DIGITAL SYSTEM PERFORMANCE SOFTWARE UTILIZING NOISE MEASUREMENT DATA

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This report summarizes techniques that use the measured instantaneous envelope statistics of arbitrary noise or interference processes to calculate the degradation these processes cause to digital communication systems. Computer implementation of the techniques are also given. The computer algorithms are designed for the data obtained from a general purpose noise measurement device, termed the DM-4 (for "distribution meter-model number 4") recently developed by NTIA/ITS. For illustration and for comparison with theoretical results, two noise examples are employed, one for "narrowband" interference, and one for "broadband" interference. These examples are taken from the noise models recently developed by Middleton.

Key Words: computer algorithms; digital system performance; non-Gaussian noise; system performance software.

1. INTRODUCTION

Most currently used receiving systems are those which are optimum in Gaussian noise. Unfortunately, the actual interference environment is almost never Gaussian in character, but usually quite different, being impulsive in nature. By "impulsive" we mean only that there are significant probabilities of quite large instantaneous values of noise, which is a more general definition in that we can, and do, have both broadband (the usual definition, e.g., automotive ignition noise) and narrowband (e.g., various combinations of interfering signals) "impulsive" processes. Recently there have been receiving systems designed to match this actual interference (e.g., Spaulding and Middleton, 1977; Middleton, 1979). However, it is the purpose of this short report to provide computer programs that will use noise measurements to calculate the performance of "normal" digital systems in arbitrary noise or interference (including, of course, Gaussian noise). By "normal" systems we mean those that are optimum in Gaussian noise, and, therefore, suboptimum in any other kind of noise or interference. The "normal" digital systems are "matched filter" or "correlation" systems. The common digital systems covered here are:

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1. Binary non-coherent frequency shift keying (NCFSK);
2. Binary differentially coherent phase shift keying (DCPSK);
3. Binary coherent phase shift keying (antipodal or CPSK);
4. Binary coherent frequency shift keying (othogonal or CFSK); and
5. Binary coherent ON-OFF keying.

In addition, the coherent signal detection system (Neyman-Person detection) is included. The performance of other systems, such as M level systems and minimal shift keying systems, can usually be obtained by appropriate extensions of the techniques summarized here. However, these extensions are not always straightforward.

Recently, a general purpose noise measurement device, termed the DM-4 (for "distribution meter-model number 4") was developed by NTIA/ITS (Matheson, 1980, DM-4 operation and maintenance manual, NTIA-TM 80-50), and the software presented here is designed to work specifically with the DM-4 measurements although no actual DM-4 measurements are used. For any received noise process, $Z(t)$, we denote the probability density function (pdf) of the instantaneous amplitude by $p_Z(z)$. Denote the envelope of this received noise process by $R(t)$ and pdf of the envelope by $p_R(r)$. The DM-4 measures the amplitude probability distribution (APD) of the envelope or Prob$[R > R_0]$, which we will denote by $P_R(r)$. Note that

$$p_R(r) = - \frac{d}{dr} P_R(r). \tag{1}$$

The DM-4 can also measure the average crossing rate characteristic of the received noise envelope, but these measurements are not considered here. While the DM-4 measures the APD in terms of the actual levels exceeded, referred to the input of the receiving system via calibration (e.g., dBm), we normally require the $P_R(r)$ or $p_Z(z)$ in normalized form so that the mean noise power is equal to 1. When we consider our desired signals, then the mean signal power is also the signal-to-noise ratio. For example, for the signal $\sqrt{2S}$ Cos $(\omega_0 t)$, S is the signal power and also the signal-to-noise ratio. The DM-4 measures the APD at 31 calibrated levels, i.e., measures Prob$[R > R_0]$ for 31 values of R_0. It uses a maximum sampling rate of 20 MHz, which means it can measure the output waveforms from systems of about 10 MHz bandwidth (IF) or less. The DM-4 is designed to work with the detected logarithmic output of modern spectrum analyzers and EMI meters, so that the 31 DM-4 levels are equally spaced in voltage, corresponding to 31 levels equally spaced in dB when referred to the receiving system input. For our purposes here we only need the calibrated 31 levels (not necessarily equally spaced) and the Prob[$R > R_0$] for each of these levels as the input data to the system performance algorithms.

For illustration and for comparison with theoretical results we make use of recently developed noise models. The models were developed for ITS by Middleton (1977, 1980) and Spaulding (1977). Two examples are selected, one for "narrowband" interference, termed Class A, and one for "broadband" interference, termed Class B. These examples of noise, from actual measurements (not DM-4 however), are presented in the next section (Section 2) and are used then to simulate DM-4 "measurements."

