were replaced by the basic transmission loss L_b computed by propagation models that take all of these propagation effects into account. Basic transmission loss is the transmission loss that would occur if the transmitter and receiver antennas were replaced by ideal isotropic loss-free antennas with the same polarization as the real antennas [3,4].

This paper describes the different types of radio-wave propagation phenomena and antenna behavior that must be considered for general radio-wave propagation models in the roadway environment in the MF band. Radio-wave propagation prediction models generally compute basic transmission loss L_b which is combined with antenna gains and transmitter power to perform engineering analyses of communication systems.

3. GROUND-WAVE PROPAGATION

The ground-wave signal can be determined using one of several models [6,7] that specifically address the propagation phenomena at these frequencies. One general ground-wave model [6] computes propagation loss, electric field strength, received power, noise, received signal-to-noise power ratio, and antenna factors over lossy Earth. The smooth-Earth and irregular-Earth (terrain dependent) propagation loss prediction methods within this model can be used over either homogeneous or mixed paths. This model combines three propagation loss prediction methods for both smooth and irregular Earth, and an antenna algorithm into a single analysis tool. The propagation loss prediction methods for the ground-wave model compute basic transmission loss and are valid from 10 kHz to 30 MHz. A model that incorporates the sky wave with the ground wave [7] specifically addresses the 150 kHz to 1705 kHz frequency range. The frequency limits of this model have been set by the valid frequency range of the sky-wave model. This model was previously available on a mainframe computer but now operates in a Windows environment on a PC.

The ground wave includes the direct line-of-sight space wave, the ground-reflected wave, and the Norton surface wave that diffracts around the curved Earth. The Norton surface wave will hereafter be referred to as a surface wave in this paper. Propagation of the ground wave depends on the relative geometry of the transmitter and receiver location and antenna heights. The radio wave propagates primarily as a surface wave when both the transmitter and receiver are near the Earth in wavelengths, because the direct and ground-reflected waves in the space wave cancel each other and as a result the surface wave is the only wave that is left. This cancellation is a result of the fact that the elevation angle is zero and the two waves (direct and reflected) are equal amplitude and opposite in phase. This is the condition that exists for the MF band. The surface wave is predominantly vertically polarized, since the ground conductivity effectively shorts out most of the horizontal electric field component. What is left of the horizontal component is attenuated at a rate many times that for the vertical component of the field. When one or both antennas are elevated above the ground to a significant height with respect to a wavelength, the space wave predominates.

The ground-wave propagation phenomena at these frequencies are basically deterministic processes. The noise, however, is a stochastic process. The surface wave propagates along and is guided by the Earth's surface. This is similar to the way that an electromagnetic wave is guided along a transmission line. Charges are induced in the Earth by the surface wave. These charges travel with the surface wave and create a current in the Earth. The Earth carrying this current can be represented by a leaky capacitor (a resistance shunted by a capacitive reactance). The characteristics of the Earth as a conductor can therefore be represented by this equivalent parallel RC circuit, where the Earth's conductivity can be simulated with a resistor and the Earth's dielectric constant by a capacitor. As the surface wave passes over the surface of the Earth, it is attenuated as a result of the energy absorbed by the Earth due to the power loss resulting from the current flowing through the Earth's resistance. Energy is taken from the surface wave to supply the losses in the ground, and the attenuation of this wave is directly affected by the ground constants of the Earth along which it travels [8].

The attenuation function is the ratio of the electric field from a short dipole over the lossy Earth's surface to that field from the same short dipole located on a flat perfectly conducting surface. At distances within the line of sight the surface wave field strength, *E*, is given by [8]:

$$E = \frac{AE_0}{d} \tag{3}$$

where E_o is the electric field strength of the surface wave at the surface of the Earth at a small distance on the order of a few wavelengths away from the transmitting antenna (the Earth's losses are negligible at this small distance, so this could also be considered the field from the short dipole located on a flat perfectly conducting surface), *d* is the distance between the transmitting antenna and reception point, and *A* is the flat-Earth attenuation function that takes into account the ground losses.

The exact equation derived by Norton [10] for the flat-Earth attenuation function is described later in this paper. Norton further simplified this exact equation into a form that is more amenable to calculation. These simplified expressions for the Norton approximations to the flat-Earth attenuation [9,10] function can be easily implemented on a programmable calculator. They are reasonably accurate for line-of-sight propagation [11].

For $p_o \leq 4.5$ and all *b*:

$$A = e^{-.43p_o + .01p_o^2} - (\sqrt{p_o/2})(\sin b)(e^{-5/8p_o}) \tag{4}$$

For $p_o > 4.5$ and all *b*:

$$A = \frac{1}{2p_o - 3.7} - (\sqrt{p_o/2})(\sin b)(e^{-5/8p_o}) \tag{5}$$

where for vertical polarization:

$$p_o = \frac{\pi R(km)\,[f(MHz)]^2 \cos b}{(54x10^2)\sigma}$$
$$b = \tan^{-1}\frac{(\varepsilon_r+1)f(MHz)}{18x10^3\sigma} \tag{6}$$

and for horizontal polarization:

$$p_o = \frac{\pi R(km)\,6x10^4\sigma}{\cos b'}$$
$$b' = \tan^{-1}\frac{(\varepsilon_r-1)f(MHz)}{18x10^3\sigma} \tag{7}$$

