Annex V

PHASE INTERFERENCE FADING AND SERVICE PROBABILITY

As a general rule, adequate service over a radio path requires protection against noise when propagation conditions are poor, and requires protection against interference from cochannel or adjacent channel signals when propagation conditions are good. Optimum use of the radio spectrum requires systems so designed that the reception of wanted signals is protected to the greatest degree practicable from interference by unwanted radio signals and by noise.

The short-term fading of the instantaneous received power within periods of time ranging from a few minutes up to one hour or more is largely associated with random fluctuations in the relative phasing between component waves. These waves arrive at the receiving antenna after propagation via a multiplicity of propagation paths having electrical lengths that vary from second to second and from minute to minute over a range of a few wavelenths. A small part of this short-term fading and usually all of the long-term variations arise from minute-to-minute changes in the root-sum-square value of the amplitudes of the component waves, i.e., in short-term changes in the mean power available from the receiving antenna. In the analysis of short-term fading, it is convenient to consider the effects of these phase and root-sum-square amplitude changes as being two separate components of the instantaneous fading. Multipath or "phase interference fading" among simultaneously occurring modes of propagation usually determines the statistical character of short-term variability.

Over most transhorizon paths, long-term variability is dominated by "power fading", due to slow changes in average atmospheric refraction, in the intensity of refractive index turbulence, or in the degree of atmospheric stratification. The distinction between phase interference fading and power fading is somewhat arbitrary, but is nevertheless extremely useful. Economic considerations, as contrasted to the requirements for spectrum conservation, indicate that radio receiving systems should be designed so that the minimum practicable transmitter power is required for satisfactory reception of wanted signals in the presence of noise. Fading expected within an hour or other convenient "short" period of time is allowed for by comparing the median wanted signal power w_m available at the receiver with the median wanted signal power w_{mr} which is required for satisfactory reception in the presence of noise. This operating sensitivity w_{mr} assumes a specified type of fading signal and a specified type of noise, but does not allow for other unwanted signals.

In the presence of a specified unwanted signal, but in the absence of other unwanted signals or appreciable noise, the fidelity of information delivered to a receiver output will increase as the ratio r_u of wanted-to-unwanted signal power increases. The degree of fidelity of the received information may be measured in various ways. For example, voice signals are often measured in terms of their intelligibility, television pictures by subjective observation, and teletype signals by the percentage of correctly interpreted received characters. A specified grade of service provided by a given wanted signal will guarantee a corresponding degree of fidelity of the information delivered to the receiver output. For example, a Grade A teletype service could be defined as one providing 99.99 percent error-free characters, while a Grade B service could be defined as one providing 99.9 percent error-free characters.

The protection ratio r_{ur} required for a given grade of service will depend upon the nature of the wanted and unwanted signals; i.e., their degree of modulation, their location in the spectrum relative to the principal and spurious response bands of the receiving system, and their phase interference fading characteristics. The use of receiving systems having the smallest values of r_{ur} for the kinds of unwanted signals likely to be encountered will permit the same portions of the spectrum to be used simultaneously by the maximum number of users. For instance, FM with feedback achieves a reduction in r_{ur} for a cochannel unwanted signal by occupying a larger portion of the spectrum. But optimum use of the spectrum requires a careful balance between reductions in r_{ur} on the same channel and on adjacent channels, taking account of other system isolation factors such as separation between channels, geographical separation, antenna directivity, and cross-polarization.

Note that the operating sensitivity w_{mr} is a measure of the required magnitude of the median wanted signal power but r_{ur} involves only the ratio of wanted to unwanted signal powers. For optimum use of the spectrum by the maximum number of simultaneous users, the transmitting and receiving systems of the individual links should be designed with the primary objective of ensuring that the various values of r_u exceed r_{ur} for a large percentage of the time during the intended periods of operation. Then sufficiently high transmitter powers should be used so that the median wanted signal power w_m exceeds w_{mr} for a large percentage of the time during the intended period of reception at each receiving location. This approach to frequency assignment problems will be unrealistic in a few cases, such as the cleared channels required for radio astronomy, but these rare exceptions merely serve to test the otherwise general rule [Norton, 1950, 1962 and Norton and Fine 1949] that optimum use of the spectrum can be achieved only when interference from other signals rather than from noise provides the ineluctable limit to satisfactory reception.