Section 3 presents the system performance algorithms, sample performance calculations, comparison with theoretical results when possible, and discussion of the algorithms. An Appendix then contains the actual computer software listings in FORTRAN.

2. THE IMPULSIVE NOISE MODEL, TEST EXAMPLES

Recent work by Middleton has led to the development of a physical-statistical model for radio noise and interference. This model has been used to develop optimum detection algorithms for a wide range of communications problems (Spaulding and Middleton, 1977). It is this model which we will use here to simulate DM-4 "measurements." The Middleton model is the only one proposed to date in which the parameters of the model are determined explicitly by the underlying physical mechanisms (e.g., source density, beam-patterns, propagation conditions, and emission waveforms). It is also the first model which treats narrowband interference processes (termed Class A), as well as the traditional broadband processes (Class B). The model is also canonical in nature in that the mathematical forms do not change with changing physical conditions. For a large number of comparisons of the model with measurements and for the details of the derivation of the model, see Middleton (1974, 1976, 1977, 1978a, 1978b) and Spaulding (1977). We only summarize the results of the model which we need here.

For the class A model, the expression for the pdf of the received noise signal, Z(t), is

$$P_Z(z) = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!\sqrt{2\pi\sigma_m^2}} \; e^{-z^2/2\sigma_m^2} \;, \qquad (2)$$

where

$$\sigma_m^2 = \frac{m/A + \Gamma'}{1 + \Gamma'} \;, \qquad (3)$$

and for the envelope, R(t),

$$P[R > R_o] = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \; e^{-R_0^2/\sigma_m^2} \;. \qquad (4)$$

The Class A model has two parameters, A and Γ'. A is termed the impulsive index, and as A becomes larger ($\sim$ 10), the noise approaches Gaussian (still narrowband) and Γ' is the ratio of the energy in the Gaussian portion of the noise to the energy in the non-Gaussian component. In the above, the rms value of Z is equal to 1, i.e., the process is already normalized.

For our sample DM-4 "measurement" of Class A noise, Figure 1 gives a measured Class A distribution and the appropriate model parameters are A=0.35 and $\Gamma'=0.5 \times 10^{-3}$. The first program, APDA, given in the Appendix, simply generates our "measurement" data. We compute $P_R(r)$ at 31 levels, starting with -59 dB (see Table 1) with the levels 3 dB apart. We further assume that the measurements after $P_R(r) = 10^{-6}$ are zero. This gives us some zero "measurements" that are likely from actual DM-4 measurements. The dynamic range covered, therefore, is 90 dB, from -59 to 31 dB. If we were actually measuring the APD of Figure 1, we would probably adjust the levels, so that the significant portion of the distribution (-50 to 20 dB, say) was more accurately covered. The above procedure (3 dB spacing), however, will be a better test of the system performance algorithms, but we want to keep in mind that accurracy can be improved by proper adjustment of the measurement levels. Once we have determined our 31 values of $P_R(r)$, we then assign these values to arbitrary (unnormalized) levels (3 dB apart) to check the normalization portion of the algorithms.

