APPENDIX A

THE REFRACTIVE INDEX OF THE NEUTRAL ATMOSPHERE FOR FREQUENCIES UP TO 1000 GHz

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A.1. INTRODUCTION

Dry air and atmospheric water vapor are major millimeter-wave absorbers; so are suspended droplets (haze, fog, cloud) and precipitating water drops that emanate from the vapor phase. A practical model (designated program code: MPM) was formulated that simulates the refractive index $\underline{n} = n' - jn''$ of the atmospheric propagation medium for frequencies up to 1000 GHz [1], [3]*. The main purpose of the program is to express the electromagnetic properties of the neutral atmosphere in terms of available and/or measurable quantities.

The free propagation of a plane electromagnetic wave at frequency f and an initial field strength E_0 in an isotropic gas medium over distance L is described by the complex transmission factor

$$\underline{\tau} \equiv E(L)/E_0 = \exp[-(2\pi f/c)(n'' + jn')L]$$

where n'(f) and n"(f) are frequency-dependent measures of delay and loss, and c is the speed of light in vacuum. Since the interaction with a neutral atmosphere is relatively weak, the refractive index is converted into a refractivity

$$\underline{N} = (\underline{n} - 1)10^6 \qquad \text{ppm},$$

in units of parts per million.

Refractivity N for moist air can be obtained, in principle, by a summation over all absorption features in a given volume element. In practice, various degrees of approximations are employed to reduce labor and computer time required, since the number of contributing spectral lines by the dominant absorbers (water vapor and oxygen) and by various trace gases (e.g., O_3) exceeds 10,000. The model MPM consists of local H_2O (30x) and O_2 (48x) lines

* [ ] See 7. REFERENCES in main text.

below 1 THz and an empirical approximation to the contributions by H_2O lines above 1 THz. To complete the model, continuum spectra for dry air, suspended water droplets (haze, fog, cloud), and rain are added. The supporting spectroscopic data base contains more than 450 coefficients.

A.1.1 Features of Program MPM

A parametric program was developed (see Section A.2) that calculates the values of the complex refractivity

$$\underline{N} = N_o + N'(f) - jN''(f) \qquad \text{ppm}$$

or corresponding path-specific quantities of particular interest to radio engineering; i.e.,

refractive delay	β_o	ns/km,
dispersive delay	$\beta(f)$	ps/km, and
attenuation	$\alpha(f)$	dB/km;

for atmospheric conditions as a function of the variables f, P, T, RH, w_A (A/B/C/D), w, and R, as listed in the scheme below:

Variable	Symbol	Validity Range	Medium
frequency	f	≤ 1000 GHz	
barometric pressure temperature relative humidity	P T RH	120-0 kPa - 50- + 50°C 0-100%	Moist Air
haze model: code A, B, C, D (or combinations thereof) plus hygroscopic aerosol reference concentration	w_A(80%RH)	RH=80-99.9% 0-1 mg/m^3	Haze
suspended water droplet concentration	w(100%RH)	0-10 g/m^3	Fog, Cloud
rainfall rate	R	0-200 mm/h	Rain

The height range 30 to 100 km is treated approximately excluding the detailed O_2-Zeeman effect and trace gas spectra (e.g., O_3, CO, N_2O, etc.).

A.1.2 Physical Variables

The purpose of this section is to relate measurable variables to N model-specific variables, whereby the state of dry air is described by a partial air pressure p in kPa, a relative inverse temperature variable is $\theta = 300/(273.15 + T)$ with T in °C, and relative humidity is given by

$$RH = (e/e_s)100 = (v/v_s)100 = 4.151 \times 10^{-9}(e/\theta^5)\exp(22.64\theta) \leq 100\%. \qquad \text{(A-1)}$$

Equation (A-1) governs physical processes taking place in the atmosphere with respect to water vapor. Absolute (v) and relative (RH) humidity are interrelated through

$$v = 1.739 \times 10^9 RH\cdot\theta^5 \exp(-22.64\theta) = 7.223\ e\ \theta \quad g/m^3 \qquad \text{(A-2)}$$

where e is the partial water vapor pressure in kPa as part of the total (barometric) pressure $P = p + e$.