This annex discusses the requirements for service of a given grade g, how to estimate the expected time availability q of acceptable service, and, finally, how to calculate the service probability Q for a given time availability.

V.1 Two Components of Fading

Both the wanted and the unwanted signal power available to a receiving system will usually vary from minute to minute in a random or unpredictable fashion. It is convenient to divide the "instantaneous received signal power" $W_\pi = 10 \log w_\pi$ into two or three additive components where w_π is defined as the average power for a single cycle of the radio frequency, so as to eliminate the variance of power associated with the time factor $\cos^2(\omega t)$:

$$W_\pi = W_m + Y_\pi = W_m(0.5) + Y + Y_\pi \quad \text{dbw} \tag{V.1}$$

W_m is that component of W_π which is not affected by the usually rapid phase interference fading and is most often identified as the short-term median of the available power W_π at the receiving antenna. $W_m(0.5)$ is the median of all such values of W_m, and is most often identified as the long-term median of W_π. In terms of the long-term median transmission loss $L_m(0.5)$ and the total power W_t radiated from the transmitting antenna:

$$W_m(0.5) = W_t - L_m(0.5) \quad \text{dbw} \tag{V.2}$$

The characteristics of long-term fading and phase interference fading, respectively, are described in terms of the two fading components Y and Y_π in (V.1):

$$Y = W_m - W_m(0.5), \quad Y_\pi = W_\pi - W_m \tag{V.3}$$

The long term for which the median power, $W_m(0.5)$, is defined may be as short as one hour or as long as several years but will, in general, consist of the hours within a specified period of time. For most continuously operating services it is convenient to consider $W_m(0.5)$ as the median power over a long period of time, including all hours of the day and all seasons of the year. Observations of long-term variability, summarized in section 10 and in annex III, show that W_m is a very nearly normally distributed random variable characterized by a standard deviation that may range from one decibel within an hour up to ten decibels for periods of the order of several years. These values of standard deviation are representative only of typical beyond-the-horizon propagation paths and vary widely for other propagation conditions.

For periods as short as an hour, the variance of Y_π is generally greater than the variance of W_m The long-term variability of W_m is identified in section 10 with the variability of hourly medians, expressed in terms of $Y(q)$:

$$Y(q) = W_m(q) - W_m(0.5) = L_m(0.5) - L_m(q)$$

where $W_m(q)$ is the hourly median signal power exceeded for a fraction q of all hours, and $L_m(q)$ is the corresponding transmission loss not exceeded for a fraction q of all hours.

Often, data for a path are available in terms of the long-term cumulative distribution of instantaneous power, W_π; that is, we know $W_\pi(q)$ versus q, but not $W_m(q)$ versus q. An approximation to the cumulative distribution function $L_o(q)$ versus q is given by

$$L_o(q) \cong L_m(0.5) \pm [Y^2(q) + Y_\pi^2(q)]^{\frac{1}{2}} . \qquad (V.5)$$

The plus sign in (V.5) is used for $q > 50$ percent, and the minus sign for $q < 50$ percent.

