The integral equation approach that is described in this report is a point-to-point prediction method and should be most useful for frequencies below 30 MHz. For higher frequencies, the variability of the ground wave in time and space becomes large, and an accurate point-to-point prediction becomes difficult. A recent report (Hufford et al., 1982) describes an area prediction model for VHF, UHF, and SHF frequencies.

The organization of this report is as follows: In Section 2, the fields of a vertical electric dipole are analyzed for the dipole either within or above a uniform slab. The effective surface impedance and the height-gain functions are derived. In Section 3, Ott's previous integral equation (1971a) is extended to include a slab above the earth by using surface impedance and height-gain approximations from Section 2. In Section 4, a two-section path is analyzed both by Kirchhoff Theory and by the integral equation method. The analytical result from the Kirchhoff Theory is useful in its own right and also provides a good check for the revised integral equation. In Section 5, the values for the equivalent slab parameters for forests, built-up areas, and snow are discussed. In Section 6, specific path calculations are performed using the integral equation approach. Section 7 includes a summary and recommendations for further work. In Appendix A, both the integral representation and the asymptotic result are derived for the case where the dipole is located within the anisotropic slab. Appendix B includes the rather involved mathematical details of the Kirchhoff integration which is required for the two-section path analysis in Section 4. Finally, Appendix C provides a user's guide, a listing, and a sample output for program WAGSLAB which is essentially an extension of Ott's program WAGNER.

Throughout the report, we present propagation results in terms of a normalized attenuation function f which is the ratio of the actual electric field to twice the free-space field. This is done in order to emphasize propagation effects and to eliminate antenna effects. Thus f is unity for a flat, perfectly conducting ground. If some form of transmission loss is desired, it can be easily computed from the free space transmission loss and the magnitude of f.

2. UNIFORM SLAB MODEL

In this section, we analyze a uniform, anisotropic slab model for a vertical electric dipole source. The asymptotic results for the uniform slab model (Wait, 1967a) can be cast in a very convenient form where the total field can be factored into a product of twice the free space field times a ground-wave attenuation function times the height-gain functions for the source and observer heights. The attenuation function depends on the surface impedance for a layered medium in the same manner that the flat-earth attenuation function for a homogeneous earth (Wait, 1962) depends on the surface impedance of a homogeneous half-space. We utilize the specific results for the surface impedance and height-gain functions of the uniform slab in Section 3 where the slab parameters are allowed to change along the path.

Although we are primarily interested in the uniform slab model as a starting point for the more general irregular terrain model of Section 3, the uniform slab model is of interest in its own right for modeling uniformly forested paths (Tamir, 1967; Dence and Tamir, 1969; and Gordon and Hoyt, 1982). In this report, we also use a slab to model built-up urban areas or snow accumulation. The specific slab parameters appropriate for forests, urban areas, and snow are discussed in Section 5.

2.1 Integral Representation

We consider first a vertical electric dipole located at a height h over a uniaxial anisotropic slab of thickness D as shown in Figure 1. The dipole has a current moment I ds, and the exp(iωt) time dependence is suppressed. The slab has horizontal and vertical relative permittivities, ε_h and ε_v, respectively, and horizontal and vertical conductivities, σ_h and σ_v, respectively. The region, $z > 0$, is assumed to be free space with permittivity ε_o and zero conductivity. The region, $z < -D$, is ground with relative permittivity ε_g and conductivity σ_g. We assume free space permeability μ_o everywhere. In the following treatment it is convenient to use the following expressions for the various relative complex dielectric constants

$$\varepsilon_{gc} = \varepsilon_g + \sigma_g/(i\omega\, \varepsilon_o) \quad ,$$

$$\varepsilon_{hc} = \varepsilon_h + \sigma_h/(i\omega\, \varepsilon_o) \quad , \tag{1}$$

and

$$\varepsilon_{vc} = \varepsilon_v + \sigma_v/(i\omega\, \varepsilon_o) \quad .$$

