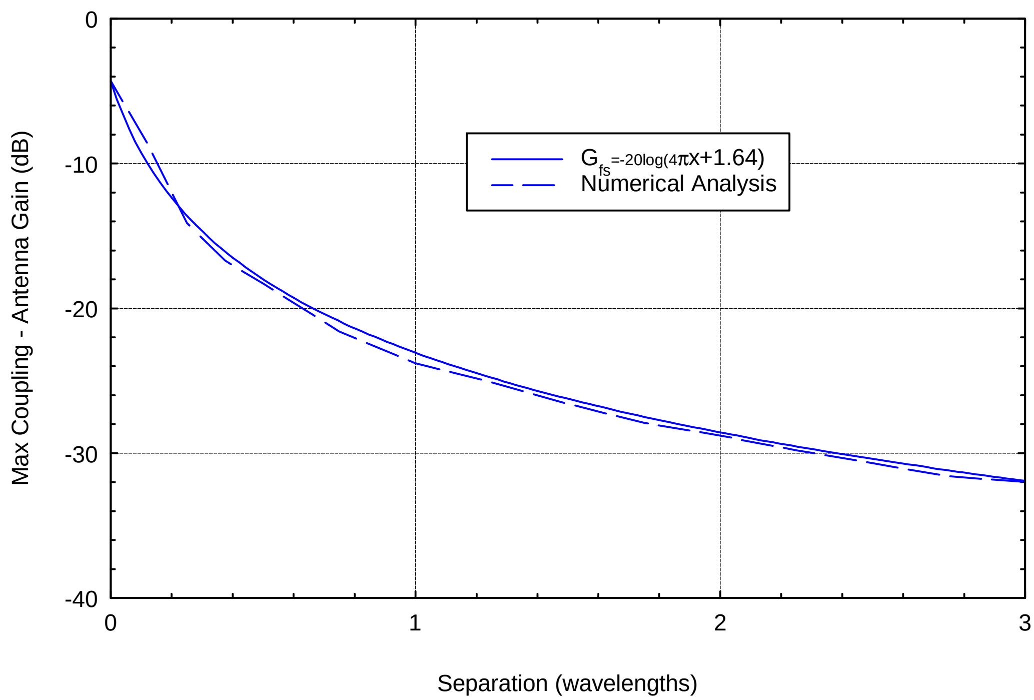

Figure 4.1 Approximation for near field antenna coupling.