V.2 The Nakagami-Rice Distribution

For studies of the operating sensitivity w_{mr} of a receiver in the presence of a rapidly fading wanted signal, and for studies of the median wanted signal to unwanted signal ratio $R_{ur}(g)$ required for a grade g service, it is helpful to consider a particular statistical model which may be used to describe phase interference fading. Minoru Nakagami [1940] describes a model which depends upon the addition of a constant signal and a Rayleigh-distributed random signal [Rayleigh 1880; Rice, 1945; Norton, Vogler, Mansfield and Short, 1955; Beckmann, 1961a, 1964]. In this model, the root-sum-square value of the amplitudes of the Rayleigh components is K decibels relative to the amplitude of the constant component. $K = +\infty$ corresponds to a constant received signal. For a Rayleigh distribution, $K = -\infty$ and the probability q that the instantaneous power, w_π, will exceed $w_\pi(q)$ for a given value of the short-term median power, w_m, may be expressed:

$$q[w_\pi > w_\pi(q) \mid w_m] = \exp[-\frac{w_\pi(q)\log_e 2}{w_m}], \ [K = -\infty] \qquad \text{(V.6a)}$$

Alternatively, the above may be expressed in the following forms:

$$q[Y_\pi > Y_\pi(q)] = \exp[-y_\pi(q)\log_e 2], \ [K = -\infty] \qquad \text{(V.6b)}$$

$$Y_\pi(q) = 5.21390 + 10\log\{\log(1/q)\}, \ [K = -\infty] \ . \qquad \text{(V.6c)}$$

Figures V.1-V.3 and table V.1 show how the Nakagami-Rice phase interference fading distribution $Y_\pi(q)$ depends on q, K, and the average $\overline{Y}_\pi$ and standard deviation σ_{Y_π} of Y_π. It is evident from figure V.1 that the distribution of phase interference fading depends only on K. The utility of this distribution for describing phase interference fading in ionospheric propagation is discussed in CCIR report [1963k] and for tropospheric propagation is demonstrated in papers by Norton, Rice and Vogler [1955], Janes and Wells [1955], and Norton, Rice, Janes and Barsis [1955]. Bremmer [1959] and Beckmann [1961a] discuss a somewhat more general fading model.

For within-the-horizon tropospheric paths, including either short point-to-point terrestrial paths or paths from an earth station to a satellite, K will tend to have a large positive value throughout the day for all seasons of the year. As the length of the terrestrial propagation path is increased, or the elevation angle of a satellite is decreased, so that the path has less than first Fresnel zone clearance, the expected values of K will decrease until, for some hours of the day, K will be less than zero and the phase interference fading for signals propagated over the path at these times will tend to be closely represented by a Rayleigh distribution. For most beyond-the-horizon paths K will be less than zero most of the time. For knife-edge diffraction paths K is often much greater than zero. When signals arrive at the receiving antenna via ducts or elevated layers, the values of K may increase to values much greater than zero even for transhorizon propagation paths. For a given beyond-the-horizon path, K will tend to be positively correlated with the median power level W_m; i.e., large values of K are expected with large values of W_m. For some within-the-horizon paths, K and W_m tend to be negatively correlated.

It is assumed that a particular value of K may be associated with any time availability q, however, few data analyses of this kind are presently available. A program of data analysis is clearly desirable to provide empirical estimates of K versus q for particular climates, seasons, times of day, lengths of recording, frequencies, and propagation path characteristics. It should be noted that K versus q expresses an assumed functional relationship.

An analysis of data for a single day's recording at 1046 MHz over a 364-kilometer path is presented here to illustrate how a relationship between K and q may be established. Figure V.4 shows for a single day the observed interdecile range $W_\pi(0.1) - W_\pi(0.9)$ $L_\pi(0.9) - L_\pi(0.1)$ for each five-minute period plotted against the median transmission loss L_m for the five-minute period. Figure V.1 associates a value of K with each value of $L_\pi(0.9) - L_\pi(0.1)$, and the cumulative distribution of L_m associates a time availability q with each value of L_m. In figure V.4, K appears to increase with increasing L_m for the hours 0000 - 1700, although the usual tendency over long periods is for K to decrease with L_m. The straight line in the figure is drawn through medians for the periods 0000 - 1700 and 1700 - 2400 hours, with linear scales for K and L_m, as shown by the inset for figure V.4. The corresponding curve of K versus q is compared with the data in the main figure.