The field in air (z>0) can be derived from the z component, Π_{oz}, of an electric Hertz vector which has the following integral form (Wait, 1967a):

$$\Pi_{oz} = \frac{I\ ds}{4\pi i\omega\, \varepsilon_o} \int_0^\infty \left[e^{-u_o|z-h|} + R_{||}(\lambda)\ e^{-u_o(z+h)} \right] \frac{\lambda}{u_o} J_o(\lambda\rho)\ d\lambda \quad , \tag{2}$$

where

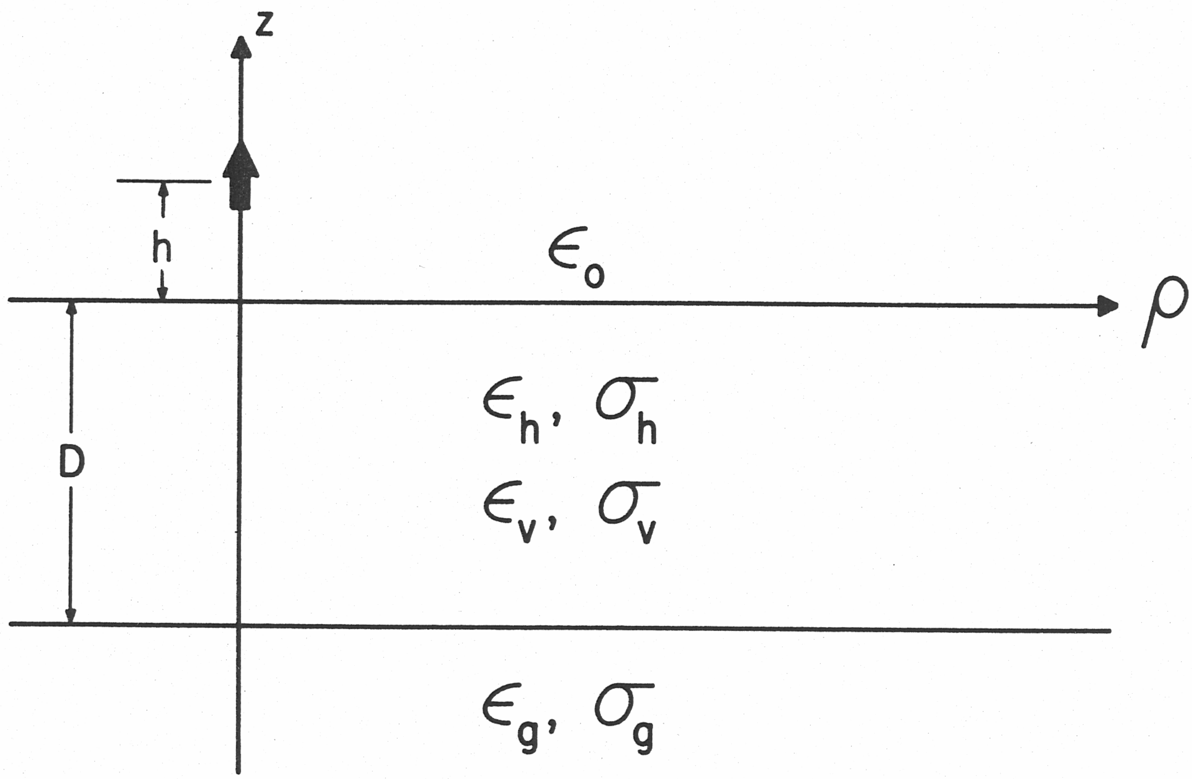

Figure 1. Vertical electric dipole over a uniaxial anisotropic slab.

