respectively over a homogeneous earth. Thus the slab result reduces to the simpler half-space result as illustrated in Figure 2.

3. INTEGRAL EQUATION APPROACH

For realistic propagation paths, the parameters will vary as a function of position along the path. Analytical methods are not general enough to handle such variations, but the integral equation approach (Hufford, 1952; Ott and Berry, 1970) has been found useful for ground wave propagation over irregular, inhomogeneous terrain. Ott's program WAGNER (Ott, 1971a and 1971b; Ott et al., 1979) has been thoroughly tested and used on a wide variety of propagation paths. In this section we extend Ott's integral equation method to include the presence of a lossy slab over homogeneous earth. As in Ott's method, the properties of the earth (and also the slab) are allowed to change along the path.

3.1 Formulation

A good description of the latest version of program WAGNER and the integral equation on which it is based is given in Ott et al. (1979). The integral equation utilizes the impedance boundary condition which we can generalize from the half-space form to the slab form. The geometry is shown in Figure 3, and the slab and ground parameters are allowed to vary with x but are independent of y.

We consider first the case where both the source and receiver heights are zero. We define the attenuation function f(x) as the ratio of the vertical electric field to E_o, twice the free-space field. The integral equation for f(x) is (Ott et al., 1979):

$$f(x) = W(x,0) - \sqrt{\frac{ik}{2\pi}} \int_0^x f(\xi)\, e^{-ik\phi(x,\xi)} \left\{ y'(\xi)\, W(x,\xi) - \frac{y(x)-y(\xi)}{x-\xi} + (\Delta(\xi)-\Delta_a)\, W(x,\xi) \right\} \left[\frac{x}{\xi(x-\xi)}\right]^{\frac{1}{2}} d\xi \quad ,(20)$$

where

$$\phi(x,\xi) = \frac{[y(x)-y(\xi)]^2}{2(x-\xi)} + \frac{y^2(\xi)}{2\xi} - \frac{y^2(x)}{2x} \quad ,$$

$$W(x,\xi) = 1 - i\sqrt{\pi p}\; w(-\sqrt{u}) \quad ,$$

$$p = -ik\Delta^2(\xi)\ (x - \xi)/2 \quad ,$$

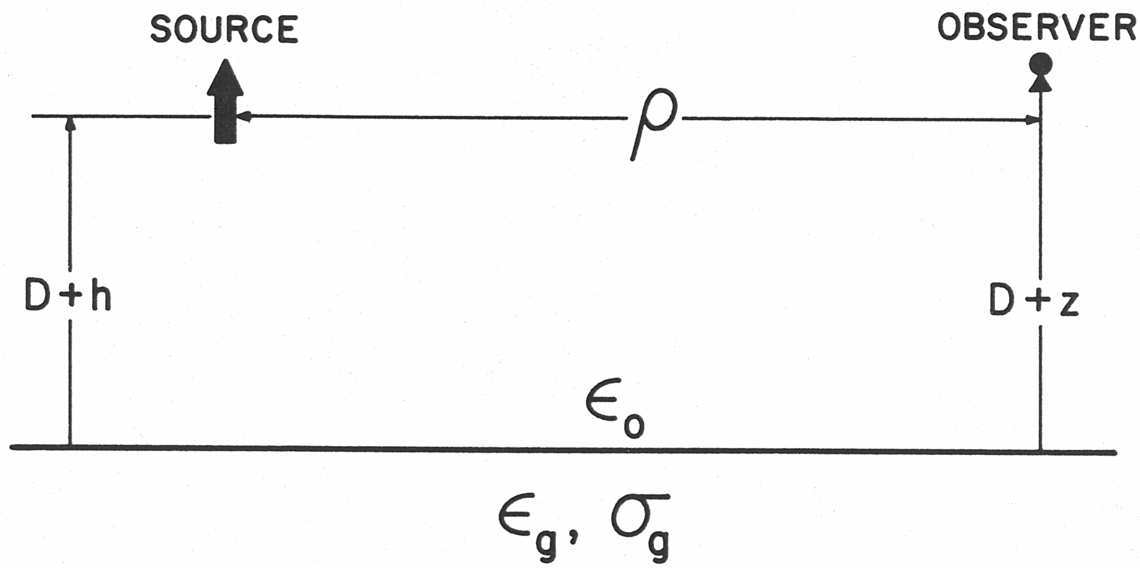

Figure 2. Equivalent geometry for the limit of a free-space slab ($\varepsilon_{hc} = \varepsilon_{vc} = 1$).