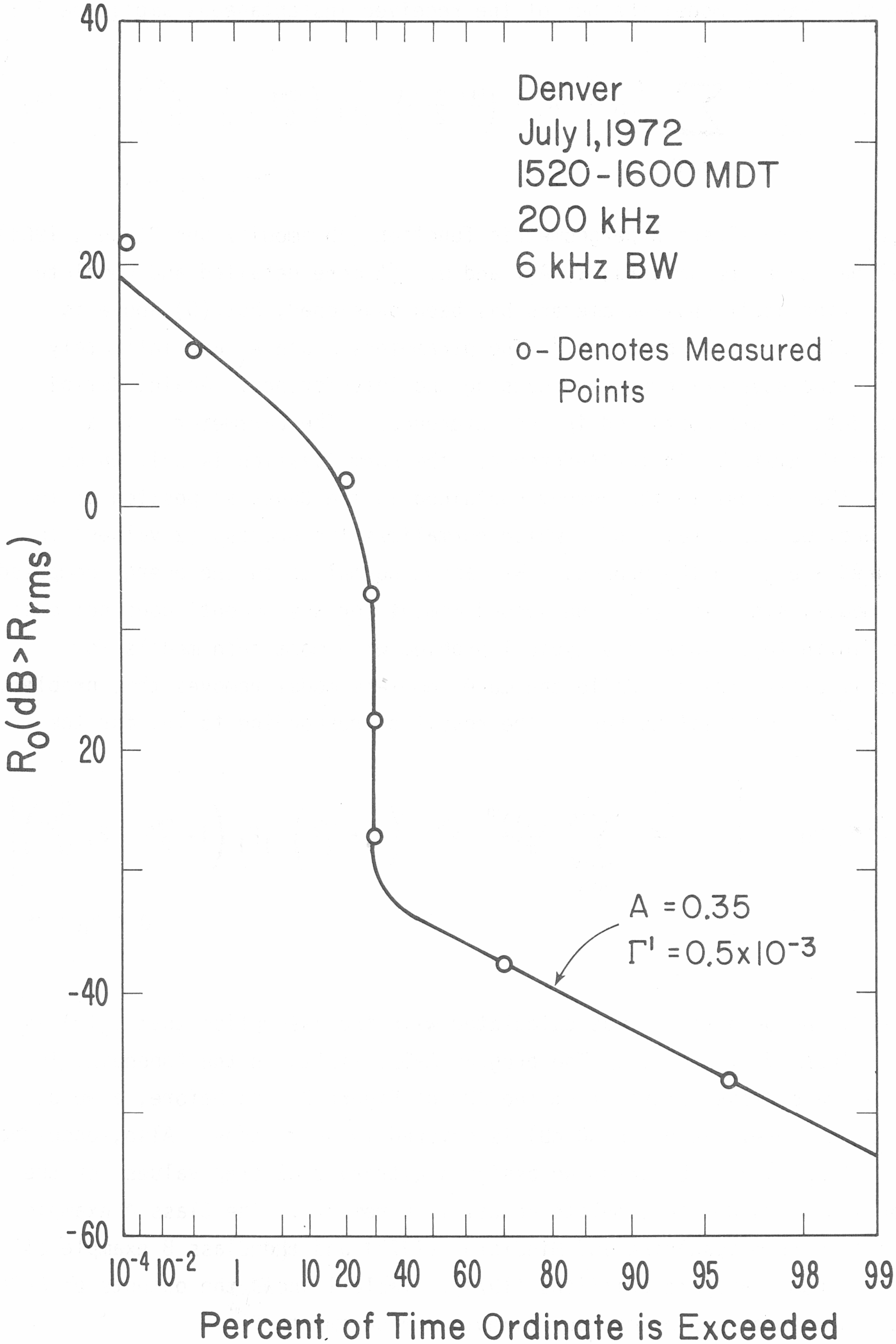

Figure 1. Example of Class A noise.

For the Class B model the pdf of the received instantaneous amplitude is:

$$p_Z(z) = \frac{e^{-z^2/\Omega}}{\pi\sqrt{\Omega}} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} A_\alpha^m \, \Gamma\left(\frac{m\alpha + 1}{2}\right) {}_1F_1\left(\frac{-m\alpha}{2} ; \frac{1}{2} ; \frac{z^2}{\Omega}\right) , \quad (5)$$

$$-\infty \leq z \leq \infty$$

where, ${}_1F_1$ is a confluent hypergeometric function (Abramowitz and Stegun, 1964). The model has three parameters, α, A_α, and Ω. [A more detailed and complete model involving additional parameters has been developed, but (5) above is quite sufficient for our purposes.] The parameters α and A_α are intimately involved in the physical processes causing the interference. Again, definitions and details are contained in the references. The parameter Ω is a normalizing parameter. In the references, the normalization is Ω=1, which normalizes the process to the energy contained in the Gaussian positon of the noise. Here, we use a value of Ω which normalizes the process (z values) to the measured energy in the process. We cannot normalize to the energy computed from the model, since for (5), the second moment (or any moment) does not exist (i.e., is infinite). This is a typical problem with most such models for broadband impulsive noise. While the more complete model removes this problem, use of (5) will not limit us here. The result corresponding to (5) for the APD is:

$$P(R > R_0) = e^{-R_0/\Omega} \left[1 - \frac{R_0^2}{\Omega} \sum_{m=1}^{\infty} \frac{(-1)^m}{m!} A_\alpha^m \, \Gamma\left(1 + \frac{m\alpha}{2}\right) {}_1F_1\left(1 - \frac{m\alpha}{2} , 2; \frac{R_0^2}{\Omega}\right)\right] \quad (6)$$

