The primary outputs are the magnitude and phase of f(x) and $f_h(x)$. We prefer to use these normalized attenuation functions because the attenuation relative to free space is an easy quantity to interpret and can be separated from the antenna effects. If some form of path loss is preferred, the additional path loss L_a to be added to the free-space path loss is

$$L_a = -20 \log_{10}(2|f_h|) \quad \text{(dB)} \quad .$$

4. TWO-SECTION PATH

The uniform slab model which was discussed in Section 2 has the advantage of yielding a rigorous analytical solution, but has the disadvantage of not providing any information regarding the effects of changes in slab parameters along the path. The integral equation approach described in Section 3 can handle arbitrary changes in slab parameters along the path, but yields only numerical results which are difficult to check. In this section, we treat the two-section path shown in Figure 4. Although this geometry has no exact solution, it is amenable to an approximate analysis which yields a fairly simple analytical result as given in Section 4.1.

The analytical result in Section 4.1 is useful in providing insight into the effect of a forest clearing on propagation. In addition, the analytical result is useful in providing a check for the integral equation method. As seen in Section 4.2, the agreement between the two methods is remarkably good.

4.1 Kirchhoff Theory

The geometry is shown in Figure 4. The section, $x < 0$, has normalized surface impedance Δ_a, and the section, $x > 0$, has normalized surface impedance Δ_b. A jump in surface height of magnitude D occurs at $x=0$. Since we employ the surface impedance boundary condition, there is no need to specify the details of the slab or ground at this point. The region, $x < 0$, could represent a slab of thickness D as in Figure 1 or it could just as well represent elevated ground as in a cliff geometry.

The geometry in Figure 4 has been treated by Monteath (1973) for the case of D=0, and here we extend his approach to allow for nonzero D. We take the source and receiving vertical electric dipoles to be located at the surface. The mutual impedance Z_{ab} between the two dipoles can be written as an integral over the vertical aperture S defined by $z > D$. If we make the Kirchhoff approximation that the aperture fields of each dipole are those of a uniform structure ($\Delta_a=\Delta_b$ and D=0), then the mutual impedance is given by (Monteath, 1973)