where σ is the conductivity of the Earth in siemens per meter, and ε_r is the relative permittivity of the Earth. The field strength at this small distance is directly proportional to the square root of power radiated by the transmitter and the directivity of the antenna in the horizontal and vertical planes. If the antenna is non-directional in the horizontal plane and has a vertical directional pattern that is proportional to the cosine of the elevation angle (this corresponds to a short vertical antenna), then the electric field at one kilometer for an effective radiated power of one kilowatt is 300 mV/m [8]. The flat-Earth attenuation function A is dependent upon frequency, distance, and the ground constants of the Earth along which the wave is traveling. A numerical distance, p_o (equations (6) and (7) above) can be computed that is a function of frequency, ground constants, and distance in wavelengths. If the numerical distance is less than one, then the attenuation function is very close to one, and as a result for distances close to the transmitting antenna, the losses in the Earth have very little effect on the electric-field strength of the surface wave. In this region, the electric field strength is inversely proportional to distance. For situations where the numerical distance becomes greater than unity, the attenuation function decreases in magnitude rapidly. When the numerical distance becomes greater than 10, the attenuation factor is also inversely proportional to distance. The combination of the attenuation factor and the unattenuated electric field both being inversely proportional to distance results in the electric field strength of the surface wave being inversely proportional to the square of the distance when the numerical distance is greater than 10.

At lower HF frequencies, AM broadcast (medium frequencies), and lower frequencies in the LF band (below 300 kHz), the Earth can be regarded as being purely resistive in nature. The equivalent circuit of the Earth is still a resistor of resistance R and capacitor of capacitance C in parallel. However, the Earth is predominantly resistive at these frequencies because of the fact that more current flows through the resistance, because $R<<1/\omega C$, so under these conditions the resistor has the major effect. The attenuation factor is then primarily dependent on the conductivity of the Earth and the frequency. For frequencies above about 10 MHz, the impedance represented by the ground is primarily capacitive, so the attenuation factor for the surface wave at a given physical distance is determined by the dielectric constant of the Earth and the frequency.

The attenuation of the surface wave is determined by the average values of the Earth conductivity and dielectric constant down to a depth to which the ground currents penetrate and still maintain an appreciable magnitude. This is similar to a skin depth phenomenon in a good conductor. The depth of penetration of the surface wave currents depends upon frequency, dielectric constant, and conductivity. This ranges from a fraction of a meter at the highest frequencies for HF communications to tens of meters at AM broadcast and lower frequencies. For this reason propagation at the lower frequencies is not particularly dependent on conditions at the actual surface of the ground. Therefore, a recent rainfall would not significantly affect propagation at MF and LF frequencies.

The depth of penetration of an electromagnetic wave into the surface of the Earth depends upon the frequency and ground constants of the Earth. The electric field strength at a distance z below the surface of the Earth is given by [12]:

$$E = E_o e^{-\alpha z} \tag{8}$$

where E_o is the electric field intensity at the surface of the Earth, z is the depth below the surface of the Earth, and α is the attenuation per meter of the electric field intensity.

The attenuation per meter α is given by:

$$\alpha = \omega\sqrt{\mu\varepsilon}\sqrt{1/2\sqrt{1+(\sigma/\omega\varepsilon)^2}-1/2} \tag{9}$$

where ω is the angular frequency and is equal to $2\pi f$, f is radio frequency in Hertz, μ is the magnetic permeability of the Earth $\mu = \mu_r \times 4\pi \times 10^{-7}$ henries/meter, μ_r is the relative permeability, ε = the permittivity of the Earth = $\varepsilon_r \bullet (8.85 \times 10^{-12})$ farads per meter, ε_r is the relative permittivity of the Earth, and σ is the conductivity of the Earth in siemens per meter.

The distance the wave must travel in a lossy medium to reduce its amplitude to e^{-1} =.368 of its value at the surface is $\delta = 1/\alpha$ meters and is called the skin depth of the lossy medium. For other values of attenuation of the electric field, $r=e^{-\alpha z}$, one can use α to determine the distance z below the surface where the electric field is attenuated to that ratio r. The ratio r is always less than or equal to 1. The distance z is given by:

$$z = -\frac{\ln r}{\alpha} \tag{10}$$

where ln$r$ is the natural logarithm of r. An example is where f=300 kHz, μ_r= 1 for a nonmagnetic Earth, ε_r= 15 for average ground, σ = .005 for average ground. The attenuation α is calculated as .0751 per meter and δ is calculated as $1/\alpha$ as 13.32 meters.

This is the distance at which the electric field is e^{-1} or .368 (36.8 percent) of its value at the surface of the Earth. If the distance z at which the electric field is .1 (10 percent) of its value at the surface is desired, then ln r is ln (.1)= -2.3026, and α = .0751, so z= $-1/\alpha$ (-2.3026) = 30.66 meters. Table 1 gives the depth of penetration for f= 300 kHz.

Table 1. Depth of Penetration in the Earth for Different Ground Conditions

Conductivity (siemens/m)	Permittivity (no units)	Media Type	Alpha (α) (per meter)	Skin Depth (δ) for $r=e^{-1}$=.368 (meters)
0.001	4.0	poor ground	0.0333	30.04
0.005	15.0	avg. ground	0.0751	13.32
0.020	25.0	good ground	0.1523	06.57
0.010	81.0	fresh water	0.1017	09.83
5.000	81.0	sea water	2.4330	00.41

