7. APPENDIX A: EQUATIONS

This appendix describes all of the equations that were used to compute the feature values in this report.

Equation (1): Single-frame Temporal Alignment

Let $i(v,h,t_i)$ be the digitized input video sequence where v is the vertical sampling index, h is the horizontal sampling index, and t_i is the input frame sampling index. Here $v = \{1, 2, ..., N_v\}$, and $h = \{1, 2, ..., N_h\}$, where N_v is the total number of vertical pixels, and N_h is the total number of horizontal pixels. Similarly, let the digitized output video sequence be represented by $o(v,h,t_o)$, where t_o is the output frame sampling index. Assume that the input video sequence and the output video sequence are sampled at the same frame rate. Then, given an output reference frame $t_o = r$, the single-frame temporal alignment problem is to find the closest corresponding input frame $t_i = m$. The single-frame temporal alignment proposed here assumes that a priori knowledge is available which gives the range of the closest corresponding input frame index (say, from $t_i = t_l$ to t_u, where t_l and t_u are the lower and upper limits, respectively) and that the input video sequence contained moving and/or changing scenes. Then the closest matching input frame $t_i = m$ can be found as the t_i that minimizes the standard deviation of the error (accumulated over all pixels in the error image) or

$$\sqrt{\left\{\frac{1}{N_v N_h}\sum_{v=1}^{N_v}\sum_{h=1}^{N_h}\left[i(v,h,t_i)-o(v,h,r)\right]^2\right\} - \left\{\frac{1}{N_v N_h}\sum_{v=1}^{N_v}\sum_{h=1}^{N_h}\left[i(v,h,t_i)-o(v,h,r)\right]\right\}^2}$$

where t_i falls within the range from t_l to t_u, inclusive. Pairing of the input and output video frame indices is performed as ..., (t_i = m-1, t_o = r-1), (t_i = m, t_o = r), (t_i = m+1, t_o = r+1), ...

Equation (2): Missing Frame Ratio (MFR)

To apply multi-frame temporal alignment to the output video frame sequence $o(v,h,t_o)$, where $t_o = \{1, 2, ..., N_o\}$, and N_o is the total number of output video frames, the method of finding the closest matching input video frame (equation 1, Appendix A) is applied to every output video frame $t_o = \{1, 2, ..., N_o\}$. The computation of the closest input video frame to output video frame $t_o = 1$, may be used to refine the estimates of the lower (t_l) and upper (t_u) input frame limits for output video frame $t_o = 2$. While performing multi-frame alignment, the closest input video frame index to each one of the output video frames is stored. Let the number of <u>unique</u> input frame indices within the set of stored indices be N_u. N_u will be less than N_o if input video frames have been omitted in the output. Then, the missing frame ratio (MFR) is calculated as

$$MFR = \frac{N_o - N_u}{N_o}$$

<u>Equation (3): Mean of the Sobel Image (M-SI)</u>

If the Sobel edge extracted image is given by s(v,h), where v and h represent the vertical and horizontal sampling indices of the video image, then the mean of s(v,h) over a given sub-regional area is given by

$$M\text{-}SI = \frac{1}{N_A}\sum_{v}\sum_{h} s(v,h)$$

where the summation is performed over the sub-regional area and N_A is the total number of pixels within the sub-regional area.

<u>Equation (4): Standard Deviation of the SI (SD-SI)</u>

Following the notation established for equation (3) in Appendix A, the standard deviation of the sub-regional Sobel image is

$$SD\text{-}SI = \sqrt{\left\{\frac{1}{N_A}\sum_{v}\sum_{h} s(v,h)^2\right\} - (M\text{-}SI)^2}$$

Equation (5): Root Mean Square of the SI (RMS-SI)

Following the notation established for equation (3) in Appendix A, the root mean square of the sub-regional Sobel image is

$$RMS\text{-}SI = \sqrt{\frac{1}{N_A}\sum_{v}\sum_{h} s(v,h)^2}$$

Equation (6): Number of Pixels Greater than Threshold of SI (NPGT-SI)