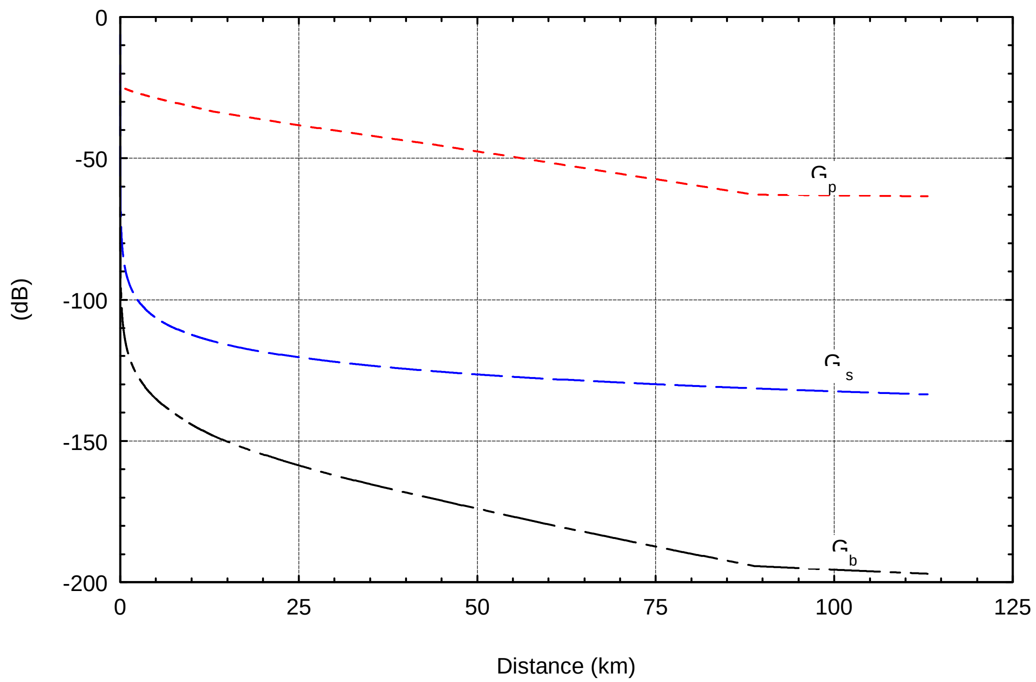

Figure 4.2. Example calculation of basic transmission gain G_b using ITM. G_p is the path gain and G_{fs} is the free-space gain. (1000 MHz, Δh=90 m, T_x = 2 m, R_x = 3 m).

Effects of Δh and Receiver Height

In area prediction mode, the statistical parameter Δh is used to characterize terrain in the geographical region of interest. The dependence of the parameter Γ_b on Δh is shown in Figure 4.3. Of note is the fact that in *flat* terrain Γ_b is more than 20 dB greater than for *hilly* terrain ($\Delta h = 90$ m).

In Figure 4.4, the parameter Γ_b is plotted as a function of receiver height. Basically, the path gain increases with increasing antenna height since terrestrial attenuation is not a factor at increasing distances from the receiver (as the receiver height increases). With increasing height, the path gain from the entire region within line-of-sight of the receiver is essentially due to free space propagation.

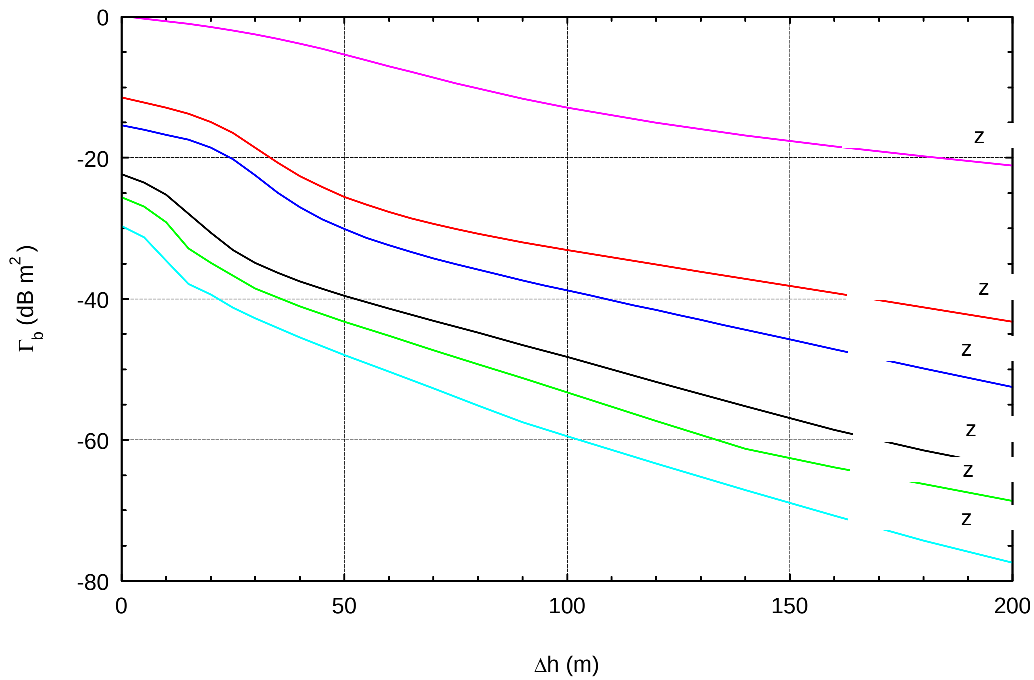

Figure 4.3. Basic areal gain Γ_b as a function of Δh and frequency. (T_x = 2 m, R_x = 3 m).

