2. DUCTING STRUCTURES

Among the broad array of features which characterize atmospheric layers, specific parameters have been defined that have a particular relevance for radio wave propagation. The most prominent of these would be the radio refractive index structure or its equivalent. For example, the bending of a radio wave's trajectory through the atmosphere results primarily from the spatial variation of the atmospheric refractive index n(h) with elevation h above the surface. However, a more sensitive measure of this n(h) structure is given by the refractivity structure N(h):

$$N(h) = [n(h) - 1]\, 10^6 \quad \text{N units,} \tag{2a}$$

and its vertical gradient

$$\frac{dN}{dh} = 10^6 \frac{dn}{dh} \quad , \quad \text{N units/km.} \tag{2b}$$

Nevertheless, in describing the effects of this structure upon radio propagation, it is often more convenient to describe the gradient in terms of the modified refractivity structure M(h),

$$M(h) = N(h) + 157h \quad \text{M units,} \tag{2c}$$

and its gradient

$$\frac{dM}{dh} = \frac{dN}{dh} + 157 \quad \text{M units/km.} \tag{2d}$$

This modified refractivity results from the geometrical transformation from a spherically stratified atmosphere above a spherical path to a planar stratification above a flattened earth. This is described in Appendix A and will be illustrated in the next section. For an elevated ducting layer ($dM/dh < 0$ M units/km), the critical angle of incidence (upon the layer from below) mentioned in (1a) is given by

$$\theta_c = \sqrt{2|\delta M|} \quad \text{mrad,} \tag{3a}$$

i.e., in milliradians [Gough, 1962]. Here the M-lapse δM is determined from

$$\delta M = \frac{dM}{dh} \cdot \delta h \; 10^{-3} \quad \text{M units,} \tag{3b}$$

for a layer thickness δh in meters and the gradient dM/dh in M units/km. There is,

of course, a corresponding negative critical angle of incidence (upon a layer from above) when the layer gradient of modified refractivity is positive ($dM/dh > 0$ M units/km). Its magnitude is still determined from (3a) and (3b). The value given by (3a) is at the layer base h_o.

2.1 Ducting

For those angles of incidence at the layer base that are equal to or less than the critical value of (3a), the incident wave from below is refracted (bent) by a trapping layer of ($dM/dh < 0$ M units/km) and may propagate efficiently (be ducted) for long distances. This is illustrated by Figure 5. To the left of Figure 5, the vertical refractivity profile N(h) depicts a layer δh meters thick with a gradient $(dN/dh)_o < -157$ N units/km and at a base height of h_o meters above ground. Above and below the layer, there are standard gradients ($dN/dh \simeq -39$ N units/km). The remainder of Figure 5 depicts the layer extending over an effective earth curvature R_e that is 4/3 that of the true earth, i.e., $R_e = 4/3\ R_o$, $R_o = 6370$ km. In this situation, a radio wave launched at the layer base height h_o at (or less than) the critical angle will follow a curved path (as illustrated, from O" to a to O or O" to a' to O') within the layer. A radio wave traveling exterior to the layer will follow a straight path.

Consider a wave trajectory (ray path) launched from point O" in Figure 5 at an initial elevation angle $\theta_o < \theta_c$. Its dashed-line trajectory is strongly refracted so that, after achieving a maximum elevation at a', it returns to the layer base at O'. It continues, emerging from the layer at an elevation angle $\theta = -\theta_o$, relative to the layer tangent at O', to travel in a straight-line trajectory past an elevation minimum at b' to again intercept the layer at its base. There is, of course, a maximum launch angle $\hat{\theta}_o$ at O" that will restrict the trajectory elevation to a maximum coinciding with the layer top at point a. The continuous-line trajectory of Figure 5 illustrates this launch angle $\theta = \hat{\theta}_o$. The minimum elevation of the trajectory occurs on the straight-line at b as the trajectory continues to a re-entry of the layer. The value of $\hat{\theta}_o$ is given by (3a), i.e., $\hat{\theta}_o = \theta_c$.