3.1 Specific Ground-wave Propagation Models

3.1.1 The Smooth-Earth Model

The simplified expressions for field strength of a ground wave described previously in equations (3) through (7) are only valid for line-of-sight radio-wave propagation. The antennas must also be located on the ground or very near it in distance with respect to a wavelength. They are the simplified expressions for the Norton [10] approximation to the Sommerfeld flat-Earth attenuation function. Sommerfeld originally solved the problem of radiation over a flat lossy Earth for a short current element in 1909 [13]. Later work by Sommerfeld and many others resulted in an extensive numerical evaluation of the flat-Earth attenuation function. One of the more prominent evaluations of the flat-Earth attenuation function was that due to Norton [9,10]. When the geometry of the receiver antenna, transmitter antenna, and the Earth are such that a flat-Earth attenuation function [14] is no longer valid, then other methods must be used. In this paper a smooth-Earth model is described that uses a variety of algorithms to account for this geometry. The smooth-Earth model described in this paper is that developed by L.A. Berry [15]. It is also described in more detail in [6,7,14,16,17,18]. The following six computation techniques are used in this smooth-Earth model to account for all possible propagation geometries: flat-Earth attenuation function, flat-Earth attenuation function with a curvature correction, the power series expansion, the residue series calculation, geometric optics, and numerical integration of the full-wave theory. The appropriate technique is selected according to the relative geometry of the transmitter and receiver antenna heights and locations with respect to the Earth. Diffraction is included where necessary in the last five of these techniques.

The details of the smooth-Earth model equations are given in Berry [15] and Stewart, et al. [16]. Some of the basic equations will be summarized in this section. Figure 1 describes the geometry for a spherical smooth Earth. The flat-Earth attenuation function is used for line-of-sight propagation when the path is short and the Earth can be assumed to be flat [19]. If in addition, the transmitter and receiver antennas are near the Earth with respect to a wavelength, then the flat-Earth attenuation function [19] is used and the electric-field strength *E(d)* in V/m is given by [15,16,19,20]:

$$E(d) = \frac{9.487\sqrt{P_E}\ A(\rho)}{d} \tag{11}$$

where P_E is the effective radiated power in watts, d is the distance between the transmitter and receiver in kilometers, and $A(\rho)$ is the flat-Earth attenuation function. The equations (3) through (7) given earlier for the flat-Earth attenuation function A are an approximation to $A(\rho)$. The following equations are the more precise expressions used to predict ground-wave loss [13,19]:

$$A(\rho) = 1 - R_0 \Delta e^{\rho^2} \mathrm{erfc}(\rho) \tag{12}$$

$$\begin{aligned} R_0 &= e^{j\pi/4}\sqrt{\pi k D/2} \\ D &= \sqrt{d^2 + (h_1 - h_2)^2} \end{aligned} \tag{13}$$

$$\rho = e^{j\pi/4}\sqrt{kD/2}\,\Delta\left(1 + \frac{(h_1 + h_2)}{\Delta D}\right) \tag{14}$$

where $d = a\theta$ (Figure 1) is the great circle path distance between the transmitter and the receiver base locations, h_1 and h_2 are the heights of the transmitter and receiver antennas respectively, D is the line of sight distance between the transmitter and the receiver, and $k = 2\pi/\lambda$, and erfc(ρ) is the complementary error function [21].

The surface impedance of the ground is given by Δ. If the polarization is vertical, then Δ is:

$$\Delta = \frac{\sqrt{\varepsilon_{gc} - 1}}{\varepsilon_{gc}} \tag{15}$$

If the polarization is horizontal, then Δ is:

$$\Delta = \sqrt{\varepsilon_{gc} - 1} \tag{16}$$

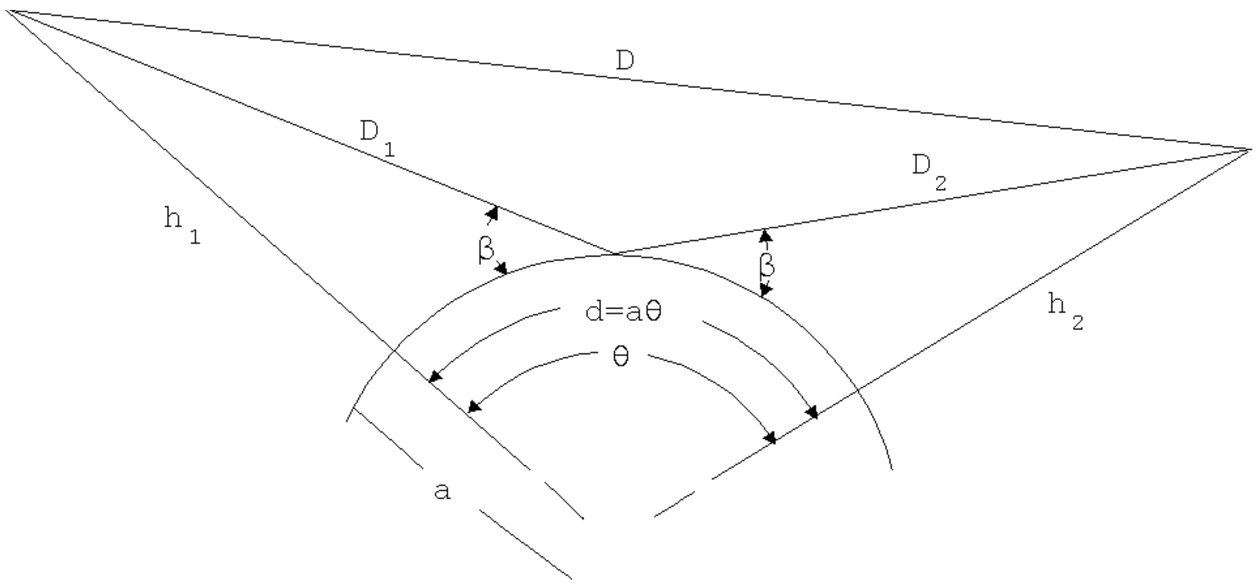

Figure 1. Spherical smooth-Earth geometry.

$$\varepsilon_{gc} = \varepsilon_g + \frac{\sigma_g}{j\omega\varepsilon_o} \tag{17}$$

where ε_g is the relative dielectric constant of ground, σ_g is the conductivity of the ground in siemens per meter, ε_o is the permittivity of free space, 8.85 x 10^{-12} farads per meter, $\omega = 2\pi f$, and f is the operating frequency in Hertz. The surface impedance, Δ, is a function of the ground constants of the Earth's surface.