Water vapor variability at sea level (e.g., P = 101 kPa, $\theta = 1.016$ or 22°C) is typically

	Dry	Normal	Humid	Saturated
v	1	10	17	20g/m^3
RH	5	50	85	100 %

Suspended hydrometeors are described by the liquid water concentration w, which relates approximately to optical (0.55 μm) visibility U(km). A schematic categorization can be made by

	Haze	Fog	Stratus Cloud	Convective Cloud
w ≤	10^{-2}	10^{-1}	1	5 g/m^3
U ≥	1.1	0.27	0.07	0.03 km

Another atmospheric ingredient is hygroscopic aerosol with a mass concentration w_A in mg/m^3. Solution droplets appear for RH > 80%, and haze conditions develop as RH approaches 100 percent. The growing haze droplets can reach values ($w < 0.1\ g/m^3$) sufficient to contribute to medium losses. Haze conditions are modeled by assuming that at the reference humidity, RH = 80%, the concentration w_A(RH=80%) is known. Any RH-dependent swelling/shrinking of w(RH) up to RH=99.9% is modeled by

$$w = w_A(C1-RH)/C2(100-RH) = w_A\ g(RH), \qquad (A-3)$$

where g(RH) is a growth function. The following values have been reported for C1, C2:

Case	Aerosol Species	C1	C2	g(RH=99.9%)
A	Rural	117	1.87	91
B	Urban	128	2.41	117
C	Maritime	183	5.31	162
D	C + Strong Wind (>10 km/h)	197	5.83	167

Average values of w_A for given stabilized climatic situations can be found in the literature. Typical values for w_A lie between 0.01 and 0.5 mg/m^3 and the increase to w can be substantial as indicated by the maximum prefog values g(99.9%).

Precipitation originates as a statistical event within clouds suspended in saturated air. Its vertical distribution is separated into two regions by the height of the 0°C isotherm, which can vary between 6 km and ground level depending on season and latitude. The lower part is mostly liquid drops, and the upper region consists of frozen particles with occasional supercooled droplet loadings by strong updrafts.

Point rain rates R have proven useful in modeling rain-induced <u>N</u> effects. Rain rate can be related to percent time, t_R, a given value occurs over the period of an "average" year; to the effective rain cell extent L_R/L; and to the instaneous suspended liquid water concentration,

$$w_R = m\ R \quad g/m^3.$$

In terms of these variables, an hypothetical local rain may be classified as follows (horizontal path, L = 10 km):

	Drizzle	Steady	Heavy	Downpour	Cloudburst	
R	1	5	20	100	250	mm/h
t_R	2	0.5	0.07	0.001	0.0001	% per yr
L_R/L	1	1	0.7	0.35	0.2	
m	0.1	0.07	0.05	0.04	0.04	

These numbers may differ substantially for given locations.

The simple coefficient scheme reveals some fundamentals of rain. Changes in the factor m indicate rain rate-dependent characteristics of drop-size distributions. Widespread steady rain occurs more uniformly and favors small drop sizes (≤1 mm diameter) that stay in the air longer. Heavy showers are more localized, favor larger drops, and occur less frequently.

A.2. MODEL FOR COMPLEX REFRACTIVITY

The complex refractivity in N units (i.e., ppm ≡ 10^{-6})

$$\underline{N}(f;P/T/RH,w_A/w,R) = N_o + N'(f) - jN''(f) \quad \text{ppm} \qquad \text{(A-4)}$$

is a macroscopic measure of interactions between radiation and absorbers. The refractivity consists of a frequency-independent term N_o plus various spectra of refractive dispersion N'(f) and absorption N"(f).