Following the notation established in equation (3) in Appendix A and letting T be the chosen threshold, the number of pixels greater than T within the sub-regional Sobel image is

$$NPGT\text{-}SI = \sum_{v}\sum_{h} u(v,h)$$

where

$$\begin{aligned} u(v,h) &= 1 \;\; \text{if } s(v,h) > T \\ &= 0 \;\; \text{otherwise} \end{aligned}$$

Equation (7): Mean of the Positive Sobel Difference Image (M-PSDI)

If the Sobel difference image (Sobel filtered input image minus the Sobel filtered output image) is given by $s_d(v,h)$, where v and h represent the vertical and horizontal sampling indices, then the mean of the sub-regional, positive part of the sobel difference image is

$$M\text{-}PSDI = \frac{1}{N_A}\sum_v \sum_h s_d(v,h) \quad , \; s_d(v,h)>0$$

where N_A is the total number of points within the sub-regional area.

<u>Equation (8): Standard Deviation of the PSDI (SD-PSDI)</u>

Following the notation of equation (7) in Appendix A, the standard deviation of the sub-regional, positive part of the Sobel difference image is

$$SD\text{-}PSDI = \sqrt{\frac{1}{N_A}\sum_v \sum_h s_d^2(v,h) - (M\text{-}PSDI)^2} \quad , \; s_d(v,h)>0$$

<u>Equation (9): Root Mean Square of the PSDI (RMS-PSDI)</u>

Following the notation of equation (7) in Appendix A, the root mean square of the sub-regional, positive part of the Sobel difference image is

$$RMS\text{-}PSDI = \sqrt{\frac{1}{N_A}\sum_v \sum_h s_d^2(v,h)} \quad , \; s_d(v,h)>0$$

<u>Equation (10): Number of Pixels Greater than Threshold of PSDI (NPGT-PSDI)</u>

Following the notation of equation (7) in Appendix A, and letting T_p be the chosen threshold, the number of pixels greater than T_p within the sub-regional, positive part of the Sobel difference image is

$$NPGT\text{-}PSDI = \sum_{v} \sum_{h} u_p(v,h)$$

where

$$u_p(v,h) = 1 \text{ if } s_d(v,h) > T_p$$
$$= 0 \text{ otherwise}$$

Equation (11): Mean of the Negative Sobel Difference Image (M-NSDI)

If the Sobel difference image (Sobel filtered input image minus the Sobel filtered output image) is given by $s_d(v,h)$, where v and h represent the vertical and horizontal sampling indices, then the mean of the sub-regional, negative part of the sobel difference image is

$$M\text{-}NSDI = \frac{1}{N_A} \sum_{v} \sum_{h} s_d(v,h) \quad , s_d(v,h) < 0$$

where N_A is the total number of points within the sub-regional area.

Equation (12): Standard Deviation of the NSDI (SD-NSDI)

Following the notation of equation (11) in Appendix A, the standard deviation of the sub-regional, negative part of the Sobel difference image is

$$SD\text{-}NSDI = \sqrt{\frac{1}{N_A} \sum_{v} \sum_{h} s_d^2(v,h) - (M\text{-}NSDI)^2} \quad , s_d(v,h) < 0$$

Equation (13): Root Mean Square of the NSDI (RMS-NSDI)

Following the notation of equation (11) in Appendix A, the root mean square of the sub-regional, negative part of the Sobel difference image is

$$RMS\text{-}NSDI = \sqrt{\frac{1}{N_A}\sum_v \sum_h s_d^2(v,h)} \quad , \; s_d(v,h) < 0$$

Equation (14): Number of Pixels Less than Threshold of NSDI (NPLT-NSDI)