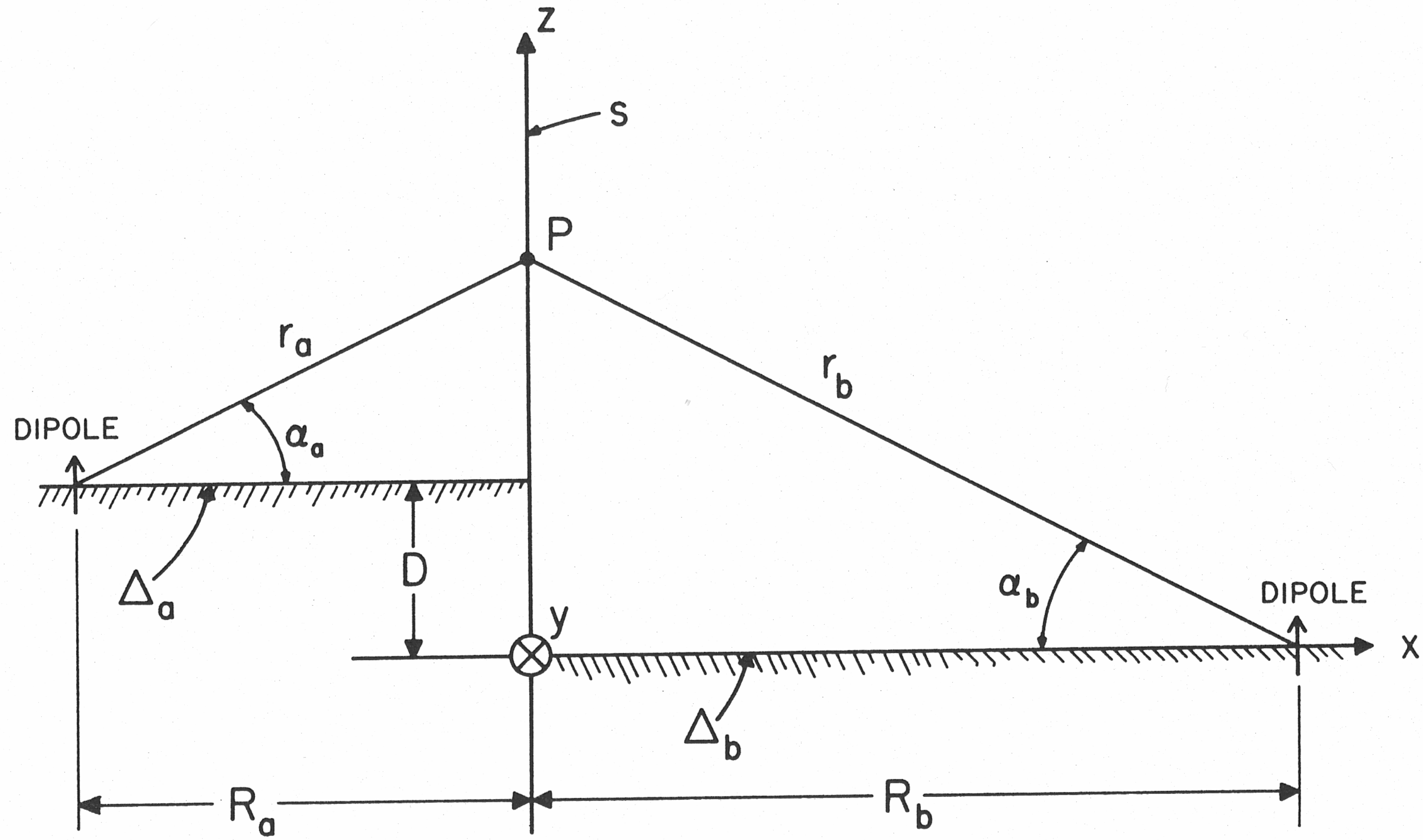

Figure 4. Geometry for a two-section path with an elevation change D. The section, $x < 0$, has a normalized surface impedance Δ_a, and the section, $x > 0$, has a normalized surface impedance Δ_b.

$$Z_{ab} = K_o \int_D^{\infty} dz \int_{-\infty}^{\infty} \frac{(1+\rho_a)\, e^{-ikr_a}}{r_a} \cdot \frac{(1+\rho_b)\, e^{-ikr_b}}{r_b}\, dy \quad , \tag{26}$$

where

$$r_a \simeq R_a + \frac{y^2 + (z - D)^2}{2\, R_a} \quad ,$$

$$r_b \simeq R_b + \frac{y^2 + z^2}{2\, R_b} \quad ,$$

and ρ_a and ρ_b are reflection coefficients for vertical polarization. K_o is a constant which depends on the dipole lengths, but the final result for the attenuation function is independent of K_o. We make the Fresnel approximation where quadratic terms are kept in the phase but not in the amplitude

$$Z_{ab} = \frac{K_o}{R_a R_b} \int_D^{\infty} dz \int_{-\infty}^{\infty} (1 + \rho_a)(1 + \rho_b) \cdot$$

$$\exp\left\{-ik\left[\frac{y^2}{2}\left(\frac{1}{R_a} + \frac{1}{R_b}\right) + \frac{z^2}{2R_b} + \frac{(z-D)^2}{2\, R_a}\right]\right\} dy \quad . \tag{27}$$

The y integration in (27) can be done by stationary phase by using the following result (Monteath, 1973)

$$\int_{-\infty}^{\infty} \exp\left\{-ik\left[\frac{y^2}{2}\left(\frac{1}{R_a} + \frac{1}{R_b}\right)\right]\right\} dy = e^{-i\pi/4} \sqrt{\frac{2\pi}{k\left(\frac{1}{R_a} + \frac{1}{R_b}\right)}} \tag{28}$$

Substituting (28) into (27) and introducing a new constant K_1, we have

$$Z_{ab} = \frac{K_1}{\sqrt{R_a R_b (R_a + R_b)}} \int_D^{\infty} (1+\rho_a)(1+\rho_b) \cdot$$

$$\exp\left\{-ik\left[\frac{z^2}{2\, R_b} + \frac{(z - D)^2}{2\, R_a}\right]\right\} dz \quad . \tag{29}$$

The reflection coefficients in (29) are given by

$$\rho_a = \frac{\sin \alpha_a - \Delta_a}{\sin \alpha_a + \Delta_a} \text{ and } \rho_b = \frac{\sin \alpha_b - \Delta_b}{\sin \alpha_b + \Delta_b} \quad . \tag{30}$$