$$R_{||}(\lambda) = \frac{K_o - Z_1}{K_o + Z_1} \quad , \quad K_o = \frac{u_o}{i\omega\ \varepsilon_o} \quad ,$$

$$u_o = (\lambda^2 - k^2)^{\frac{1}{2}} \quad , \quad k = \omega/c \quad ,$$

$$Z_1 = K_1 \frac{K_2 + K_1 \tanh(vD)}{K_1 + K_2 \tanh(vD)} \quad ,$$

$$K_1 = v/(i\omega\ \varepsilon_o\varepsilon_{hc}) \quad , \quad K_2 = u/(i\omega\ \varepsilon_o\varepsilon_{gc}) \quad ,$$

$$v = (\lambda^2\kappa + \gamma^2)^{\frac{1}{2}} \quad , \quad \kappa = \varepsilon_{hc}/\varepsilon_{vc} \quad ,$$

$$\gamma^2 = -\omega^2\mu_o\ \varepsilon_o\varepsilon_{hc} \quad , \quad u = (\lambda^2 + \gamma_g{}^2)^{\frac{1}{2}} \quad ,$$

and

$$\gamma_g^2 = -\omega^2\mu_o\ \varepsilon_o\varepsilon_{gc} \quad .$$

Cylindrical coordinates (ρ,ϕ,z) are used. The electric and magnetic fields in air $(z>0)$ are given by

$$E_{o\rho} = \frac{\partial^2\Pi_{oz}}{\partial\rho\partial z} \quad , \quad E_{oz} = (k^2 + \frac{\partial^2}{\partial z^2})\Pi_{oz} \quad ,$$

$$H_{o\phi} = -i\omega\ \varepsilon_o \frac{\partial\Pi_{oz}}{\partial\rho} \quad , \tag{3}$$

and

$$E_{o\phi} = H_{o\rho} = H_{oz} = 0 \quad .$$

An integral expression for the fields in the slab $(0 > z > -D)$ can be derived and is similar in form to (2). The fields in the ground $(z < -D)$ are not normally of interest. Expressions for the fields both in air and within the slab when the dipole source is located within the slab are derived in Appendix A.

2.2 The Lateral Wave

In general an evaluation of the fields involves a numerical integration as indicated by (2). However, when $k\rho$ is large and $|z|$ is not large, integrals of the type in (2) can be evaluated asymptotically. The result for the dipole in air ($h>0$) was given by Wait (1967a), and the result for the dipole in the slab ($0 > h > -D$) is derived in Appendix A. In either case, the vertical electric field E_z either in air or in the slab can be cast into the following convenient form

$$E_z \simeq E_o \; f(p) \; G(h) \; G(z) \quad . \tag{4}$$

The result in (4) has the following interpretation. E_o is twice the field of a dipole in free space or by image theory is the field when both the dipole and the observer are located on a flat, perfect conductor:

$$E_o = \frac{-i\omega\mu_o \; I \; ds}{2\pi\rho} \quad . \tag{5}$$

The Sommerfeld attenuation function $f(p)$ accounts for the imperfect conductivity of the ground and the presence of the slab. For a perfectly conducting ground, $p=0$ and $f(0)=1$. In our case, the magnitude of the numerical distance p is large, and $f(p)$ is given by (Wait, 1962)

$$f(p) \simeq -1/(2p) \quad , \tag{6}$$

where

$$p = -ik\rho\Delta^2/2 \quad .$$

The normalized surface impedance Δ will be defined and discussed in the following section. Note that $E_o \; f(p)$ has the expected ρ^{-2} variation which is associated with the "lateral wave" (Tamir, 1967). The point we wish to make here is that the "ground wave" has the same ρ^{-2} variation when $|p|$ is large. The "lateral wave" and the "ground wave" are simply different names for the dominant part of the total field near the ground when the horizontal distance is large. The height gain product, $G(h) \; G(z)$, in (4) is independent of ρ and will be discussed in Section 2.4.

2.3 Normalized Surface Impedance

We define the surface impedance as the ratio of the tangential electric and magnetic fields at the upper surface of the slab ($z=0$). From (2) and (3), we derive the following

$$\left.\frac{-E_{o\rho}}{H_{o\phi}}\right|_{z=0} = Z_1 \tag{7}$$

where Z_1 is given by (2). The surface impedance Z_1 has units of ohms, and from (2) it can be seen that it depends on the horizontal wavenumber λ.