Following the notation of equation (11) in Appendix A, and letting T_n be the chosen threshold, the number of pixels less than T_n within the sub-regional, negative part of the Sobel difference image is

$$NPLT\text{-}NSDI = \sum_v \sum_h u_n(v,h)$$

where

$$\begin{aligned} u_n(v,h) &= 1 \;\; \textit{if } s_d(v,h) < T_n \\ &= 0 \;\; \textit{otherwise} \end{aligned}$$

Equation (15): Temporal Root Mean Square Position Error (TRMS-PE)

Let the input, or reference, vertical and horizontal positions of the moving object be represented by $v_i(t_i)$ and $h_i(t_i)$, where t_i represents the frame sampling index such that $t_i = \{1, 2, 3, ..., N_i\}$, and N_i is the total number of time samples for the input object path. Similarly, let the output vertical and horizontal positions of the moving object be represented by $v_o(t_o)$ and $h_o(t_o)$, where $t_o = \{1, 2, 3, ..., N_o\}$, and N_o is the total number of time samples for the output object path. In order to measure TRMS-PE, the input and output motion paths have to be aligned to compensate for the absolute video delay of the device under test. The alignment procedure described here corresponds to what a viewer would observe if that viewer was insensitive to the absolute video delay. Assume that the output motion path corresponds to some portion of the input path, and is thus contained completely within the input motion path (i.e., $N_o < N_i$). Then, the TRMS-PE feature is computed as the minimum root mean square position error of the output motion path with respect to the input motion path, where the minimization is performed over all possible time shifts $s = \{0, 1, 2, ..., N_i\text{-}N_o\}$ of the two motion paths. In equation form, the computation may be written as

$$TRMS\text{-}PE = \underset{s}{MIN} \sqrt{\frac{1}{N_o} \sum_{t_o=1}^{t_o=N_o} [v_i(t_o+s) - v_o(t_o)]^2 + [h_i(t_o+s) - h_o(t_o)]^2}$$

As in equation (1) in Appendix A, a priori knowledge of the absolute video delay may be used to narrow the range of time shifts.

<u>Equation (16): Standard Deviation of Difference Image (SD-DI)</u>

Assume that the input and output video sequences have been aligned using single frame temporal alignment given in equation (1) in Appendix A, and thus each output video frame has been paired with some input video frame. Let each pair of video frames be represented by the index p = {1, 2, 3, ..., N_p), where N_p is the total number of input/output pairs. Let the difference image (input image minus output image) of each input/output pair be represented by $d_p(v,h)$, where v and h are the vertical and horizontal sampling indices. Then, the standard deviation of the difference image over a sub-regional area is given by

$$(SD\text{-}DI)_p = \sqrt{\{\frac{1}{N_A}\sum_v \sum_h d_p^2(v,h)\} - \{\frac{1}{N_A}\sum_v \sum_h d_p(v,h)\}^2}$$

where N_A is the total number of points within the sub-regional area.

<u>Equation (17): Temporal Mean of SD-DI (TM-SD-DI)</u>

Following the notation of equation (16) in Appendix A, the temporal mean of the time history of SD-DI is given as

$$TM\text{-}SD\text{-}DI = \frac{1}{N_p} \sum_{p=1}^{p=N_p} (SD\text{-}DI)_p$$

Equation (18): Temporal Standard Deviation of SD-DI (TSD-SD-DI)

Following the notation of equation (16) and (17) in Appendix A, the temporal standard deviation of the time history of SD-DI is given as

$$TSD\text{-}SD\text{-}DI = \sqrt{\left\{ \frac{1}{N_p} \sum_{p=1}^{p=N_p} (SD\text{-}DI)_p^2 \right\} - (TM\text{-}SD\text{-}DI)^2}$$

Equation (19): Temporal Root Mean Square of SD-DI (TRMS-SD-DI)

Following the notation of equation (16) in Appendix A, the temporal root mean square of the time history of SD-DI is given as

$$TRMS\text{-}SD\text{-}DI = \sqrt{\frac{1}{N_p} \sum_{p=1}^{p=N_p} (SD\text{-}DI)_p^2}$$