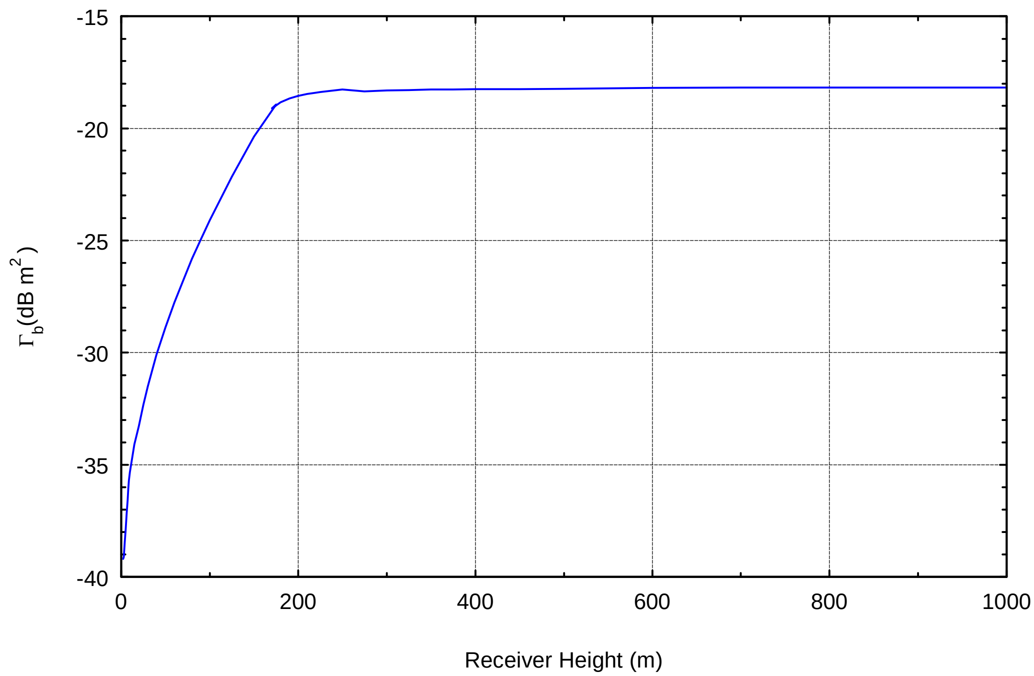

Figure 4.4. Basic areal gain Γ_b as a function of receiver height. (1000 MHz, Δh=90 m).

Table 4.1 Parameters for ITM Calculations

ITM Parameter	Value
Frequency	Various
Receiver Antenna Height	3 m
Transmitter Antenna Height	2 m
Polarization	Vertical
Terrain Irregularity Parameter Δh	90 m, 30 m, 0
Ground Electrical Constants	.005 S/m, $\epsilon_r = 15$
Surface Refractivity	301 N-units
Climate	Continental Temperate
Siting Criteria	Random
Time and Location Variability	50%
Confidence	50%

Estimated Interference Power Levels for a Half-wave Dipole Receiver

Referring to Equation 4.9, azimuthal dependence of the receiver gain in the direction of the UWB transmitters can be explicitly included in the analysis. The quantity Γ_r defined in Equation 4.10 is just the *average* gain in the azimuthal direction. To give a simple example, a half-wave dipole has a constant azimuthal gain of 2.15 dBi, hence $\Gamma_r = 2.15\ dBi$.

Assuming that the UWB transmitters are short dipoles and $\bar{G}_t = 1.76$, the power at the receiver per watt of transmitted power is

$$W_r = 3.91 + \mathrm{P} + \Gamma_b \ dBW \qquad , \tag{4.16}$$

where P is the average density in dB per unit area, and Γ_b is *area average of path gain*. Calculated values for various frequencies and terrain parameters associated with so called *flat* ($\Delta h = 0$), *plains* ($\Delta h = 30\ m$), and *hills* ($\Delta h = 90\ m$) environments are given in Table 4.2 below.