Figure V.5 shows for $f = 2$ GHz and 30-meter antenna heights over a smooth earth a possible estimate of K versus distance and time availability. These crude and quite speculative estimates are given here only to provide the example in the lower part of figure V.5 which shows how such information with (V.6) may be used to obtain curves of $L_\pi(q)$ versus q. The solid curves in the lower part of figure V.5 show how $L_\pi(q)$ varies with distance for q = 0.0001, 0.01, 0.5, 0.99 and 0.9999, where L_π is the transmission loss associated with the instantaneous power, W_π.

Table V.1 Characteristics of the Nakagami-Rice
Phase Interference Fading Distribution $Y_\pi(q)$

$Y_\pi > Y_\pi(q)$ **with Probability** q; $Y_\pi(0.5) \equiv 0$

K	$\overline{Y}_\pi$	$\sigma_{Y\pi}$	$Y_\pi(0.005)$	$Y_\pi(0.01)$	$Y_\pi(0.02)$	$Y_\pi(0.05)$	$Y_\pi(0.1)$	$Y_\pi(0.9)$	$Y_\pi(0.95)$	$Y_\pi(0.98)$	$Y_\pi(0.99)$	$Y_\pi(0.995)$	$Y_\pi(0.1) - Y_\pi(0.9)$
db	db	db	db	db	db	db	db	db	db	db	db	db	db
40	-0.0002	0.061	0.1568	0.1417	0.1252	0.1004	0.0784	-0.0790	-0.1016	-0.1270	-0.1440	-0.1596	0.1574
35	-0.0007	0.109	0.2768	0.2504	0.2214	0.1778	0.1352	-0.1411	-0.1815	-0.2272	-0.2579	-0.2860	0.2763
30	-0.0022	0.194	0.4862	0.4403	0.3898	0.3136	0.2453	-0.2525	-0.3254	-0.4082	-0.4638	-0.5151	0.4978
25	-0.0069	0.346	0.8460	0.7676	0.6811	0.5496	0.4312	-0.4538	-0.5868	-0.7391	-0.8421	-0.9374	0.8850
20	-0.0217	0.616	1.4486	1.3184	1.1738	0.9524	0.7508	-0.8218	-1.0696	-1.3572	-1.5544	-1.7389	1.5726
18	-0.0343	0.776	1.7840	1.6264	1.4508	1.1846	0.9332	-1.0453	-1.3660	-1.7416	-2.0014	-2.2461	1.9785
16	-0.0543	0.980	2.1856	1.9963	1.7847	1.4573	1.1558	-1.3326	-1.7506	-2.2463	-2.5931	-2.9231	2.4884
14	-0.0859	1.238	2.6605	2.4355	2.1829	1.7896	1.4247	-1.7028	-2.2526	-2.9156	-3.3872	-3.8422	3.1275