These two elevation extremes, $h_a = h_o + \delta h$ at the top of the ducting layer and h_b well below the base of the layer, constitute the bounds on wave trajectories for trapped radio waves. The <u>duct</u> width or thickness D exceeds the thickness of the layer δh,

$$D = h_a - h_b = \delta h + [h_o - h_b] \quad . \tag{4}$$

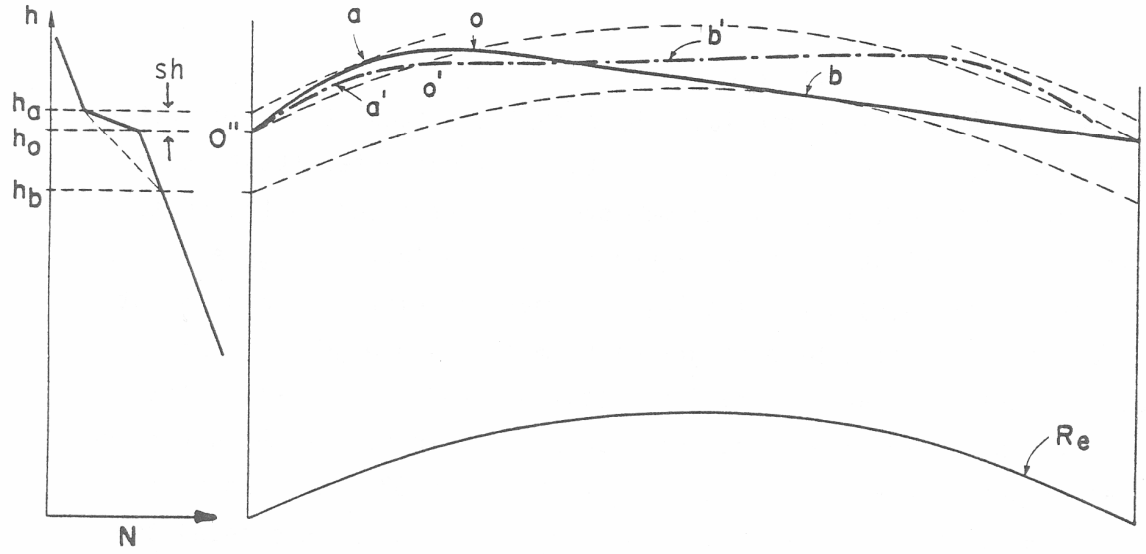

Figure 5. Elevated layer with a ducting gradient, its refractivity profile, and two trapped trajectories. The reference elevation is for an effective earth radius R_e.

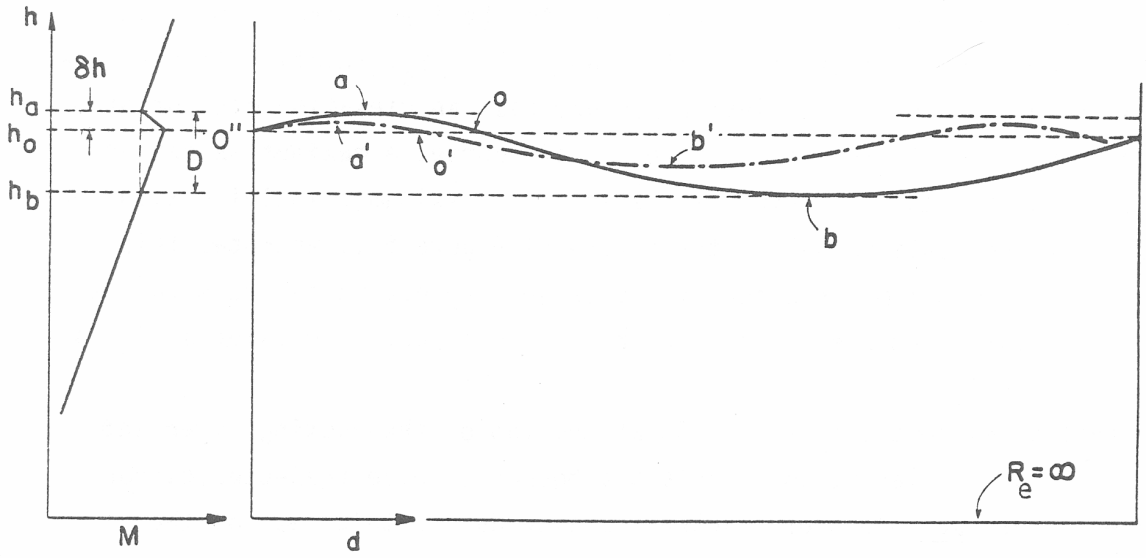

Figure 6. Elevated layer with a ducting gradient, its modified refractivity profile, and two trapped trajectories. The reference elevation is for a flattened earth. D is the duct thickness; δh is the layer thickness.

The trajectories of Figure 5 have been called earth-detached ducting modes (Wait, 1962).

2.2 Duct Width

Figure 6 is a replotting of Figure 5 for the flattened earth. The modified refractivity profile M(h) is shown at the left. Note that the top of the duct at h_a and the bottom of the duct at h_b have the same M value,

$$M(h_a) = M(h_b) \tag{5}$$

The remainder of Figure 6 illustrates the conventional symmetrical trajectories associated with duct propagation. The trajectories are well approximated as parabolic within a layer of constant gradient [Millington, 1957; Dougherty and Hart, 1979]. See Appendix A.