Table 4.2 Γ_b as a Function of Δh and Frequency (Based on Parameters Given in Table 4.1)

	$\Gamma_b \; dB\,m^2$		
Frequency (MHz)	$\Delta h = 0$	$\Delta h = 30m$	$\Delta h = 90m$
100	0.14	-2.51	-11.61
500	-11.46	-18.56	-31.97
1000	-16.84	-27.48	-39.11
1500	-20.03	-32.02	-43.35
2000	-22.32	-34.88	-46.53
2500	-24.10	-36.93	-49.13
3000	-25.56	-38.53	-51.26
3500	-26.81	-39.84	-53.11
4000	-27.89	-40.97	-54.74
4500	-28.83	-41.96	-56.24
5000	-29.69	-42.77	-57.51

4.3.2 Example Calculation Assuming Free Space Propagation to the Radio Horizon

When the victim receiver is located high above the earth, as with an aircraft receiver, the transmission path to the radio horizon is largely unaffected by the earth (see Figure 4.4). In such cases, the interfering signal power can be estimated by assuming free space propagation to all devices located within the radio horizon. It should be noted that the methodology described below neglects the effects of line-of-sight propagation in the troposphere and that due to diffraction and tropospheric scatter from beyond the radio horizon. The over-the-horizon diffracted and scattered signals will be minimal in most cases of interest.

The areal gain Γ_b is calculated from

$$\Gamma_b = 10\log_{10} \frac{\lambda^2}{8\pi} \int_0^{r_{horiz}} \frac{r\,dr}{(h_r - h_t)^2 + r^2} \quad , \tag{4.17}$$

where h_r is the height of the receiver, h_t is the height of the transmitter, and r_{horiz} is the distance to the radio horizon which can be calculated using the following approximate expression [2]

$$r_{horiz} = \sqrt{2h_r/\gamma_e} + \sqrt{2h_t/\gamma_e} = \sqrt{2a_e h_r} + \sqrt{2a_e h_t} \quad . \tag{4.18}$$

The earth's effective curvature γ_e is the reciprocal of the earth's effective radius a_e and is normally determined from the surface refractivity using the empirical formula [2]

$$\begin{aligned} \gamma_e &= \gamma_a/K \\ a_e &= Ka \\ K &= \left(1 - 0.04665\, e^{N_s/N_1}\right)^{-1} \quad , \end{aligned} \tag{4.19}$$

where K is the *effective earth radius factor*, N_s is the surface refractivity, $N_1 = 179.3$ N-units, and $\gamma_a = 1/a = 157 \times 10^{-9}$ m^{-1} = 157 N-units/km [2].

Evaluating the integral in Equation 4.17 gives

$$\Gamma_b = 10\log_{10}\left(\frac{\lambda^2}{16\pi} \log_e\left(\frac{(h_r - h_t)^2 + r_{horiz}^2}{(h_r - h_t)^2} \right) \right) \quad . \tag{4.20}$$

When the receiver height is much greater than the transmitter height the result can be reduced to

$$\Gamma_b = 32.52 - 20\log_{10} f_{MHz} + 10\log_{10}\log_e\left(1 + \frac{2Ka}{h_r} \right) \quad dB\,m^2 \quad . \tag{4.21}$$

Assuming a standard four thirds earth ($K = 4/3$), Figure 4.5 shows the areal gain as a function of frequency and receiver antenna height as compared with Γ_b calculated using the ITM up to its recommended limit of 1 km. Note that at 1 km, the results are within about 0.5 dB. Using Equation 4.21, Figure 4.6 shows Γ_b for various frequencies as a function of receiver height up to 10 km.