12	-0.136	1.569	3.2136	2.9491	2.6507	2.1831	1.7455	-2.1808	-2.9119	-3.8143	-4.4715	-5.1188	3.9263
10	-0.214	1.999	3.8453	3.5384	3.1902	2.6408	2.1218	-2.7975	-3.7820	-5.0372	-5.9833	-6.9452	4.9193
8	-0.334	2.565	4.5493	4.1980	3.7975	3.1602	2.5528	-3.5861	-4.9287	-6.7171	-8.1418	-9.6386	6.1389
6	-0.507	3.279	5.3093	4.9132	4.4591	3.7313	3.0307	-4.5714	-6.4059	-8.9732	-11.0972	-13.4194	7.6021
4	-0.706	4.036	6.0955	5.6559	5.1494	4.3315	3.5366	-5.7101	-8.1216	-11.5185	-14.2546	-17.1017	9.2467
2	-0.866	4.667	6.8613	6.3811	5.8252	4.9219	4.0366	-6.7874	-9.6278	-13.4690	-16.4258	-19.4073	10.8240
0	-0.941	5.094	7.5411	7.0246	6.4248	5.4449	4.4782	-7.5267	-10.5553	-14.5401	-17.5512	-20.5618	12.0049
-2	-0.953	5.340	8.0697	7.5228	6.8861	5.8423	4.8088	-8.0074	-11.0005	-15.0271	-18.0527	-21.0706	12.8162
-4	-0.942	5.465	8.4231	7.8525	7.1873	6.0956	5.0137	-8.0732	-11.1876	-15.2273	-18.2573	-21.2774	13.0869
-6	-0.929	5.525	8.6309	8.0435	7.3588	6.2354	5.1233	-8.1386	-11.2606	-15.3046	-18.3361	-21.3565	13.2619
-8	-0.922	5.551	8.7394	8.1417	7.4451	6.3034	5.1749	-8.1646	-11.2893	-15.3349	-18.3669	-21.3880	13.3395
-10	-0.918	5.562	8.7918	8.1881	7.4857	6.3341	5.1976	-8.1753	-11.3005	-15.3466	-18.3788	-21.4000	13.3729
-12	-0.916	5.567	8.8155	8.2090	7.5031	6.3474	5.2071	-8.1792	-11.3048	-15.3512	-18.3834	-21.4046	13.3863
-14	-0.916	5.569	8.8258	8.2179	7.5106	6.3531	5.2112	-8.1804	-11.3065	-15.3529	-18.3852	-21.4064	13.3916
-16	-0.915	5.570	8.8301	8.2216	7.5136	6.3552	5.2128	-8.1811	-11.3072	-15.3537	-18.3860	-21.4072	13.3929
-18	-0.915	5.570	8.8319	8.2232	7.5149	6.3561	5.2135	-8.1813	-11.3075	-15.3540	-18.3863	-21.4075	13.3948
-20	-0.915	5.570	8.8326	8.2238	7.5154	6.3565	5.2137	-8.1814	-11.3076	-15.3541	-18.3864	-21.4076	13.3951
$-\infty$	-0.915	5.570	8.8331	8.2242	7.5158	6.3567	5.2139	-8.1815	-11.3077	-15.3542	-18.3865	-21.4077	13.3954