From (3b), (4), and (5), the duct width is given by

$$D = \delta h \left[1 - \left\{ \left(\frac{dM}{dh} \right)_o \Big/ \left(\frac{dM}{dh} \right)_b \right\} \right] \tag{6}$$

where $(dM/dh)_o$ is the (negative) modified refractivity gradient across the ducting layer and $(dM/dh)_b$ is the (positive) modified refractivity gradient below the ducting layer.

2.3 Duct Definitions

In addition to the trajectories illustrated in Figure 6 with positive launch angles, there is also an array of trajectories that could be launched at 0" for negative angles $\theta > -|\theta_c|$. Inspection of Figures 5 and 6 should make it clear that any trapped wave trajectory will cross the duct axis (the optimum elevation h_o) at an angle whose magnitude has the limit $|\theta| \leq |\theta_c|$ relative to the layer tangent at that point. For the situation illustrated in Figures 5 and 6 where $h_b \geq 0$, a trapped trajectory continues as earth-detached. If the ducting layer were lowered ($h_b \leq 0$) and if the earth's surface were sufficiently smooth, a trapped trajectory could strike and be reflected from the surface. Then the lower boundary of the duct would be provided by the smooth surface; the duct would be a ground-based duct, although the layer would still be elevated ($h_o > 0$). Duct propagation then would involve both earth-detached and earth reflected trajectories.

Figure 7 illustrates the three categories of ducts. Note that a ground-based layer ($h_o = 0$) will also constitute a ground-based duct; then, the layer thickness δh equals the duct thickness D and ground reflection is essential to duct propagation.

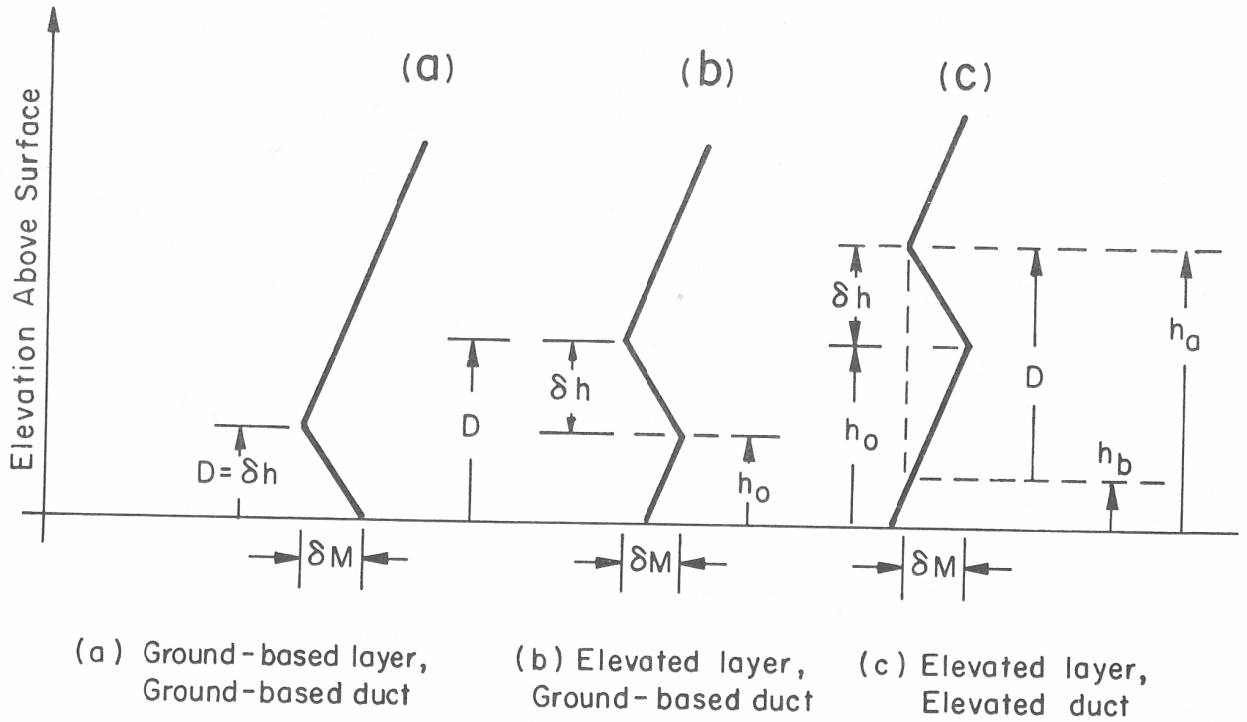

Figure 7. Elevated and surface duct parameters.