In radio engineering it is customary to express the imaginary part of (A-4) as power attenuation α and the real part as group delay β (with reference to vacuum) as follows:

$$\alpha = 0.1820\ f\ N''(f) \quad \text{dB/km} \qquad \text{(A-5a)}$$

and

$$\beta = 3.336[N_o + N'(f)] \quad \text{ps/km,} \qquad \text{(A-5b)}$$

where frequency f is in gigahertz (GHz) throughout.

Radio refractivity is defined to be $\underline{N} = N_o$ at f = 0 and consists of four terms; i.e.,

$$N_o = N_p^o + N_e^o + N_w^o + N_R^o.$$

The individual contributions are described for dry air by

$$N_p^o = 2.588\ p\ \theta, \qquad \text{(A-6)}$$

for water vapor by

$$N_e^o = 2.39\ e\ \theta + 41.6\ e\ \theta^2, \qquad \text{(A-7)}$$

for the SWD term N_w^o by (A-16c), and for the rain term N_R^o by (A-18c). The results (A-6) and (A-7) have been determined experimentally at microwave frequencies where dispersive contributions N(f) are negligible.

A calculation of the spectrum $\underline{N}(f)$ for frequencies up to 1000 GHz consists of several additive parts:

- resonance information of n_a=48 oxygen lines and n_b=30 water vapor lines
- nonresonant O_2 and pressure-induced N_2 absorption ($\underline{N}_p$)
- continuum absorption from far-wing contributions of strong H_2O lines falling in the frequency range 1-30 THz ($\underline{N}_e$)
- suspended water droplet SWD term ($\underline{N}_w$)
- rain-effect approximation($\underline{N}_R$)

Absorption and dispersion spectra are obtained from line-by-line calculations plus various continuum spectra $\underline{N}_{p,e,w,R}$ according to

$$N''(f) = \underbrace{\sum_{i=1}^{n_a} (SF'')_i + N''_p}_{\text{Dry Air}} + \underbrace{\sum_{i=1}^{n_b} (SF'')_i + N''_e}_{\cdot\text{Water Vapor}} + \underbrace{N''_w}_{\cdot\text{SWD}} + \underbrace{N''_R}_{\cdot\text{Rain}} \qquad \text{(A-8a)}$$

$$N'(f) = \sum_{i=1}^{n_a} (SF')_i + N'_p + \sum_{i=1}^{n_b} (SF')_i + N'_e + N'_w + N'_R \qquad \text{(A-8b)}$$

where S is the line strength in kilohertz, and F' and F" are real and imaginary parts of a line shape function in GHz^{-1}.

A.2.1 Local Line Absorption and Dispersion

The Van Vleck-Weisskopf function is modified to describe to first-order line overlap effects, which leads to local absorption and dispersion line profiles in the form

$$F''(f) = \left[\frac{1}{X} + \frac{1}{Y} - \frac{\delta}{\gamma}\left(\frac{\nu_o - f}{X} + \frac{\nu_o + f}{Y}\right)\right] A \qquad \text{(A-9a)}$$

and

$$F'(f) = \frac{Z - f}{X} + \frac{Z + f}{Y} - \frac{2}{\nu_o} + \delta\left(\frac{1}{X} - \frac{1}{Y}\right) A \qquad \text{(A-9b)}$$

with the abbreviations $A = \gamma f/\nu_o$,

$$X = (\nu_o - f)^2 + \gamma^2, \quad Y = (\nu_o + f)^2 + \gamma^2, \quad Z = (\nu_o^2 + \gamma^2)/\nu_o .$$

Standard line shapes F"(f), including the modified Van Vleck-Weisskopf function (A-9), predict in frequency regions of local line dominance about the same results for N"(f) as long as F"(f) exceeds by 0.1% the peak value at $f = \nu_o$. Far-wing contributions of smaller magnitude depend very much upon the chosen shape function. For $f \to \infty$, the wing response of (A-9a) becomes non-physical and is cut off; i.e., F" = 0 when $f \geq (\nu_o + 40\gamma)$. So far, no line shape has been confirmed that predicts absorption intensities over ranges 10^{-3} to $< 10^{-6}$ of $F''(\nu_o)$. Far-wing contributions from strong infrared water vapor lines, where $\alpha(\nu_o)$ can exceed 10^6 dB/km, are accounted for summarily by empirical correction [see (A-15a)].