THE NAKAGAMI-RICE PROBABILITY DISTRIBUTION OF THE INSTANTANEOUS FADING ASSOCIATED WITH PHASE INTERFERENCE

q IS THE PROBABILITY THAT $Y_\pi = (W_\pi - W_m)$ EXCEEDS $Y_\pi(q)$.
AS K DECREASES WITHOUT LIMIT, THE NAKAGAMI-RICE DISTRIBUTION APPROACHES THE RAYLEIGH DISTRIBUTION

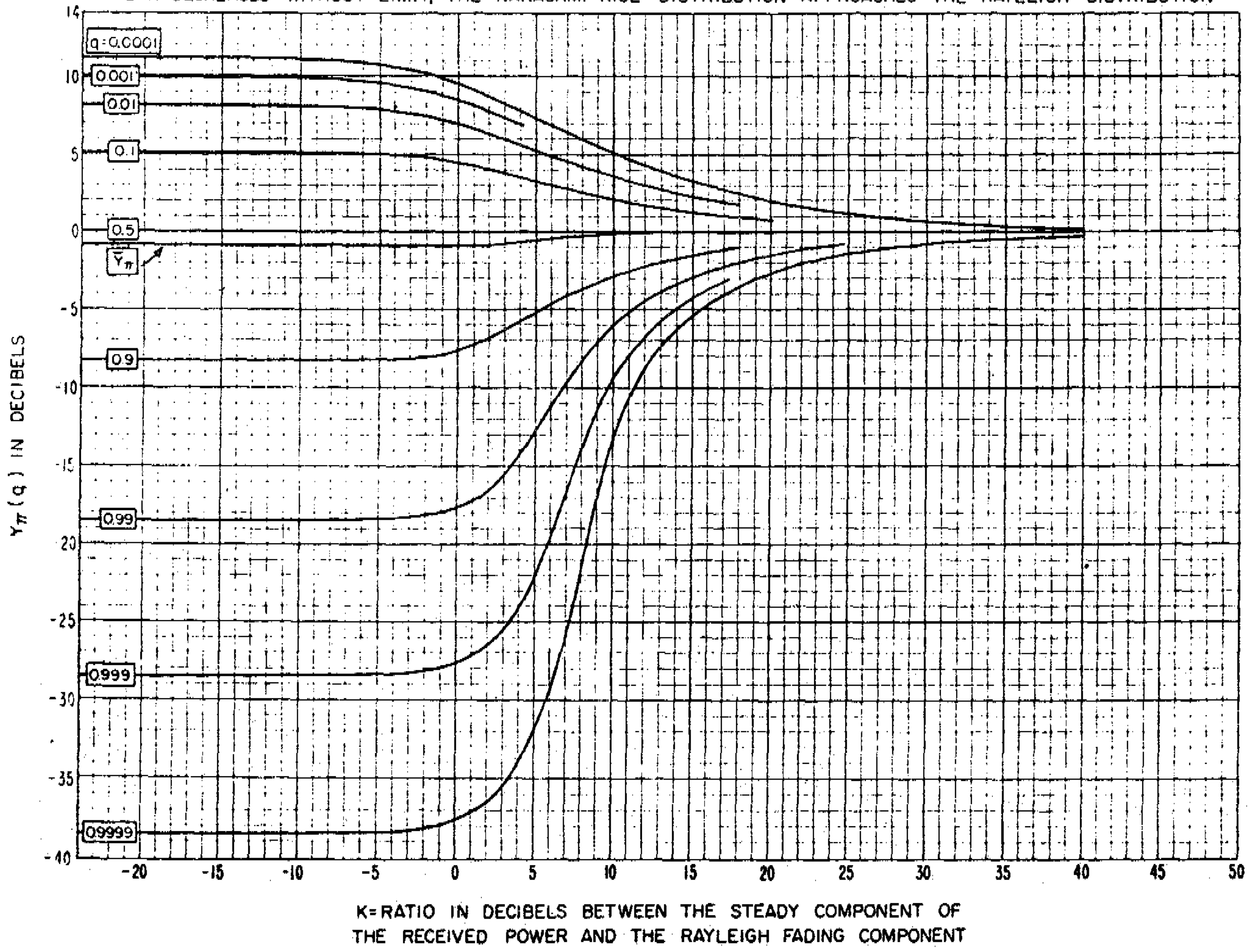
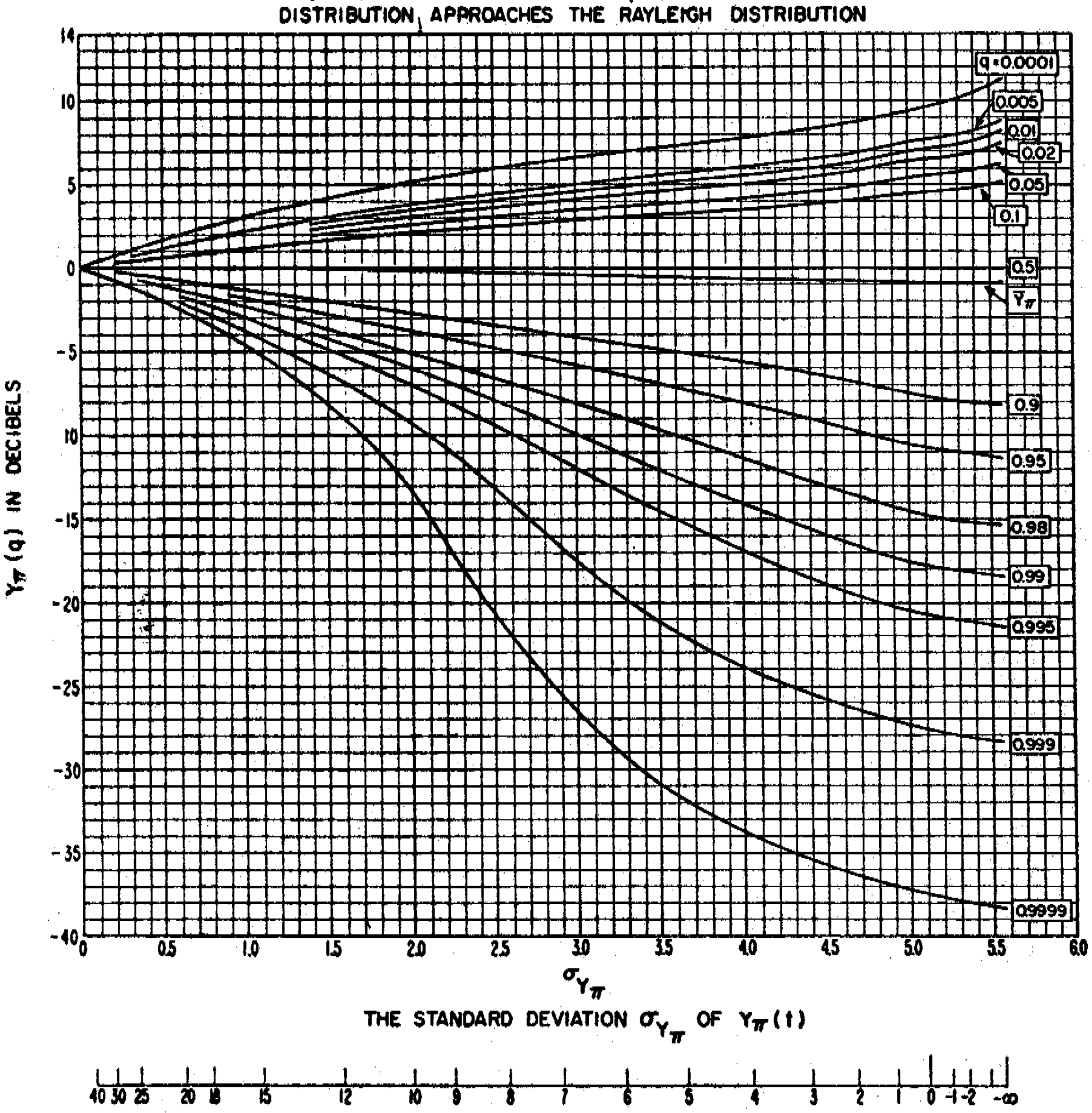