When the transmitter and receiver antenna locations are high enough such that an observer positioned at either the receiver or transmitter location is well above the radio horizon when viewed from the other, the field strength computation involves the use of geometrical optics [15,16]. The electric field E in V/m is given by:

$$E = 9.487\sqrt{P_E}\frac{e^{-jk(D-d)}}{2D}(1 + R_g e^{-jk(D_1 + D_2 - D)}) \tag{18}$$

where R_g is the appropriate ground reflection coefficient for vertical or horizontal polarization. The direct line-of-sight distance D between the transmitter and receiver antennas is given by [15,16]:

$$D=\sqrt{[2a^2+2a(h_1+h_2)](1-\cos\theta)+h_1^2+h_2^2-2h_1h_2\cos\theta} \tag{19}$$

The radius of the Earth is denoted as a. Closed form solutions for D_1 and D_2 are not possible, but Stewart et al. [16] gives details on calculating these distances using an iterative method.

When the receiving antenna is near the radio horizon of the transmitting antenna, but not beyond it, then the field depends on diffraction effects in addition to the direct wave, and in this case the computation is performed by numerical integration of the full-wave theory integral [15]. The expression for calculating the electric field using the full-wave theory integral is [15,16,19]:

$$E=9.487\sqrt{P_E}\sqrt{\frac{v}{12\sin\theta}}\frac{e^{j3\pi/4}}{a}\int_{\Gamma} e^{-jkt}F_I(q,t)\,dt \tag{20}$$

where $v=(ka/2)^{1/3}$, $x=v\theta$, and $q=-jv\Delta$. Γ is a contour enclosing the poles of $F_I(q,t)$.

$$F_I(q,t)=\frac{H_1(h_1)H_2(h_2)}{\frac{W_1'(t)}{W_1(t)}-q} \tag{21}$$

$H_1(h_1)$ and $H_2(h_2)$ are height gain functions given by:

$$H_1(h)=\frac{W_1(t-y)}{W_1(t)} \tag{22}$$

$$H_2(h_2)=-0.5jW_2(t-y)\,[W_1'(t)-qW_1(t)]-W_1(t-y)\,[W_2'(t)-qW_2(t)] \tag{23}$$

where $y=kh_i/v$, and $W_1'(t)$ denotes the differential of $W_1(t)$ with respect to t. The functions $W_n(t)$ are Airy functions [19] that satisfy the differential equation:

$$\begin{aligned} &W_n''(t)=tW_n(t)\\ &W_1(t)=\frac{1}{\sqrt{\pi}}\int_{\Gamma_1} e^{(st-s^3/3)}ds\\ &W_2(t)=\frac{1}{\sqrt{\pi}}\int_{\Gamma_2} e^{(st-s^3/3)}ds \end{aligned} \tag{24}$$

where the contour Γ_1 is taken as the straight line segment from $\infty e^{j2\pi/3}$ to the origin and out along the real axis to ∞, and contour Γ_2 is taken as the straight line segment from $\infty e^{-j2\pi/3}$ to the origin and out along the real axis to ∞.

The poles of the function $F_I(q,t)$ occur at points t_i and they satisfy the differential equation [19,22]:

$$W_1'(t) - qW_1(t) = 0 \tag{25}$$

For long paths, the Earth cannot be considered flat. If, in addition, the geometry is such that a straight line connecting the transmitter and receiver antennas intersects the curved Earth so that the transmitter and receiver antennas are beyond line of sight of each other and propagation is beyond the horizon, then the full-wave theory integral must be evaluated using the residue series [19,22]. It is necessary to calculate the series by summing the residues at the poles t_i of $F_I(q,t)$ in equation (21) above [19,22]. The electric field is given by [19]:

$$E = 9.487\sqrt{P_E}\frac{\pi}{a}\sqrt{\frac{2v}{6\sin\theta}}e^{-j\pi/4}\sum_i e^{-jxt_i}\frac{H_1(h_1)H_1(h_2)}{t_i - q^2} \tag{26}$$

For cases where the antennas are close to the Earth and the path lengths are long enough such that the Earth cannot be considered flat, the field-strength computation is performed using either a flat-Earth attenuation function with a small-Earth curvature correction or a power series expansion. These two techniques reduce the need to use the numerical integration of the full-wave theory integral (equation 20), since it is a very time consuming computation [23]. These two techniques bridge the gap for loss computation between the case where the Earth is flat (flat-Earth attenuation function) and that where the receiving antenna is near the radio horizon of the transmitting antenna.

The computation technique (either flat-Earth attenuation function with curvature correction or power series expansion) is selected depending on whether the magnitude of the factor q (already defined) is small or large [24]. Recall that q is given by:

$$q = -j(ka/2)^{1/3}\Delta \tag{27}$$

where $k=2\pi/\lambda$, λ is the wavelength of the radio wave in meters, a is the radius of the Earth in meters, and Δ is the normalized surface impedance of the ground below the antenna in question. The expressions for Δ are the same as given in equations (15) through (17). The surface impedance Δ is a function of the ground constants of the Earth's surface.