The line parameters are calculated by the expressions below:

Symbol	O_2 Lines in Air	H_2O Lines in Air	Eq.
S, kHz	$a_1 p\ \theta^3 \exp[a_2(1-\theta)]$	$b_1 e\ \theta^{3.5} \exp[b_2(1-\theta)]$	(A-10)
γ, GHz	$a_3(p\theta^{(0.8-a_4)} + 1.1e\theta)$	$b_3(p\theta^{0.6} + 4.80e\theta^{1.1})$	(A-11)
δ	$a_5 p\ \theta^{a_6}$	0	(A-12)

Line center frequencies ν_o and the spectroscopic coefficients $a_1(\geq 10^{-7}$ Hz/Pa) to a_6 and $b_1(\geq 10^{-3}$ Hz/Pa) to b_3 for strength S, width γ, and overlap correction δ are listed in the Line Data File of MPM (see Appendix B).

For the O_2 lines in air, (A-9) to (A-12) are valid for altitudes $h \leq 35$km ($p > 0.7$ kPa), where lines are pressure-broadened. Higher up, Zeeman-splitting and Doppler-broadening of the Zeeman components must be taken into account (Liebe and Gimmestad, 1978). An estimate for h > 35 km is made by geometrically adding to the pressure proportional width γ_a in (A-11) a second term

$$\gamma_h = \left[\gamma_a^2 + (25H)^2\right]^{0.5} \quad \text{GHz} \tag{A-11a}$$

where H is the scalar Earth magnetic field strength in Tesla, ranging from 0.2 to 0.9 x 10^{-4}. The O_2 spectrum vanishes around H ≈ 90 km.

For the H_2O lines in air, Doppler-broadening has to be considered at altitudes above 60 km ($p < 0.07$ kPa). An approximation is made by replacing the width γ_b in (A-11) with

$$\gamma_h = (\gamma_b^2 + \gamma_D^2)^{0.5} \quad \text{GHz} \tag{A-11b}$$

where $\gamma_D^2 = 2.14\nu_o^2 \times 10^{-12}/\theta$ is the squared Doppler width. In applications requiring the detailed mesospheric line shape it is necessary to apply the more correct Voigt line shape.

A.2.2 Continuum Spectra for Air

Continuum spectra in (A-8) identify dry air and water vapor terms $\underline{N}_p + \underline{N}_e$ and must be added to the selected group of local (MPM) O_2 and H_2O resonance lines described by (A-9). Continuum absorption increases monotonically with frequency.

The dry air continuum

$$N_p''(f) = f\left(2a_o\{\gamma_o[1 + (f/\gamma_o)^2]\}^{-1} + a_p p\, \theta^{1.5}\right) p\theta^2 \tag{A-12a}$$

and

$$N'_p(f) = a_o\{[1 + (f/\gamma_o)^2]^{-1} - 1\}\, p\, \theta^2 \tag{A-12b}$$

make a small contribution at ground level pressures due to the nonresonant O_2 spectrum below 10 GHz and a pressure-induced N_2 spectrum that is effective above 100 GHz. A width parameter for the Debye spectrum of O_2 is formulated in accordance with (A-11) to be $\gamma_o = 4.8 \times 10^{-3}(p + 1.1e)\theta^{0.8}$(GHz) [10]. The continuum coefficients are $a_o = 3.07 \times 10^{-4}$ and $a_p = 1.40(1-1.2f^{1.5}10^{-5})10^{-10}$.