If the reflecting surface has a slope, the additional tilt imparted to the reflected trajectory may permit it to escape the duct.

2.4 Duct Frequency Dependence

Early work in duct propagation defined a minimum (cut-off) frequency of propagation by an analogy to waveguide transmission [Kerr, 1951]. However, experimental studies have demonstrated that the "cut-off" effect occurs over a range of frequencies rather than abruptly at a specific frequency. The Wait and Spies (1969) full-wave solution of duct propagation showed that propagation normally occurs with very low losses for earth-detached modes. However, the loss coefficient, α in dB/km of path length, increases rapidly for decreasing frequencies approaching a critical value. A.S. Ratner (private communications) showed that for frequencies greater than trapping value

$$f_t = \frac{1572}{D^{1.8}} \quad \text{GHz} \tag{7}$$

radio energy will propagate with a loss coefficient $\alpha < 0.03$ dB/km. For lower frequencies, the coefficient would be expected to increase rapidly [Neesen and deHaas, 1980]. In (7), the duct thickness D is in meters.

2.5 Propagation Losses for Ducts

Propagation within a duct has long been the subject of theoretical as well as experimental investigation; evaluation of propagation losses dates to Kerr [1951]. Wait has placed the very wide variety of solutions into a common context [Wait, 1962]; they all assume horizontally uniform ducts (ducts whose characteristics, h_o, D, f_t, etc., do not vary horizontally). Since, the full-wave solutions have been extended to piecewise uniform ducts, a significantly closer approach to atmospheric ducts [Bahar and Wait, 1965; Cho and Wait, 1978]. Currently there are widespread efforts to relate the full-wave modal solutions for duct propagation to the ray-trajectory formulation [Cho et al., 1979; Migliora et al., 1980; Ott, 1980]. This could provide the advantage of leading to engineering-type solutions. For the present, however, there are empirical bounds that can be given for the role of duct propagation in interference fields [Dougherty and Hart, 1979].

Commonly for tropospheric radio wave propagation, the free-space field or the free-space basic transmission loss is taken as a reference. That assumes an inverse-square dependence upon the propagation path length, since the field expands in the two dimensions normal to the direction of propagation. However, in duct propagation, because of the trapping constraint on spreading in one (vertical)

dimension, the reference loss within the duct will have only an inverse-distance dependence on distance. However, there is still the loss, $\alpha \leq 0.03$ dB/km. For propagation between two terminals within a duct, i.e., when both terminals are immersed within the duct, the basic transmission loss is

$$L_b \geq 92.45 + 20 \log f + 10 \log d_o + \alpha d_o + \sum A \quad \text{dB}, \tag{8}$$

for the frequency $f > f_t$, in gigahertz and the path length within the duct, d_o in kilometers. For the earth-detached modes illustrated by the wave trajectories of Figures 5 and 6, $\alpha \leq 0.03$ dB/km; for the $h_b \leq 0$ and ground-reflected modes, the appropriate α would be much larger [CCIR, 1978c]. The summation term in (8) is the sum of coupling losses discussed below. Note that the first three terms of (8) differ from the expression for the free-space basic transmission loss

$$L_{bo} = 92.45 + 20 \log f + 20 \log d \quad \text{dB}. \tag{9}$$

only in the coefficient 10 rather than 20 for the third term. For f in megahertz, the first term in (8) and (9) would be the familiar 32.45 [CCIR, 1978d].

For an actual telecommunications system with non-isotropic antennas, the minimum transmission loss would be given by (8) minus the antenna power gains in decibels above an isotropic dBi). Since only that energy would be trapped that lies within the range $-\theta_c \leq \theta \leq \theta_c$, there is for each terminal an antenna-media coupling loss

$$\begin{aligned} A_c &= -10 \log (2|\theta_c|/\Omega) \quad \text{dB}, \quad 2|\theta_c| < \Omega , \\ &= 0.0 \quad \text{dB} \qquad\qquad , \quad 2|\theta_c| > \Omega , \end{aligned} \tag{10}$$

for an antenna lobe's half-power beamwidth of Ω in milliradians directed along the duct.

When the terminal antenna's center of radiation is at h_o (the optimum coupling elevation), θ_c is given by (3a); at other elevations, see Appendix A. There is a similar coupling loss when the antenna's center of radiation is within the same elevation range of the duct but just beyond the duct's horizontal extent. Then the θ_c of (10) would be replaced by D/d_L where D is the duct width of (6) and d_L is the distance from the end of the duct to the terminal beyond (exterior to) the duct. When an antenna is external to and above or below the duct, the antenna-media coupling loss of (10) is replaced by

a), a basic transmission loss given by (9) for that portion of the path length that is exterior to the duct,