If the magnitude of q is small (<0.1), then a power series expansion is used for the attenuation function $f(x)$. The electric field E at a great circle distance d on the Earth's surface is $E_0 \bullet f(x)$. E_0 is the electric field of the same dipole source located on a flat perfect conductor. The power series expansion for $f(x)$ for small q (<0.1) is given by [24-30]:

$$f(x) = \sum_{m=0}^{\infty} A_m [e^{j\pi/4} q\, x^{1/2}]^m \tag{28}$$

where

$$x = d/a(ka/2)^{1/3} \tag{29}$$

$$\begin{aligned}
A_0 &= 1 \\
A_1 &= -j\pi^{1/2} \\
A_2 &= -2 \\
A_3 &= j\pi^{1/2}(1+1/4q^3) \\
A_4 &= 4/3(1+1/2q^3) \\
A_5 &= -j\pi^{1/2}/4(1+3/4q^3) \\
A_6 &= -8/15(1+1/q^3+7/32q^6) \\
A_7 &= j\pi^{1/2}/6(1+5/4q^3+27/32q^6) \\
A_8 &= 16/105(1+3/2q^3+27/32q^6 \\
A_9 &= -j\pi^{1/2}/24(1+7/4q^3+5/4q^6+21/64q^9) \\
A_{10} &= -(32/945+64/945q^3+11/189q^6+1/270q^9)
\end{aligned} \tag{30}$$

Higher order coefficients are not available in the literature, but the accuracy is adequate with the coefficients given above [24].

If the magnitude of q is large (≥ 0.1), then a small curvature expansion is more appropriate for the attenuation function [25,27,28]. The expression for the small curvature expansion is given by:

$$\begin{aligned}
f(x) = F(p) + \frac{1}{4q^3}[1-j[\pi p]^{1/2} - (1+2p)F(p)] + \frac{1}{4q^6}[1-j[\pi p]^{1/2}(1-p) - 2p \\
+ \frac{5p^2}{6} + (\frac{p^2}{2} - 1)F(p)]
\end{aligned} \tag{31}$$

$$\begin{aligned}
F(p) &= 1 - j[\pi p]^{1/2} e^{-p} \operatorname{erfc}(jp)^{1/2} \\
p &= jxq^2 \equiv -jkd\Delta^2/2
\end{aligned} \tag{32}$$

The erfc is the complementary error function defined previously [21]. The implementation of these two techniques reduces the need for the numerical integration technique and reduces computation time considerably.

If the terrain contour is “smooth” or the terrain irregularities are much smaller than a wavelength, then the smooth-Earth method is mathematically and numerically accurate for ground-wave predictions in a frequency range from 10 kHz to 100 MHz; however above 30 MHz the irregularities of the atmosphere make statistical methods more appropriate. All of the models discussed so far are part of the smooth-Earth method.

When the terrain contour is smooth, the smooth-Earth model is valid for all combinations of antenna heights, frequency, and dielectric constants by virtue of the computation techniques contained within its structure. It should be used only out to the maximum distances considered useful for ground-wave propagation at each frequency, since the sky wave will become significant from those distances to points beyond.

3.1.2 The Smooth-Earth Mixed-Path Model

A mixed path is one where the ground constants change along the propagation path between the transmitter and receiver. The path can be described by a series of finite length segments, each with different ground constants. The smooth-Earth mixed-path model is a specific sequence of smooth-Earth model runs over each of the segments that are then combined in a particular order as determined by the Millington algorithm [31]. The Millington algorithm is based on reciprocity considerations. This smooth-Earth, mixed-path model is valid for the same frequency and distance ranges as the smooth-Earth method. The antenna heights are set to zero for each smooth-Earth run over each of the segments and combination of segments required by the algorithm. A height-gain function is then applied to the transmitter and receiver antennas using the ground constants under each antenna and the antenna heights. The result is the propagation loss over a mixed path with compensation for antenna heights. The Millington algorithm implemented in the smooth-earth, mixed-path model will be discussed for the three section mixed path of Figure 2. Expansion to more sections is a straightforward process.

The first step involves the calculation of losses using the smooth-Earth model over single sections and combinations of sections using the different ground constants. With a transmitter at T as a source, (Figure 2), compute the loss L_{tr} in decibels (dB).

$$L_{tr}(dB) = L_1(d_1) - L_2(d_1) + L_2(d_1+d_2) - L_3(d_1+d_2) + L_3(d_1+d_2+d_3) \qquad (33)$$

where $L_1(d_1)$ is the loss in dB over distance d_1 using σ_1 and ε_1, $L_2(d_1)$ is the loss in dB over distance d_1 using σ_2 and ε_2, $L_2(d_1+d_2)$ is the loss in dB over distance d_1+d_2 using σ_2 and ε_2, $L_3(d_1+d_2)$ is the loss in dB over distance d_1+d_2 using σ_3 and ε_3, and $L_3(d_1+d_2+d_3)$ is the loss in dB over distance $d_1+d_2+d_3$ using σ_3 and ε_3.

With a transmitter at R as a source (Figure 2), compute the loss L_{rt} in decibels.

$$L_{rt}(dB) = L_3(d_3) - L_2(d_3) + L_2(d_3+d_2) - L_1(d_3+d_2) + L_1(d_3+d_2+d_1) \qquad (34)$$

where $L_3(d_3)$ is the loss in dB over distance d_3 using σ_3 and ε_3, $L_2(d_3)$ is the loss in dB over