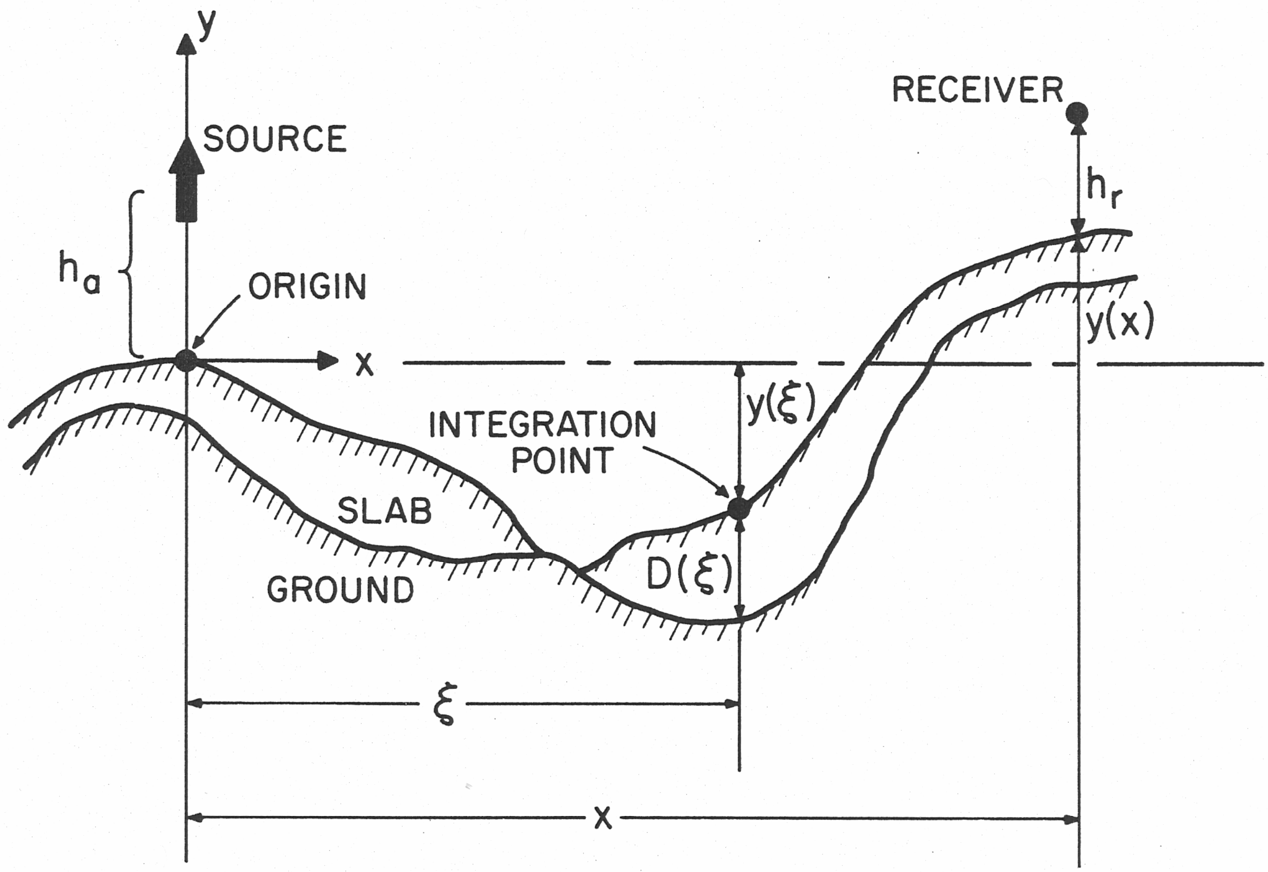

Figure 3. Geometry for integral equation. The slab and ground parameters can vary as a function of x, but are constant in y.

$$u = p\left[1 - \frac{y(x)-y(\xi)}{\Delta(x-\xi)}\right]^2 ,$$

$$w(-\sqrt{u}) = e^{-u}\,\mathrm{erfc}(i\sqrt{u}) = \frac{1}{i\pi}\int_{-\infty}^{\infty}\frac{e^{-t^2}\,dt}{\sqrt{u}+t} ,$$

and the properties of w are discussed by Abramowitz and Stegun (1964). The quantities x, ξ, $y(x)$, and $y(\xi)$ are defined in Figure 3, and $y'(\xi)$ is the slope $dy/d\xi$. The normalized surface impedance $\Delta(\xi)$ is given by (9). As indicated it is a function of distance ξ along the path because it depends on ground constants, ε_g and σ_g, and slab parameters, D, ε_h, σ_h, ε_v, and σ_v, which are a function of distance ξ along the path. The normalized surface impedance Δ_a is evaluated at the source, $\xi=0$.