$$0 \leq R_0 < \infty$$

On Figure 2, the parameter Ω was calculated with the assumption that $P_R(r)$ is zero for values of R_0 > 40 dB. The program APDB, listed in the Appendix, is used to generate "measurement" from the APD of Figure 2. As before, a 90 dB dynamic range (here, -40 dB to 50 dB) is covered in 3-dB steps. Also, once the 31 values of $P_R(r)$ are obtained, we assign arbitrary 3-dB step values for the corresponding R_0 values to simulate actual measurements. The Class A example of Figure 1 is from Spaulding and Middleton (1977) and the Class B example of Figure 2 is from Evans and Griffiths (1974). Table 1 shows the outputs of APDA and APDB, and these then become our example "measurements."

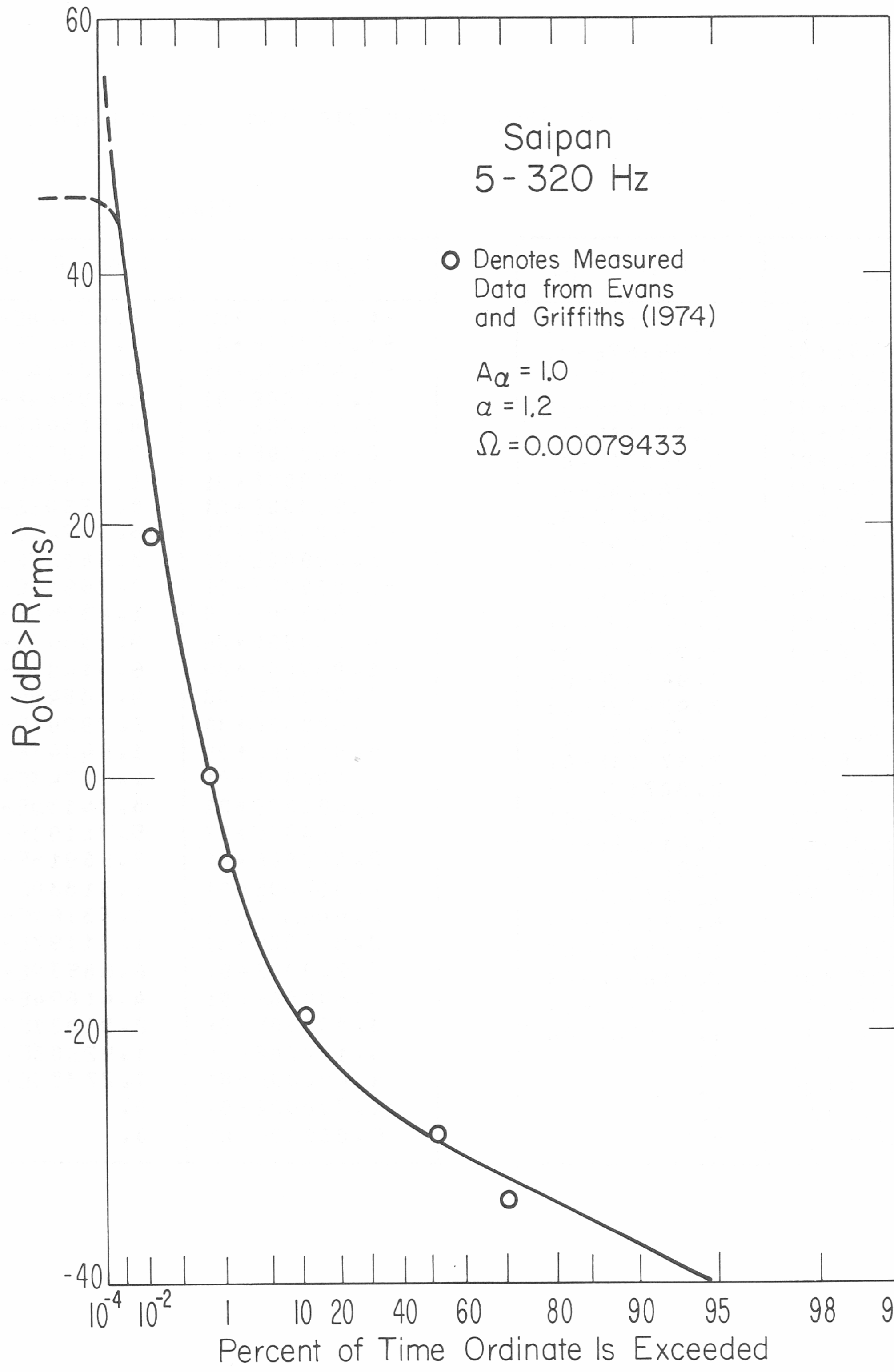

Figure 2. Example of Class B noise.