A convenient method of normalization is to consider the case of flat perfectly conducting ground ($\Delta_a = \Delta_b = D = 0$). In this case, the mutual impedance Z_{abo} is given by (Monteath, 1973)

$$Z_{abo} = \frac{4\ K_1}{\sqrt{R_a R_b (R_a + R_b)}} \int_0^\infty \exp\left[\frac{-ikz^2}{2}\left(\frac{1}{R_a} + \frac{1}{R_b}\right)\right] dz$$

$$= \frac{2\sqrt{2\pi}\ K_1}{(ik)^{\frac{1}{2}}\ (R_a + R_b)} \quad . \tag{31}$$

In a manner consistent with the rest of this report, we define the attenuation function f as the ratio of the mutual impedance Z_{ab} to that for perfect ground Z_{abo}. Thus f is given by

$$f = \frac{Z_{ab}}{Z_{abo}} = \frac{(ik)^{\frac{1}{2}}\ \sqrt{R_a + R_b}}{2\sqrt{2\pi}\ \sqrt{R_a R_b}} \int_D^\infty (1+\rho_a)(1+\rho_b) \cdot$$

$$\exp\left\{-ik\left[\frac{z^2}{2\ R_b} + \frac{(z-D)^2}{2\ R_a}\right]\right\} dz \quad . \tag{32}$$

From (30), the reflection coefficient terms in (32) are given by

$$1 + \rho_a = \frac{2\ \sin\ \alpha_a}{\sin\ \alpha_a + \Delta_a} \simeq \frac{2\ \sin\ \alpha_a}{\Delta_a}$$

and

$$1 + \rho_b = \frac{2\ \sin\ \alpha_b}{\sin\ \alpha_b + \Delta_b} \simeq \frac{2\ \sin\ \alpha_b}{\Delta_b} \quad , \tag{33}$$

where we have assumed that the angles α_a and α_b are small. Thus we can approximate the sine terms

$$\sin\ \alpha_a \simeq \frac{z - D}{R_a} \text{ and } \sin\ \alpha_b \simeq \frac{z}{R_b} \quad . \tag{34}$$

Then (33) becomes

$$1 + \rho_a \simeq \frac{2(z - D)}{\Delta_a R_a} \text{ and } 1 + \rho_b \simeq \frac{2z}{\Delta_b\ R_b} \quad . \tag{35}$$

Substituting (35) into (32), we have

$$f = \sqrt{\frac{2}{\pi}}\, \frac{\sqrt{ik(R_a + R_b)}}{\Delta_a \Delta_b (R_a R_b)^{3/2}} \int_D^\infty z(z - D) \cdot$$

$$\exp\left\{-ik\left[\frac{z^2}{2\,R_b} + \frac{(z-D)^2}{2\,R_a}\right]\right\} dz \quad . \tag{36}$$

The evaluation of the integral in (36) is rather tedious and is given in terms of complementary error functions in Appendix B. However, for small values of D, f is given by

$$f \simeq \frac{S\ e^{-H^2}}{ik(R_a+R_b)\ \Delta_a\Delta_b} \quad , \tag{37}$$

where

$$S \simeq 1 + \frac{2H}{\sqrt{\pi}}\left(\frac{1}{\sqrt{A}} - \sqrt{A}\right) + \text{order } H^2$$

$$H = \frac{D\sqrt{ik/2}}{\sqrt{R_a + R_b}} \text{ and } A = \frac{R_a}{R_b} \quad .$$

The leading term in f varies as $(R_a + R_b)^{-1}$ and is independent of D. This is actually the same result which Monteath (1973) obtained for D=0:

$$f\Big|_{D=0} = \frac{1}{ik(R_a+R_b)\Delta_a\Delta_b} \quad . \tag{38}$$

For our purposes of modeling a discontinuous forest region, D is normally on the order of 10 m and $R_a + R_b$ is on the order of kilometers. Thus for the HF band, S in (37) is well approximated by unity, and f is well approximated by (38). This is equivalent to saying that the change in surface impedance has a larger effect on the propagation than does the change in elevation. This is consistent with some recent mode matching calculations for a step change in height over a spherical earth (Hill and Wait, 1982). In the case of $\Delta_a=\Delta_b=\Delta$, (38) reduces to the result for a uniform path as given by (6).