To account for nonzero source and receiver heights, h_a and h_r, we utilize the height-gain functions discussed in Section 2.4. Thus we define an attenuation function $f_h(x)$ which is the ratio of the vertical electric field to twice the free-space field in terms of the normalized attenuation function $f(x)$ and the height-gain functions:

$$f_h(x) = f(x)\; G(h_a)\; G(h_r) \quad . \tag{21}$$

G is defined in (13), and the heights h_a and h_r must be greater than $-D$. That is, the source and observer can be either in the slab or in air. $G(h_a)$ is a function of the ground and slab parameters at the source ($\xi=0$), and $G(h_r)$ is a function of the parameters at the observer ($\xi=x$). For the source and observer in air at the surface of the slab ($h_a=h_r=0^+$), both height-gain functions are unity, and the two attenuation functions are equal

$$f_h(x)\Big|_{h_a=h_r=0^+} = f(x) \quad . \tag{22}$$

3.2 Integral Equation Solution

Before discussing the general solution of (20), it is useful to examine the case of a uniform path. In this case, $y(\xi)=y(x)=y'(\xi)=0$ and $\Delta(\xi)=\Delta_a$. Thus the integrand is zero, and $f(x)$ is simply

$$\begin{aligned} f(x) &= W(x,0) \\ &= 1 - i\sqrt{\pi p_a}\; w(-\sqrt{p_a}) \end{aligned} \tag{23}$$

where

$$p_a = -ik\Delta_a^2 x/2 \quad .$$

This result is the expected flat-earth attenuation function (Wait, 1962), and p_a is the numerical distance. When $|p_a|$ is very small, then $f(x) \simeq 1$. For example, p_a is zero for a perfectly conducting upper boundary ($\Delta_a = 0$). For large $|p_a|$, the asymptotic form in (6) holds

$$f(x) \simeq \frac{-1}{2\,p_a} \quad . \tag{24}$$

When either y or Δ varies along the path, then the integral equation (20) must be solved numerically. We use the same method of solution as Ott (1971a) which is a forward stepping solution in x (Wagner, 1953). Thus values of f (or f_h) are obtained at discrete values of x along the profile. Since (20) is a Volterra integral equation of the second kind, the value of f(x) depends only on the previously computed values of $f(\xi)$ for $\xi < x$. Physically this means that backscatter is neglected.

For a given path calculation, the computer run time t_r is roughly proportional to the square of the number of x points, N_x. The number of points required to maintain precision along a given path is proportional to length of the path L and the radio frequency. Thus the run time can be written

$$t_r \propto N_x^2 \propto L^2 \cdot \mathrm{freq}^2 \quad . \tag{25}$$

The constant of proportionality depends on how rapidly y and Δ vary along the path and is difficult to quantify.

Appendix C contains a user's guide, a listing, and a sample output for program WAGSLAB. The program is an extension of program WAGNER (Ott et al., 1979). The primary differences are that WAGSLAB requires additional input data for the slab parameters along the path and that the normalized surface impedance Δ is the form for a slab over ground as given by (9) rather than the form for a homogeneous earth given by (10). The slab parameters are allowed to change discontinuously along the path, and as a result Δ will also change discontinuously. However, a linear height interpolation scheme (Ott et al., 1979) is used to determine the height $y(\xi)$ and slope $y'(\xi)$ so that y and y' will be well behaved.

The primary outputs are the magnitude and phase of $f(x)$ and $f_h(x)$. We prefer to use these normalized attenuation functions because the attenuation relative to free space is an easy quantity to interpret and can be separated from the antenna effects. If some form of path loss is preferred, the additional path loss L_a to be added to the free-space path loss is

$$L_a = -20 \log_{10}(2|f_h|) \quad \text{(dB)} \quad .$$

4. TWO-SECTION PATH

The uniform slab model which was discussed in Section 2 has the advantage of yielding a rigorous analytical solution, but has the disadvantage of not providing any information regarding the effects of changes in slab parameters along the path. The integral equation approach described in Section 3 can handle arbitrary changes in slab parameters along the path, but yields only numerical results which are difficult to check. In this section, we treat the two-section path shown in Figure 4. Although this geometry has no exact solution, it is amenable to an approximate analysis which yields a fairly simple analytical result as given in Section 4.1.

The analytical result in Section 4.1 is useful in providing insight into the effect of a forest clearing on propagation. In addition, the analytical result is useful in providing a check for the integral equation method. As seen in Section 4.2, the agreement between the two methods is remarkably good.

4.1 Kirchhoff Theory

The geometry is shown in Figure 4. The section, $x < 0$, has normalized surface impedance Δ_a, and the section, $x > 0$, has normalized surface impedance Δ_b. A jump in surface height of magnitude D occurs at $x=0$. Since we employ the surface impedance boundary condition, there is no need to specify the details of the slab or ground at this point. The region, $x < 0$, could represent a slab of thickness D as in Figure 1 or it could just as well represent elevated ground as in a cliff geometry.

The geometry in Figure 4 has been treated by Monteath (1973) for the case of $D=0$, and here we extend his approach to allow for nonzero D. We take the source and receiving vertical electric dipoles to be located at the surface. The mutual impedance Z_{ab} between the two dipoles can be written as an integral over the vertical aperture S defined by $z > D$. If we make the Kirchhoff approximation that the aperture fields of each dipole are those of a uniform structure ($\Delta_a=\Delta_b$ and $D=0$), then the mutual impedance is given by (Monteath, 1973)