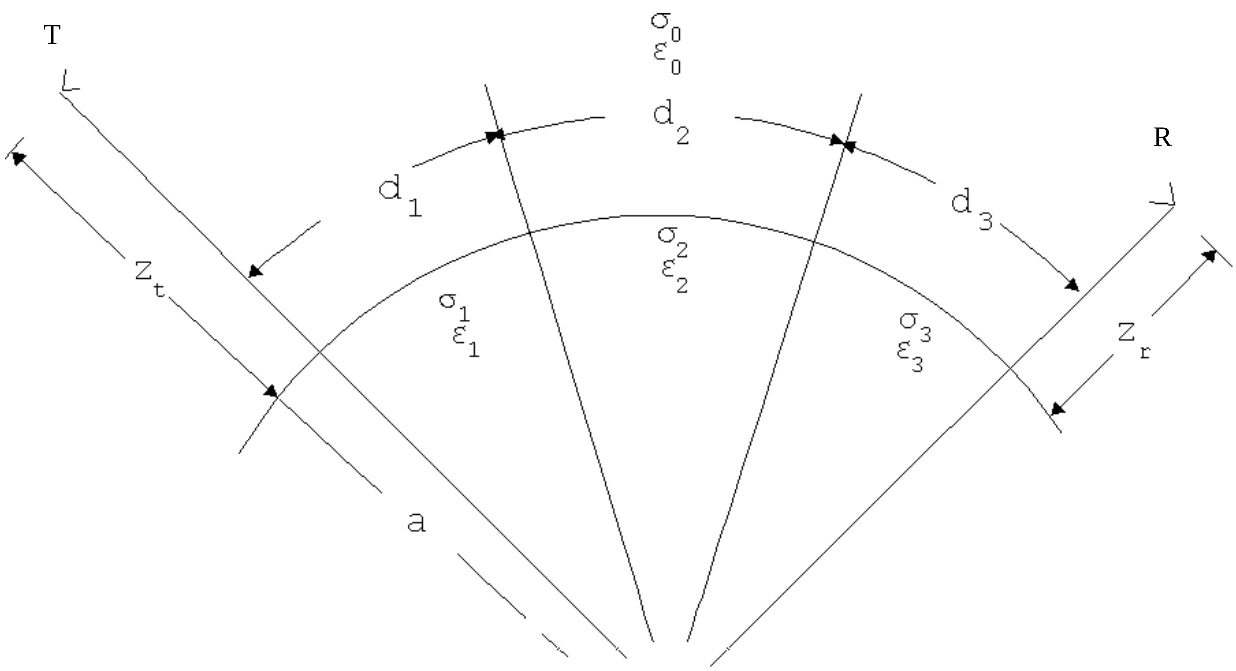

Figure 2. Spherical smooth-Earth mixed-path geometry.

distance d_3 using σ_2 and ε_2, $L_2(d_3+d_2)$ is the loss in dB over distance d_3+d_2 using σ_2 and ε_2, $L_1(d_3+d_2)$ is the loss in dB over distance d_3+d_2 using σ_1 and ε_1, and $L_1(d_3+d_2+d_1)$ is the loss in dB over distance $d_3+d_2+d_1$ using σ_1 *and* ε_1.

The total loss $L_t(dB)$ is then computed:

$$L_t(dB) = \frac{L_{tr}(dB) + L_{rt}(dB)}{2} - 20\log(|G(z_t)| * |G(z_r)|) \qquad (35)$$

$|G(z_t)|$ and $|G(z_r)|$ denote the magnitude of the height-gain functions for the transmitter and receiver antennas respectively.

The height-gain function is given approximately by the first two terms of the Taylor series expansion for the exact height-gain function arising from the smooth-Earth diffraction series [19,22,28,32]:

$$G(z) = 1 + jk\Delta z \tag{36}$$

where z is the transmitter height, z_t, or receiver antenna height, z_r, in meters. The factor $k = 2\pi/\lambda$, and λ is the wavelength in meters of the radio wave. The normalized surface impedance of the ground below the antenna in question is Δ as defined previously. The imaginary part of a complex number is denoted by j.

The function is valid up to about the first maximum in the height-gain pattern [19,29]. This height gain function is used in the smooth-Earth mixed-path model and the irregular-Earth mixed-path model discussed in the next section.

3.1.3 The Irregular-Earth Mixed-Path Model

Irregularities in the terrain have a greater effect at higher frequencies, so an irregular terrain model is more appropriate when terrain irregularities become appreciable in size with respect to a wavelength. The smooth-Earth model is much more computationally efficient and many orders of magnitude faster than the irregular-Earth model, so in cases where the terrain is smooth enough, the smooth-Earth model can be used with minimal sacrifice in accuracy. Specific comparisons of smooth-Earth and irregular-Earth predictions with actual measurements have been made for different terrain irregularities [6].

For the MF band it was demonstrated that for terrain variations along the path of less than a wavelength, the smooth-Earth and irregular-Earth models were in close agreement. The irregular-Earth, mixed-path model in this ground-wave model uses an integral equation [33,34] to compute the propagation loss of a vertically polarized electromagnetic wave over irregular terrain. This approach is a point-to-point prediction method valid for frequencies between 10 kHz and 30 MHz. Later versions [33,34] of this model simulate terrain that is covered with forests, buildings, or snow, where the terrain cover is modeled as a slab of user-specified thickness, length, conductivity, and dielectric constant. Antenna heights of the transmitter and receiver antennas without a slab are included in the irregular-Earth propagation loss computations using the same height-gain functions as the smooth-Earth, mixed-path model. When a slab is included, a special height-gain function [33] is used for the antennas within or above the slab for the irregular-Earth mixed-path model.

The approach used by Ott [34] was to solve the irregular-Earth mixed-path model integral equation numerically using a technique based on Wagner's method [35]. The integral equation is a solution to a parabolic differential equation [34]. The complete derivation of the integral equation as well as the actual numerical evaluation of the integral equation can be found in the literature [36,37]. The parabolic wave equation can be derived from the general wave equation [34,36]. The general wave equation is given by:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + k^2\varphi = -2\pi\tau(x,y) \quad , y > y(x) \tag{37}$$

The solution to this equation satisfies an impedance boundary condition [36]:

$$\frac{\partial\varphi}{\partial n}=\frac{jk\Delta\varphi}{\sqrt{1+(y')^2}}\ ,\ y=y(x),\ y'=\frac{dy}{dx} \tag{38}$$

where φ is the vertical component of the electric field for vertical polarization or the vertical component of the magnetic field for horizontal polarization with time dependence $e^{j\omega t}$ [36]. The function $\tau(x,y)$ is the source distribution.