Although the derivation has been carried out for each dipole antenna at the surface as in Figure 4, we can generalize the results to nonzero heights. If the antennas are located at heights h_a and h_b, then a new attenuation function f_h is given by

$$f_h = f\ G(h_a)\ G(h_b) \tag{39}$$

where $G(h_a)$ is the height-gain function for the left section of the path and $G(h_b)$ is the height-gain function for the right section of the path. In either case, the form of G is given by (13).

4.2 Comparison with Integral Equation

In this section, we compare the approximate solution of (39) with the integral equation solution of (21). The same height-gain functions are used in each solution so that the comparison really involves the attenuation function f as calculated by (37) for the aperture solution and by (20) for the integral equation solution. Actually both solutions incorporate a number of approximations, but the approximations employed in the integral equation approach are less severe.

The specific case which we consider involves a forest-to-clearing path. For the ground constants over the entire path, we take the conductivity $\sigma_g = 10^{-2}$ S/m and the relative permittivity $\varepsilon_g = 10$. For the forest slab parameters, we take (Ott and Wait, 1973): $\sigma_h = 10^{-4}$ S/m, $\sigma_v = 2.5 \times 10^{-4}$ S/m, $\varepsilon_h = 1.1$, $\varepsilon_v = 1.25$, and D=10 m.

In Figure 5, results are shown for propagation from a forest to a clearing. Both transmitting and receiving antennas are located at the surface of the earth. Results are shown for three theories: the vertical aperture theory of Section 4.1, the integral equation of Section 3, and an analytic continuation method of Tamir (1977). For antenna separations less than 2 km, both antennas are in the forest and the results are those of the uniform slab because all three theories neglect backscatter. In this region, the three theories agree. For the receiving antenna just beyond the forest, the vertical aperture and analytic continuation theories predict a jump in the field strength and are not valid. However, for larger distances, the aperture theory and the integral equation are in good agreement. The Tamir (1977) analytic continuation theory gives a value which is too high. It is possible to show that Tamir's theory is roughly equivalent to replacing f in (37) by

$$f \simeq \frac{1}{ik(R_a+R_b)\ \Delta_b^{\,2}} \ . \tag{40}$$

In other words the product of the two surface impedances $\Delta_a\ \Delta_b$ is replaced by $\Delta_b^{\,2}$. Since in this case the forest surface impedance Δ_a has a greater magnitude than the ground surface impedance Δ_b, Tamir's result yields a larger field. Only in the limit of a very thin forest does the Tamir result agree with the integral equation and the aperture theory. The increase in field strength in going from forest to clearing was predicted earlier (Office of Telecommunications Technical Memorandum