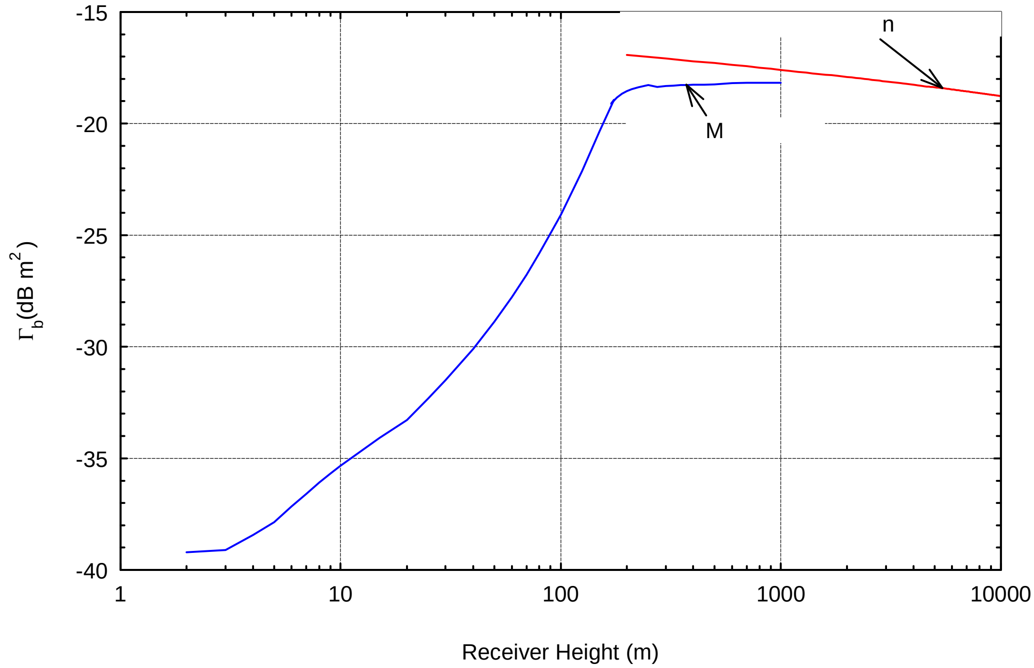

Figure 4.5. Basic areal gain Γ_b as a function of receiver height using ITM compared with a direct calculation assuming free-space propagation. (1000 MHz, Δh=90 m).

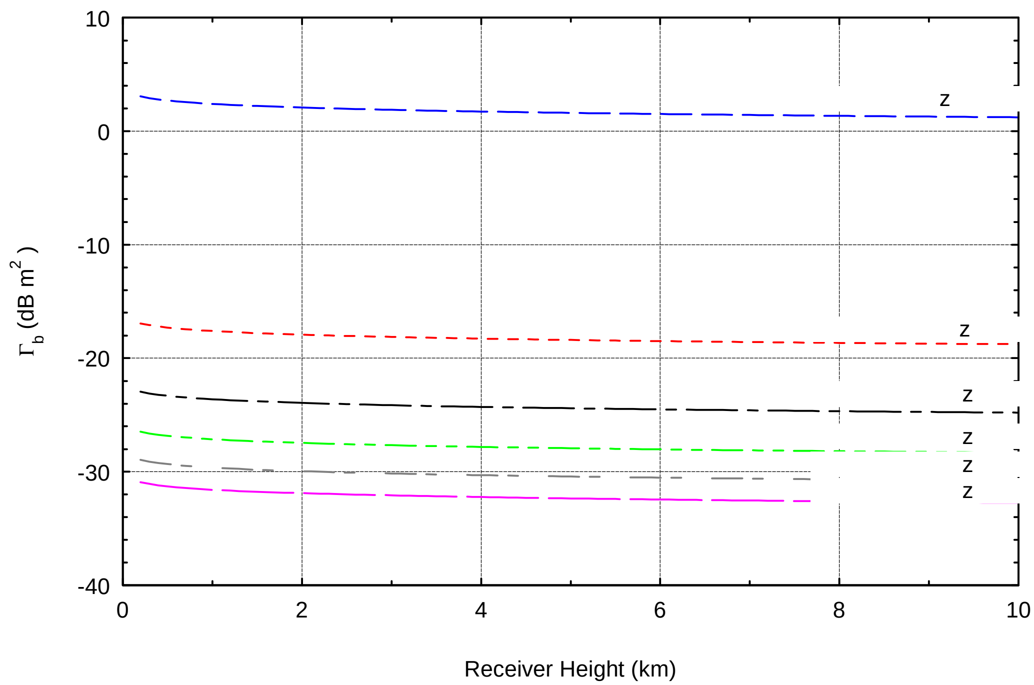

Figure 4.6. Basic areal gain Γ_b as a function of receiver height assuming free-space gain to the radio horizon.

4.4 References

[1] Harald Cramer, *Mathematical Methods of Statistics*, Princeton NJ: Princeton University Press, 1945.

[2] G.A. Hufford, A.G. Longely, and W. Kissick "A guide to the use of the ITS irregular terrain model in the area prediction mode," NTIA Report 82-100, April 1982.

[3] Constantine A. Balains, *Antenna Theory Analysis and Design*, 2nd Edition, New York: John Wiley and Sons, 1997, p 442.