If we let $\varphi(x,y)=e^{-jkx}\psi(x,y)$, and substitute this into equation (37), then equation (37) is transformed into:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}-2jk\frac{\partial\psi}{\partial x}=-2\pi\tau(x,y)e^{jkx} \tag{39}$$

If it is then assumed that the term $\partial^2\psi/\partial x^2$ is much smaller than the other terms in the equation, then the equation becomes:

$$\frac{\partial^2\psi}{\partial y^2}-2jk\frac{\partial\psi}{\partial x}=-2\pi\tau(x,y)e^{jkx} \tag{40}$$

This is a parabolic differential equation. The parabolic equation may be used to approximate the scalar wave equation if the polarization of the wave remains constant along the path and if the direction of propagation is at low angles and nearly horizontal [38]. The solution to this parabolic differential equation is an integral equation, where the path of integration is along the irregular ground on a line between the transmitter and the receiver. The integral equation has the form [34]:

$$\begin{aligned} f(x)=g(x,y)W(x,0)-\sqrt{j/x}\int_0^x f(\xi)e^{-k\phi(x,\xi)}\ \bullet \\ \left[y'(\xi)W(x,\xi)-\frac{y(x)-y(\xi)}{x-\xi}+(\Delta(\xi)-\Delta_r)W(x,\xi)\right]\bullet\sqrt{\frac{x}{\xi(x-\xi)}}\,d\xi \end{aligned} \tag{39}$$

where $f(x)$ is the field normalized to twice the free-space field, and Δ_r is the normalized surface impedance at the transmitter antenna. The normalized surface impedance can be computed from σ,ε, and the frequency f using the expressions given earlier in this paper in the smooth-Earth discussion. $\Delta(\xi)$ is a function of these parameters at the integration point on the path ξ, and $\Delta(\xi)-\Delta_r$ is zero for a homogeneous path. The surface impedance $\Delta(\xi)$ along the path can be continuous or have abrupt changes along the path.

$$W(x,\xi) = 1 - j\sqrt{\pi P}\; w(-\sqrt{u}) \tag{42}$$

$$P = -jk[\Delta(\xi)]^2 \frac{(x-\xi)}{2} \tag{43}$$

$$y'(\xi) = \frac{dy}{d\xi} \tag{44}$$

$$u = P\,[1 - \frac{y(x)-y(\xi)}{\Delta(x-\xi)}]^2 \qquad for \;\; \xi < x \tag{45}$$

$$\phi(x,\xi) = \frac{[y(x)-y(\xi)]^2}{2(x-\xi)} + \frac{y^2(\xi)}{2} - \frac{y^2(x)}{2} \tag{46}$$

$$w(-\sqrt{u}) = e^{-u}\mathrm{erfc}(j\sqrt{u}) \tag{47}$$

$$w(\sqrt{-u}) = \frac{1}{j\pi}\int_{-\infty}^{\infty} \frac{e^{-t^2}dt}{\sqrt{u}+t} \tag{48}$$

This integral definition of the complementary error function (equation 48) is defined in [21]. Figure 3 shows the geometry for the irregular-Earth mixed-path model. The symbol *x* denotes the distance from the transmitter antenna at which the receiver antenna is located, *y(x)* denotes the height of the receiver antenna with respect to the transmitter antenna height, ξ is the distance of the integration point measured from the transmitter, *y(* ξ *)* denotes the height of the integration point with respect to the transmitter height, and *g(x,y)* is the antenna pattern factor for the transmitter antenna. The transmitter terrain is at zero height and serves as a reference height since all heights are used as program input only after the terrain height at the transmitter is subtracted from each terrain height.

