APPENDIX A: ANTENNA WITHIN THE SLAB

In this appendix, we consider the case where the vertical electric dipole source is located within the slab. The geometry is shown in Figure 1, but the antenna height h is now negative: $0 > h > -D$.

Following the notation in Section 2.1, the fields in air ($z > 0$) can be derived from the z component, Π_{oz}, of an electric Hertz vector of the following form:

$$\Pi_{oz} = \frac{I\ ds}{4\pi i\omega\varepsilon_o} \int_o^\infty T_o(\lambda)\ e^{-u_o z} \frac{\lambda}{u_o} J_o(\lambda\rho)\ d\lambda \quad , \tag{A-1}$$

where $T_o(\lambda)$ is an unknown transmission coefficient and all other symbols are defined in Section 2.1. The three nonzero field components, $E_{o\rho}$, E_{oz}, and $H_{o\phi}$, are given by (3).

In the ground ($z < -D$), the z component, Π_{gz}, of the Hertz vector can be written

$$\Pi_{gz} = \frac{I\ ds}{4\pi i\omega\varepsilon_o\varepsilon_{gc}} \int_o^\infty T_g(\lambda)\ e^{u(z+D)} \frac{\lambda}{u} J_o(\lambda\rho)\ d\lambda \quad , \tag{A-2}$$

where $T_g(\lambda)$ is an unknown transmission coefficient and all other symbols are defined in Section 2.1. The three nonzero field components, $E_{g\rho}$, E_{gz}, and $H_{g\phi}$, are given by

$$E_{g\rho} = \frac{\partial^2 \Pi_{gz}}{\partial\rho\ \sigma z} \quad , \quad E_{gz} = \left(-\gamma_g^2 + \frac{\partial^2}{\partial\ z^2}\right)\Pi_{gz} \quad , \tag{A-3}$$

and

$$H_{g\phi} = -i\omega\varepsilon_o\varepsilon_{gc} \frac{\partial\ \Pi_{gz}}{\partial\rho} \quad .$$

Within the anisotropic slab ($-D < z < o$), it is most convenient to work directly with the azimuthal magnetic field, H_ϕ. Clemmow (1966) has given the expression for the field in an infinite medium which we can generalize to the appropriate form within a slab:

$$H_\phi = \frac{I\ ds\ \kappa}{4\pi} \int_o^\infty J_1(\lambda\rho)\left[e^{-v|z-h|} + A(\lambda)\ e^{vz} + B(\lambda)\ e^{-vz}\right]\left(\frac{\lambda^2}{v}\right) d\lambda \quad , \tag{A-4}$$

where $A(\lambda)$ and $B(\lambda)$ are unknown coefficients and all other symbols are defined in Section 2.1. From Maxwell's curl equation, the two nonzero components of the electric field, E_ρ and E_z, can be written

$$E_\rho = \frac{-1}{i\omega\varepsilon_o\varepsilon_{hc}} \frac{\partial H_\phi}{\partial z}$$

and

$$E_z = \frac{\kappa}{i\omega\varepsilon_o\varepsilon_{hc}\rho} \frac{\partial}{\partial\rho} (\rho\ H_\phi) \quad . \tag{A-5}$$

The four unknowns, $T_o(\lambda)$, $T_g(\lambda)$, $A(\lambda)$, and $B(\lambda)$, can be determined from the continuity of tangential electric and magnetic fields at the interfaces:

$$(E_{o\rho} - E_\rho)\Big|_{z=0} = 0 \quad ,$$

$$(H_{o\phi} - H_\phi)\Big|_{z=0} = 0 \quad ,$$

$$(E_\rho - E_{g\rho})\Big|_{z=-D} = 0 \quad ,$$

and

$$(H_\phi - H_{g\phi})\Big|_{z=-D} = 0 \quad . \tag{A-6}$$

By substituting the field expressions into (A6) and performing some algebra, we obtain the following expressions for the unknowns:

$$A(\lambda) = R_o\ e^{vh} \left[1 + R_g\ e^{-v(2D+2h)}\right] /S \quad ,$$

$$B(\lambda) = R_g\ e^{-v(2D+h)} \left[1 + R_o\ e^{2vh}\right] /S \quad ,$$

$$T_o(\lambda) = \frac{\kappa\ u_o(1+R_o)\ e^{vh}}{v\ S} \left[1 + R_g\ e^{-v(2D+2h)}\right] \quad , \tag{A-7}$$

$$T_g(\lambda) = \frac{\kappa\ u\ e^{-v(D+h)}}{v\ S} \left[1 + R_o\ e^{2vh}\right] \quad ,$$

where $S = 1 - R_oR_g\ e^{-2vD}$,

$$R_o = \frac{K_1 - K_o}{K_1 + K_o} \quad , \quad R_g = \frac{K_1 - K_2}{K_1 + K_2} \quad ,$$

and all other symbols are defined in Section 2.1. This completes the formal integral representation of the fields in all three regions.