As indicated in Appendix A, the main contribution to the integral comes from the region $\lambda \simeq k$ when ρ is large and the source and observer heights are small. This is equivalent to saying that we can use the surface impedance value for a plane wave at grazing incidence ($\lambda \simeq k$) in ground wave propagation. In addition it is convenient to normalize the surface impedance by dividing by the impedance of free space η_o ($=\sqrt{\mu_o/\varepsilon_o}$). Thus, we define the normalized surface impedance Δ as

$$\Delta = Z_1/\eta_o\Big|_{\lambda=k} \quad . \tag{8}$$

From (2) we can write Δ in the following form

$$\Delta = \Delta_1 \frac{\Delta_2 + \Delta_1 \tanh(v_o D)}{\Delta_1 + \Delta_2 \tanh(v_o D)} \quad , \tag{9}$$

where

$$\Delta_1 = \sqrt{\varepsilon_{hc} - \kappa}\,/\varepsilon_{hc} \quad ,$$

$$\Delta_2 = \sqrt{\varepsilon_{gc} - 1}\,/\varepsilon_{gc} \quad ,$$

and

$$v_o = ik\sqrt{\varepsilon_{hc} - \kappa} \quad .$$

Although (9) is derived for the case of a uniform slab, we use the same expression for Δ in Section 3 where the slab parameters are allowed to vary along the path.

There are a number of limiting cases which provide some insight into the dependence of Δ on the various parameters. If the parameters of the slab approach those of the ground ($\varepsilon_{hc} = \varepsilon_{vc} = \varepsilon_{gc}$), then

$$\Delta = \Delta_2 = \sqrt{\varepsilon_{gc} - 1}\,/\varepsilon_{gc} \quad . \tag{10}$$

The same limit (10) is obtained by letting D approach zero. In either case we are back to a homogeneous half-space, and (10) is the known result for a homogeneous half-space.

Another interesting limit is to let the slab thickness D approach infinity. Then if v_o is complex, tanh $(v_o D)$ approaches unity and Δ simplifies:

$$\Delta\Big|_{D=\infty} = \Delta_1 \quad . \tag{11}$$

This is simply the surface impedance for an anisotropic half-space.

In many cases, the slab parameters are close to those of free space. So the limit of ε_{hc} and ε_{vc} approaching unity is also of interest. In this case, the following result is obtained

$$\Delta\Big|_{\varepsilon_{hc}=\varepsilon_{vc}=1} = \Delta_2/(1+ikD\Delta_2) \quad . \tag{12}$$

This limit will be found useful in Section 2.5.

2.4 Height-Gain Functions

The height-gain function G(z) which appears in (4) takes on different forms depending on whether z is positive or negative (Wait, 1967a):

$$G(z) = \begin{cases} G_o(z) & , \quad z \geq 0 \\ G_s(z) & , \quad 0 > z > -D \end{cases} \tag{13}$$

where

$$G_o(z) = 1 + ikz\Delta \quad ,$$

$$G_s(z) = \frac{1}{\varepsilon_{vc}} \frac{e^{v_o z} + R\, e^{-v_o(2D+z)}}{1 + R\, e^{-2v_o D}} \quad ,$$

$$R = \frac{\Delta_1 - \Delta_2}{\Delta_1 + \Delta_2} \quad ,$$

and v_o, Δ_1, and Δ_2 are given by (9). As shown in Appendix A, the same height-gain function applies to both the source and observer locations. This is simply a consequence of reciprocity since even the anisotropic slab is a reciprocal medium (Monteath, 1973).

The limit of $G_o(z)$ as z approaches zero is unity

$$G_o(0) = 1 \quad . \tag{14}$$

However, the limit of $G_s(z)$ as z approaches zero is

$$G_s(0) = 1/\varepsilon_{vc} \quad . \tag{15}$$

This is a consequence of the continuity of normal current density at z=0.