Table 1. Class A and Class B Noise Data from Figures 1 and 2

Class A

R_0(dB)	Prob [$R > R_0$]
-5.90000E+01	9.98227E-01
-5.60000E+01	9.96467E-01
-5.30000E+01	9.92968E-01
-5.00000E+01	9.86038E-01
-4.70000E+01	9.72418E-01
-4.40000E+01	9.46038E-01
-4.10000E+01	8.96434E-01
-3.80000E+01	8.08473E-01
-3.50000E+01	6.69554E-01
-3.20000E+01	4.94632E-01
-2.90000E+01	3.51942E-01
-2.60000E+01	2.99700E-01
-2.30000E+01	2.94870E-01
-2.00000E+01	2.94368E-01
-1.70000E+01	2.93432E-01
-1.40000E+01	2.91573E-01
-1.10000E+01	2.87901E-01
-8.00000E+00	2.80719E-01
-5.00000E+00	2.66942E-01
-2.00000E+00	2.41529E-01
1.00000E+00	1.98120E-01
4.00000E+00	1.34301E-01
7.00000E+00	6.37192E-02
1.00000E+01	1.67017E-02
1.30000E+01	2.11605E-03
1.60000E+01	1.04690E-04
1.90000E+01	1.07174E-06
2.20000E+01	0.
2.50000E+01	0.
2.80000E+01	0.
3.10000E+01	0.

Class B

R_0(dB)	Prob [$R > R_0$]
-4.00000E+01	9.43528E-01
-3.70000E+01	8.91427E-01
-3.40000E+01	7.98434E-01
-3.10000E+01	6.49084E-01
-2.80000E+01	4.51644E-01
-2.50000E+01	2.63070E-01
-2.20000E+01	1.44504E-01
-1.90000E+01	8.57505E-02
-1.60000E+01	5.38743E-02
-1.30000E+01	3.46832E-02
-1.00000E+01	2.25913E-02
-7.00000E+00	1.48055E-02
-4.00000E+00	9.73559E-03
-1.00000E+00	6.41403E-03
2.00000E+00	4.23040E-03
5.00000E+00	2.79201E-03
8.00000E+00	1.84343E-03
1.10000E+01	1.21743E-03
1.40000E+01	8.04130E-04
1.70000E+01	5.31191E-04
2.00000E+01	3.50915E-04
2.30000E+01	2.31830E-04
2.60000E+01	1.53161E-04
2.90000E+01	1.01190E-04
3.20000E+01	6.68539E-05
3.50000E+01	4.41694E-05
3.80000E+01	2.91821E-05
4.10000E+01	1.92803E-05
4.40000E+01	1.27383E-05
4.70000E+01	0.
5.00000E+01	0.

Finally, white Gaussian noise is a special case, and the performance of digital systems in white Gaussian noise has been treated in great detail. Here, we will occasionally refer to the well-known results of system performance in white Gaussian noise for comparison. The pdf for the instantaneous amplitude for Gaussian noise (mean noise power = 1) is:

$$p_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad , \tag{7}$$

and for the corresponding envelope

$$p_R(r) = r e^{-r^2/2} \quad , \text{ and} \tag{8}$$

$$P_R(r) = e^{-r^2/2} \quad . \tag{9}$$

3. SYSTEM PERFORMANCE CALCULATIONS

In this Section we want to present the results which are most advantageous for our use. We want to develop system performance algorithms which do not require particularly sophisticated numerical analysis techniques and which can be used on small scale computers.

We start with the simplest. For arbitrary additive interference which is independent from an integration period (bit length) to the next and which has uniformly distributed phase, Montgomery (1954) has shown that the probability of binary bit error, P_e, for NCFSK (non-coherent frequency shift keying) is given by:

$$P_e = \frac{1}{2} \text{ Prob [noise envelope > rms signal level]}. \tag{10}$$

While Montgomery's result for NCFSK is in terms of the noise and signal envelopes at the input to an ideal discriminator, it has been shown (White, 1966) that the result is also applicable to most common FSK receivers (bandpass-filter discriminator receivers and matched-filter envelope detection receivers). Using (10), then, the performance for NCFSK can be obtained instantly from the APD measurements, once the APD has been normalized to its rms level. For example, if the two NCFSK waveforms are given by