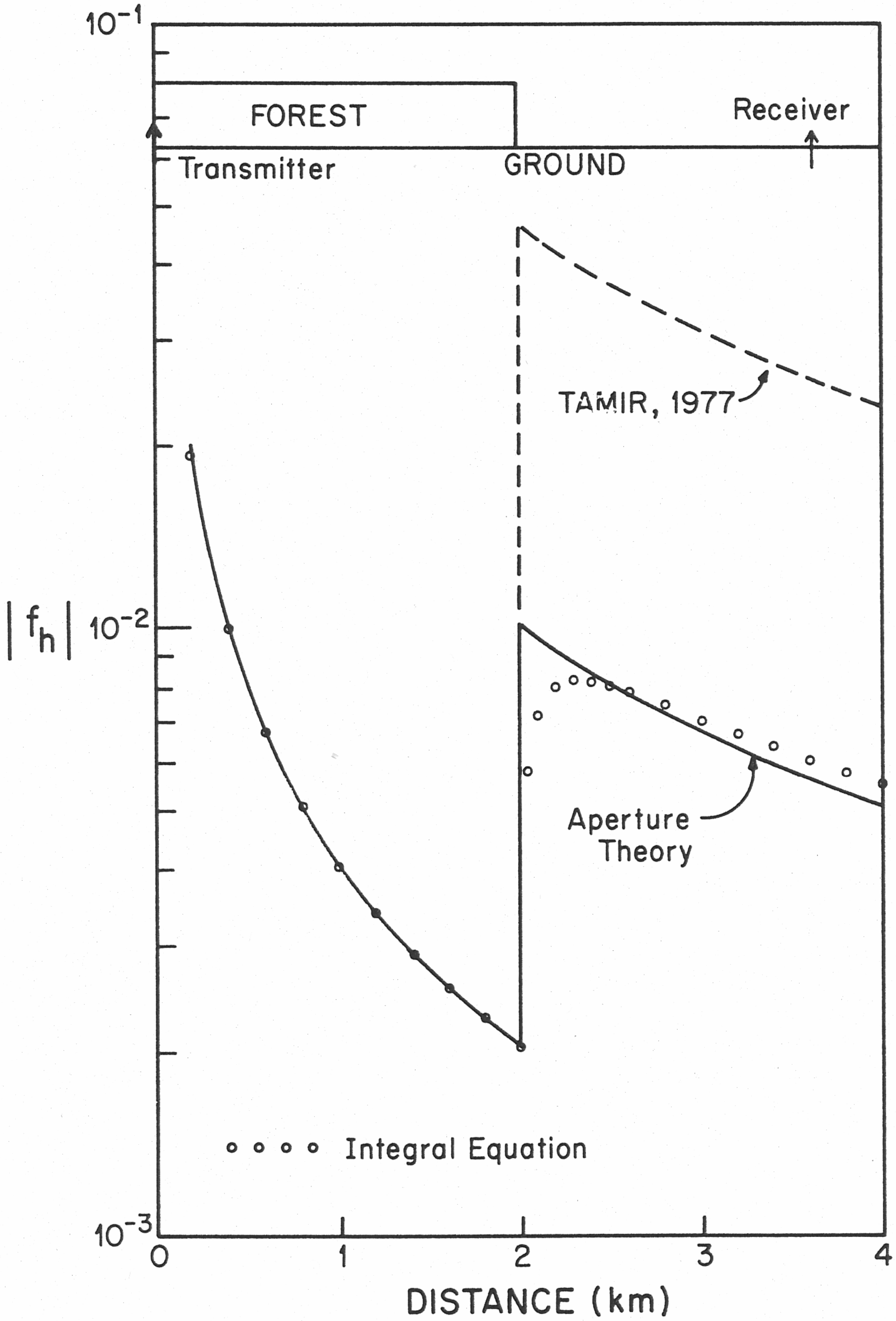

Figure 5. Propagation from a forest to a clearing for a frequency of 10 MHz.

73-154, by Ott and Wait, limited distribution) and is analogous to the "recovery effect" which has been calculated (Wait and Walters, 1963; Hill and Wait, 1981a) and measured (Millington, 1949) for propagation from land to sea.

The problem of propagating from the clearing to the forest is equally interesting, and the three theories are shown in Figure 6. In this case, both antennas are located 10 m above the ground even with the top of the forest. For short distances, the agreement with the integral equation is not perfect because the numerical distance p as defined by (6) is not large. For distances well beyond the forest transition, the aperture and integral equation theories are in good agreement, but the Tamir (1977) analytic continuation result again is high. Actually for the case of the antennas even with the top of the forest, Tamir's analytic continuation result is independent of the forest parameters.

Other cases for various slab parameters and frequencies have been studied, and the trends are qualitatively similar. In order to use the integral equation approach, we actually replace the abrupt height change from the ground to the forest with a linear transition. This is done automatically by program WAGSLAB so that the terrain slope remains finite. A similar linear height interpolation method has been used in program WAGNER (Ott et al., 1979).

5. EQUIVALENT SLAB PARAMETERS

The program WAGSLAB requires the user to supply the slab parameters along the path. The five slab parameters are as pictured in Figure 1: height (D), horizontal conductivity (σ_h), horizontal relative permittivity (ε_h), vertical conductivity (σ_v), and vertical relative permittivity (ε_v). In the HF band, the wavelength ranges from 10 m to 100 m. Consequently, objects such as trees and buildings are not necessarily electrically small. However, the equivalent slab representation seems to be the only tractable model for forests and urban areas at this time.

In this section, we review the literature and make some recommendations regarding the equivalent parameters for forests, urban areas, and snow cover. It is hoped that future comparisons of theory and measurements for propagation through forests and built-up areas will aid in the determination of the appropriate equivalent parameters. It might turn out that a multilayer slab (Ott and Wait, 1973a; Cavalcante et al., 1982) is more appropriate for modeling forests and built-up areas, but it is felt that our knowledge of the equivalent slab parameters is not precise enough to justify such additional complexity at this time. However, such an extension is possible simply by modifying the surface impedance Δ and the height-gain function G.