2.5 Limit of Vanishing Slab

In the case of thin forest (Dence and Tamir, 1969), the slab parameters differ only slightly from free space. Thus the limiting case when the slab actually vanishes ($\varepsilon_{hc} = \varepsilon_{vc} = 1$) is of some practical interest as well as being a good check on the asymptotic solution given by (4). By physical reasoning we should recover the result for a dipole at a height D+h and an observer at a height D+z over homogeneous ground when ε_{hc} and ε_{vc} are both equal to unity.

When both h and z are positive, (4) becomes

$$E_z \simeq E_o \left(\frac{-1}{2p}\right) G_o(h)\ G_o(z) \quad . \tag{16}$$

For the case where ε_{hc} and ε_{vc} are unity, Δ reduces to the value in (12). E_o is not a function of ε_{hc} and ε_{vc}. From (6) and (12), the second factor in (16) reduces to

$$\left(\frac{-1}{2p}\right)\Bigg|_{\varepsilon_{hc}=\varepsilon_{vc}=1} = \frac{-(1+ikD\Delta_2)^2}{2\ p_2} \quad , \tag{17}$$

where $p_2 = -ik\rho\Delta_2{}^2/2$.

From (12) and (13), $G_o(h)$ reduces to

$$G_o(h)\Bigg|_{\varepsilon_{hc}=\varepsilon_{vc}=1} = 1 + \frac{ikh\Delta_2}{(1+ikD\Delta_2)} \quad . \tag{18}$$

By substituting (17) and (18) into (16), the following result is obtained

$$E_z\Bigg|_{\varepsilon_{hc}=\varepsilon_{vc}=1} = E_o \left(\frac{-1}{2\ p_2}\right) \cdot \Big[1+ik\Delta_2(D+h)\Big] \cdot \Big[1+ik\Delta_2(D+z)\Big] \quad . \tag{19}$$

The second factor in (19) is the attenuation function for a homogeneous earth. The third and fourth factors are the height-gain factors for the source and observer respectively over a homogeneous earth. Thus the slab result reduces to the simpler half-space result as illustrated in Figure 2.

3. INTEGRAL EQUATION APPROACH

For realistic propagation paths, the parameters will vary as a function of position along the path. Analytical methods are not general enough to handle such variations, but the integral equation approach (Hufford, 1952; Ott and Berry, 1970) has been found useful for ground wave propagation over irregular, inhomogeneous terrain. Ott's program WAGNER (Ott, 1971a and 1971b; Ott et al., 1979) has been thoroughly tested and used on a wide variety of propagation paths. In this section we extend Ott's integral equation method to include the presence of a lossy slab over homogeneous earth. As in Ott's method, the properties of the earth (and also the slab) are allowed to change along the path.

3.1 Formulation

A good description of the latest version of program WAGNER and the integral equation on which it is based is given in Ott et al. (1979). The integral equation utilizes the impedance boundary condition which we can generalize from the half-space form to the slab form. The geometry is shown in Figure 3, and the slab and ground parameters are allowed to vary with x but are independent of y.

We consider first the case where both the source and receiver heights are zero. We define the attenuation function f(x) as the ratio of the vertical electric field to E_o, twice the free-space field. The integral equation for f(x) is (Ott et al., 1979):

$$f(x) = W(x,0) - \sqrt{\frac{ik}{2\pi}} \int_0^x f(\xi)\, e^{-ik\phi(x,\xi)} \left\{ y'(\xi)\, W(x,\xi) - \frac{y(x)-y(\xi)}{x-\xi} + (\Delta(\xi)-\Delta_a)\, W(x,\xi) \right\} \left[\frac{x}{\xi(x-\xi)}\right]^{\frac{1}{2}} d\xi \quad ,(20)$$

where

$$\phi(x,\xi) = \frac{[y(x)-y(\xi)]^2}{2(x-\xi)} + \frac{y^2(\xi)}{2\xi} - \frac{y^2(x)}{2x} \quad ,$$

$$W(x,\xi) = 1 - i\sqrt{\pi p}\, w(-\sqrt{u}) \quad ,$$

$$p = -ik\Delta^2(\xi)\,(x - \xi)/2 \quad ,$$