The integral forms for the fields can be evaluated asymptotically when the horizontal distance ρ is large. We are primarily interested in the vertical electric field, and from (A-1) and (3), the vertical electric field in air ($z > 0$) can be written

$$E_{oz} = \frac{I\, ds}{4\pi i\omega\varepsilon_o} \int_0^\infty T_o(\lambda)\, e^{-u_o z} \left(\frac{\lambda^3}{u_o}\right) J_o(\lambda\rho)\, d\lambda \quad . \tag{A-8}$$

The Bessel function in (A-8) can be replaced by Hankel functions (Abramowitz and Stegun, 1964), and the integration range in (A-8) can be extended to $-\infty$ to yield the following form (Wait, 1967b):

$$E_{oz} = \frac{I\, ds}{4\pi i\omega\varepsilon_o} \int_{-\infty}^{\infty} \frac{T_o(\lambda)}{2}\, e^{-u_o z} \left(\frac{\lambda^3}{u_o}\right) H_o^{(2)}(\lambda\rho)\, d\lambda \quad . \tag{A-9}$$

We follow the method which Wait (1967b) used for a dipole above an isotropic dielectric slab. The integral in (A-9) can be deformed around the branch cuts which result from branch points at k and $k_g(=-i\,\gamma_g)$ as shown in Figure A.1. In addition, there may be one or more surface wave poles at $\lambda=k_s$ which result from poles of $T_o(\lambda)$ (= zeros of S) as given by (A-6). The contributions from the contour C_g and any surface wave poles are exponentially attenuated for large ρ, and we keep only the branch cut integral C_o:

$$E_{oz} \simeq \frac{I\, ds}{4\pi j\omega\varepsilon_o} \int_{C_o} \frac{T_o(\lambda)}{2}\, e^{-u_o z}\, \frac{\lambda^3}{u_o}\, H_o^{(2)}(\lambda\rho)\, d\lambda \quad . \tag{A-10}$$

In order to evaluate (A-10), we first rewrite $T_o(\lambda)$ in a form which is equivalent to that in (A-7):

$$T_o(\lambda) = \frac{2\, K_o}{K_o + Z_1} \quad \frac{e^{vh}[1 + R_g\, e^{-v(2D+2h)}]}{\varepsilon_{vc}[1+R_g\, e^{-2vD}]} \quad . \tag{A-11}$$

Near $\lambda=k$, (A-11) can be approximated

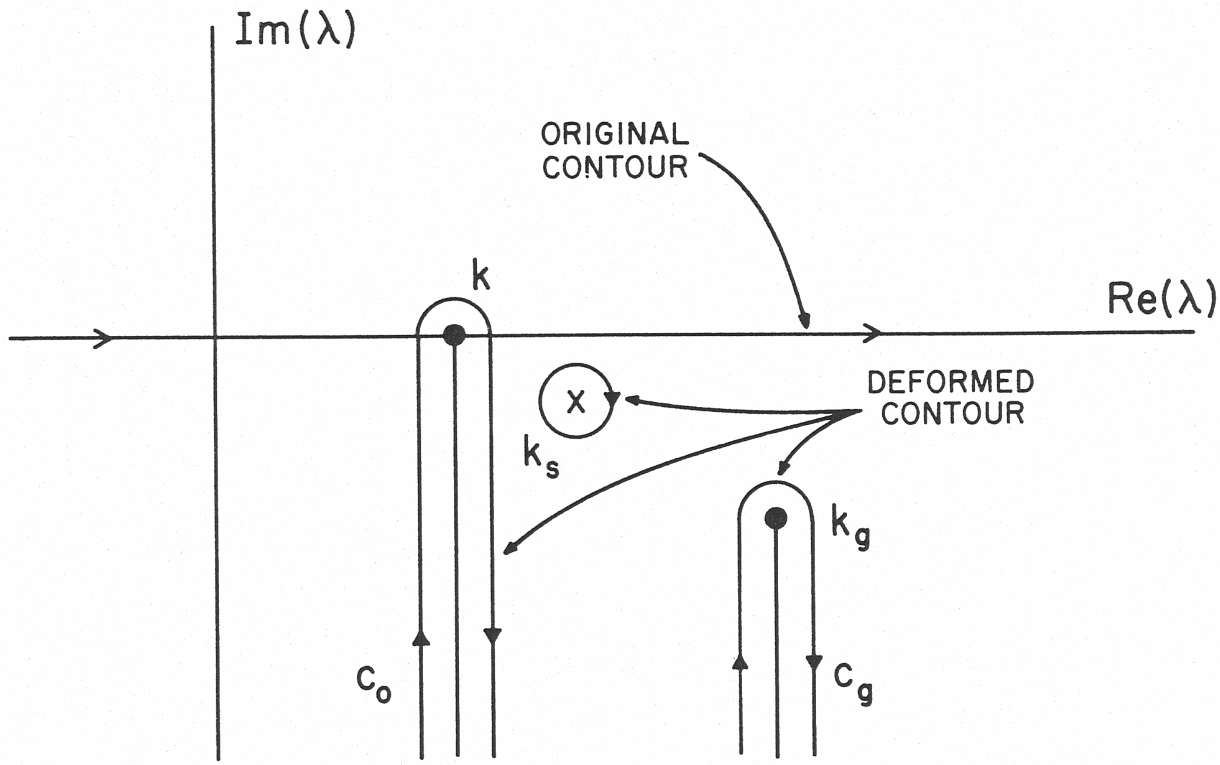

Figure A.1. Integration contours in the complex λ plane.