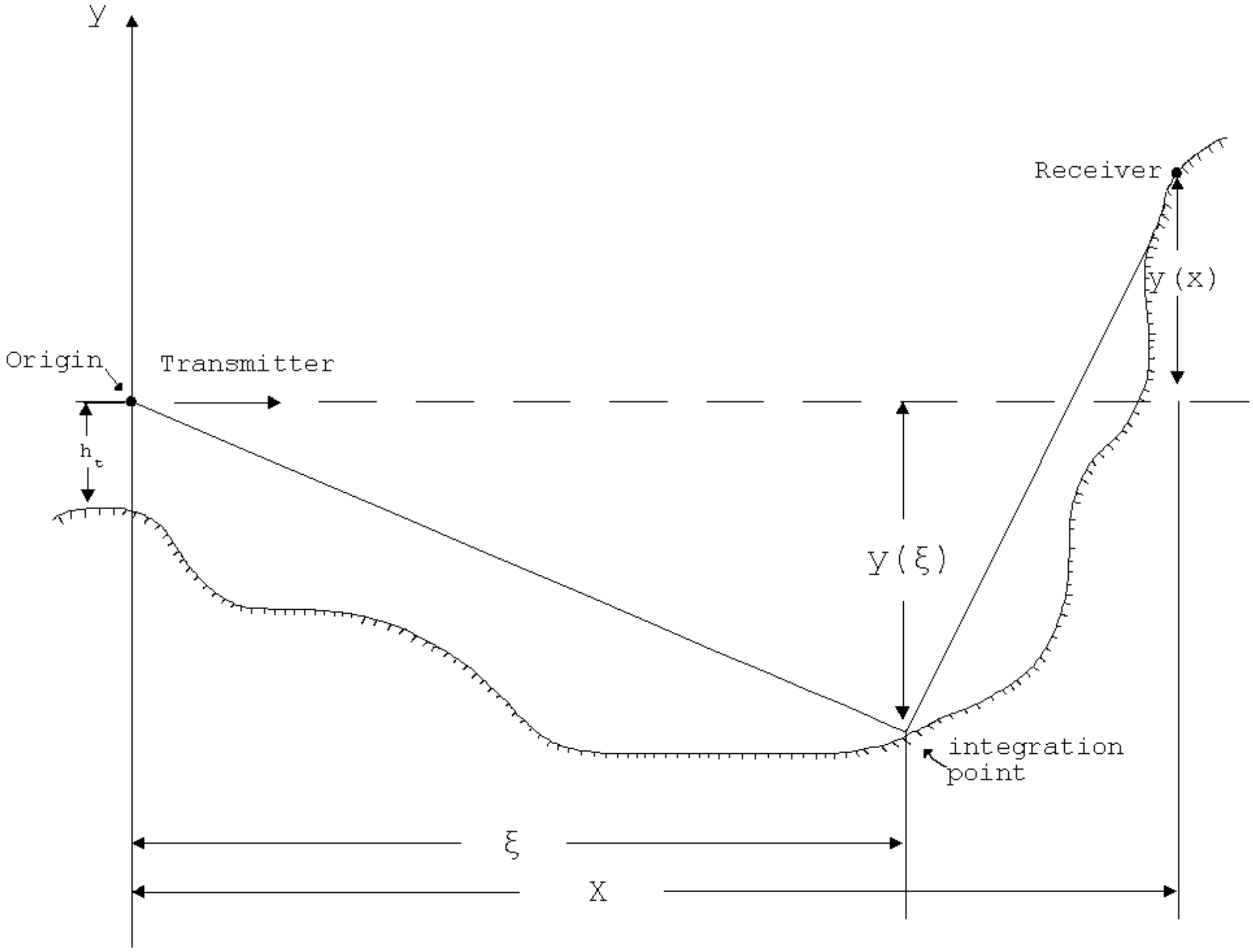

Figure 3. Geometry for integral equation irregular-Earth model along the great circle path.

W(x,ξ) is the flat-Earth attenuation function. The terms within the bracket under the integral sign represent the surface impedance, height, and slope of the ground at the integration point *ξ*. The integral equation computes the relative field *f(x)* at a point *x* along the path in terms of *f (ξ)*, its value at all previous points along the path.

The numerical solution of the integral equation is obtained by dividing the path up into discrete intervals. The field *f(ξ)* must be known initially at a series of discrete points. The field at the very next point is determined by fitting a second order polynomial in each of the intervals and then performing a numerical integration. The initial points are determined using the Sommerfeld attenuation function, and then the integral term of the equation is evaluated. The computer program implementation calculates the electric field at all desired points between the transmitter and receiver. The integral is evaluated over the irregular ground between the terminals, and the ground is considered inhomogeneous in the direction of propagation along the path between the transmitter and receiver.

It has also been assumed that the electrical properties of the ground may be described by its surface impedance which is a function of the ground constants σ and ε. The integral equation method can be used with any variations of terrain represented as heights along the propagation path. The terrain heights can change abruptly or continuously. The terrain does not need to be described by standard geometrical features or canonical shapes. The terrain can be represented by a completely arbitrary profile. The program assumes that the terrain varies linearly between points input by the terrain file or user. If the program decides that it needs additional terrain points between those given, then linear interpolation is used to determine terrain heights between available terrain heights. The program automatically chooses the spacing so that the terrain is sampled frequently enough for an accurate representation of the terrain variation, and so that the numerical integration of the integral equation is sufficiently accurate, but a compromise is also made so as to prevent excessive computation time. The distance between points should be long so as to minimize computation time, since the computation time is proportional to the square of the number of computation points, but it should be short to accurately represent the terrain and provide an accurate numerical integration.

The analytical details of the integral equation and its derivation are described in the references [33,34]. In both of these references, good agreement is found between this method and other analytical computation methods. Comparisons of calculations with measurements have also been made [6,39,40]. It has been found that for terrain variations smaller than a wavelength, the smooth-Earth and smooth-Earth mixed-path models result in comparable accuracy to the irregular-Earth mixed-path model, so it may be more efficient to use one of the smooth-Earth models.

4. SKY-WAVE PROPAGATION

A medium frequency sky wave will be returned back to Earth by the ionosphere if the degree of ionization in the appropriate regions is sufficient to refract and reflect the incident electromagnetic wave. Ionospheric propagation models for medium frequencies can predict this degree of ionization in the different layers to determine the amount of signal that is refracted and reflected and hence the system performance. The two regions that are responsible for the refraction and reflection of medium frequencies are the D region and the E region. The first region encountered by the sky wave is the D region which extends in a layer that is 50 to 90 km above the Earth's surface [41,42]. It is a region of low electron density whose degree of ionization is determined primarily by solar photoionization. This region usually exists during the daytime. This region has a low electron density and the electrons collide with predominantly neutral gases, so this region absorbs the energy in the MF radio waves that pass through it during the daytime hours [41,42]. The MF sky wave is therefore highly attenuated as it passes through the D layer during the daytime.

At night in the absence of the photoionization created by the sunlight, the ionization in the D region is at a much lower level or is nonexistent, so the D region no longer absorbs the energy from the MF sky wave passing through it. The MF sky wave proceeds to the E region above this D region where it is reflected and refracted. The E-region ionization is from multiple sources that exist all of the time, so it is active during both the daytime and the nighttime. E-region ionization in the daytime