$$\frac{T_o(\lambda)}{2} \simeq \frac{1}{\varepsilon_{vc}} \left. \frac{e^{vh}[1+R_g\; e^{-v(2D+2h)}]}{1 + R_g\; e^{-2vD}} \right|_{\lambda=k} \cdot \frac{K_o}{K_o + Z_1} \quad . \tag{A-12}$$

The first factor in (A-12) is identically equal to $G_s(h)$ as given by (13). Thus, (A-12) can be written

$$\frac{T_o(\lambda)}{2} \simeq \frac{K_o}{K_o + Z_1} G_s(h) \quad . \tag{A-13}$$

Substituting (A-13) into (A-10) and approximating λ^2 by k^2, we have

$$E_{oz} \simeq \frac{I\; ds\; k^2\; G_s(h)}{4\pi i\omega\varepsilon_o} \int_{C_o} \frac{K_o\; e^{-u_o z}}{K_o + Z_1} \frac{\lambda}{u_o} H_o^{(2)}\; (\lambda\rho)\; d\lambda \quad . \tag{A-14}$$

Wait (1967b) has evaluated the integral over C_o:

$$\int_{C_o} \simeq \frac{1 + ik\Delta z}{ik_o\Delta^2\rho^2} e^{-ik\rho} \quad . \tag{A-15}$$

Substituting (A-15) into (A-14), we can write E_{oz} in the final form

$$E_{oz} = E_o\; f(p)\; G_s(h)\; G_o(z) \quad , \tag{A-16}$$

where E_o is given by (5), $f(p)$ is given by (6), and G_s and G_o are given by (13). This form is consistent with the result in (4),, but disagrees with the result for the dipole in the slab given by Wait (1967a). He has an extra factor of $\kappa^{\frac{1}{2}}$ multiplying $G_s(h)$ which leads to a height-gain function which is different for the source and observer in the slab. More recently, Wait (private communication) has agreed that the factor $\kappa^{\frac{1}{2}}$ is spurious and that (A-16) is the correct result. Thus the height-gain function in (13) applies to both the source and observer, and reciprocity is satisfied.

For the case of both the dipole and the observer located within the slab, E_z is obtained from (A-4) and (A-5):

$$E_z = \frac{I\ ds}{4\pi i\omega\varepsilon_o\varepsilon_{vc}} \int_0^\infty J_o(\lambda\rho) \left[e^{-v|z-h|} + A(\lambda)\ e^{vz} + B(\lambda)\ e^{-vz}\right] \frac{\lambda^3}{v} d\lambda \qquad \text{(A-17)}$$

$$= \frac{I\ ds}{\pi i\omega\varepsilon_o\varepsilon_{vc}} \int_{-\infty}^\infty H_o^{(2)}(\lambda\rho) \left[e^{-v|z-h|} + A(\lambda)\ e^{vz} + B(\lambda)\ e^{-vz}\right] \frac{\lambda^3}{v} d\lambda \quad .$$

For large ρ, we again include only the C_o contour in Figure 27. Also, the first factor in (A-17) does not contribute, and E_z is approximately given by

$$E_z \simeq \frac{I\ ds}{8\pi i\omega\varepsilon_o\varepsilon_{vc}} \int_{C_o} H_o^{(2)}(\lambda\rho) \left[A(\lambda)\ e^{vz} + B(\lambda)\ e^{-vz}\right] \frac{\lambda^3}{v} d\lambda \quad . \qquad \text{(A-18)}$$

By expanding $A(\lambda)$ and $B(\lambda)$ about $\lambda=k$, (A-18) can be put into a form similar to (A-14):

$$E_z \simeq \frac{I\ ds\ k^2\ G_s(h)\ G_s(z)}{4\pi i\omega\varepsilon_o} \int_{C_o} \frac{K_o}{K_o + Z_1} \frac{\lambda}{u_o} H_o^{(2)}(\lambda\rho)\ d\lambda \quad . \qquad \text{(A-19)}$$

In this case, the integral over C_o is given by

$$\int_{C_o} \simeq \frac{e^{-ik\rho}}{i\ k_o\ \Delta^2\rho^2} \quad . \qquad \text{(A-20)}$$

From (A-19) and (A-20), E_z can be written

$$E_z \simeq E_o\ f(p)\ G_s(h)\ G_s(z) \quad .$$

As expected, the same height-gain function G_s appears for both the dipole height h and observer height z since both are located in the slab.

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