Figure V.1

THE NAKAGAMI-RICE PROBABILITY DISTRIBUTION OF THE INSTANTANEOUS FADING ASSOCIATED WITH PHASE INTERFERENCE

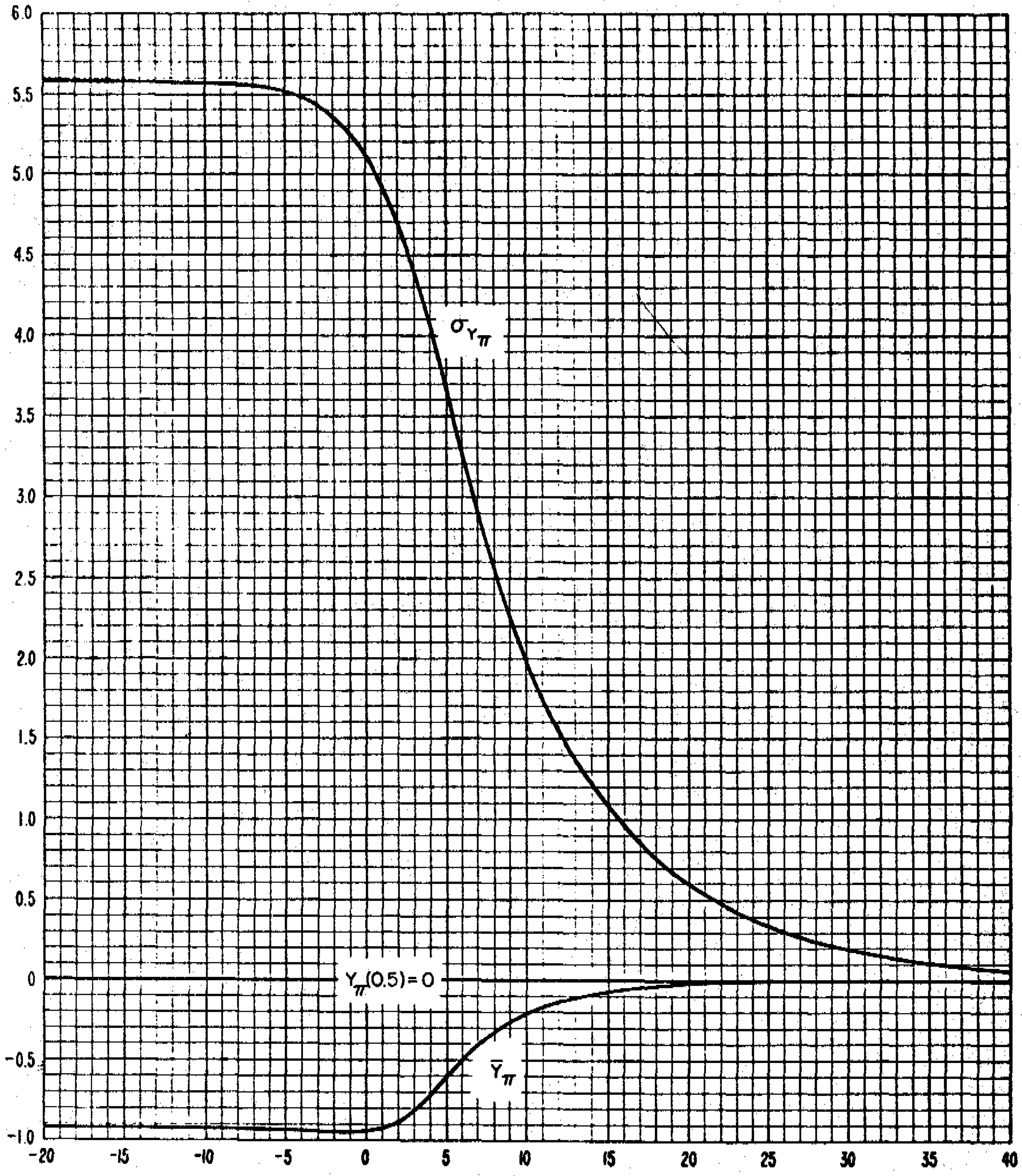
q IS THE PROBABILITY THAT $Y_\pi = (W_\pi - W_m)$ EXCEEDS $Y_\pi(q)$.
AS K DECREASES WITHOUT LIMIT, THE NAKAGAMI-RICE DISTRIBUTION APPROACHES THE RAYLEIGH DISTRIBUTION



K = RATIO IN DECIBELS BETWEEN THE STEADY COMPONENT OF THE RECEIVED POWER AND THE RAYLEIGH FADING COMPONENT

Figure V.2

STANDARD DEVIATION σ_{Y_π} AND MEAN $\bar{Y}_\pi$ FOR THE NAKAGAMI-RICE PHASE INTERFERENCE DISTRIBUTION



K=RATIO IN DECIBELS BETWEEN THE STEADY COMPONENT OF THE RECEIVED POWER AND THE RAYLEIGH FADING COMPONENT

Figure V.3

CHEYENNE MOUNTAIN, COLORADO TO GARDEN CITY, KANSAS
24 FEBRUARY 1953
1046 MHz d = 364.5 km

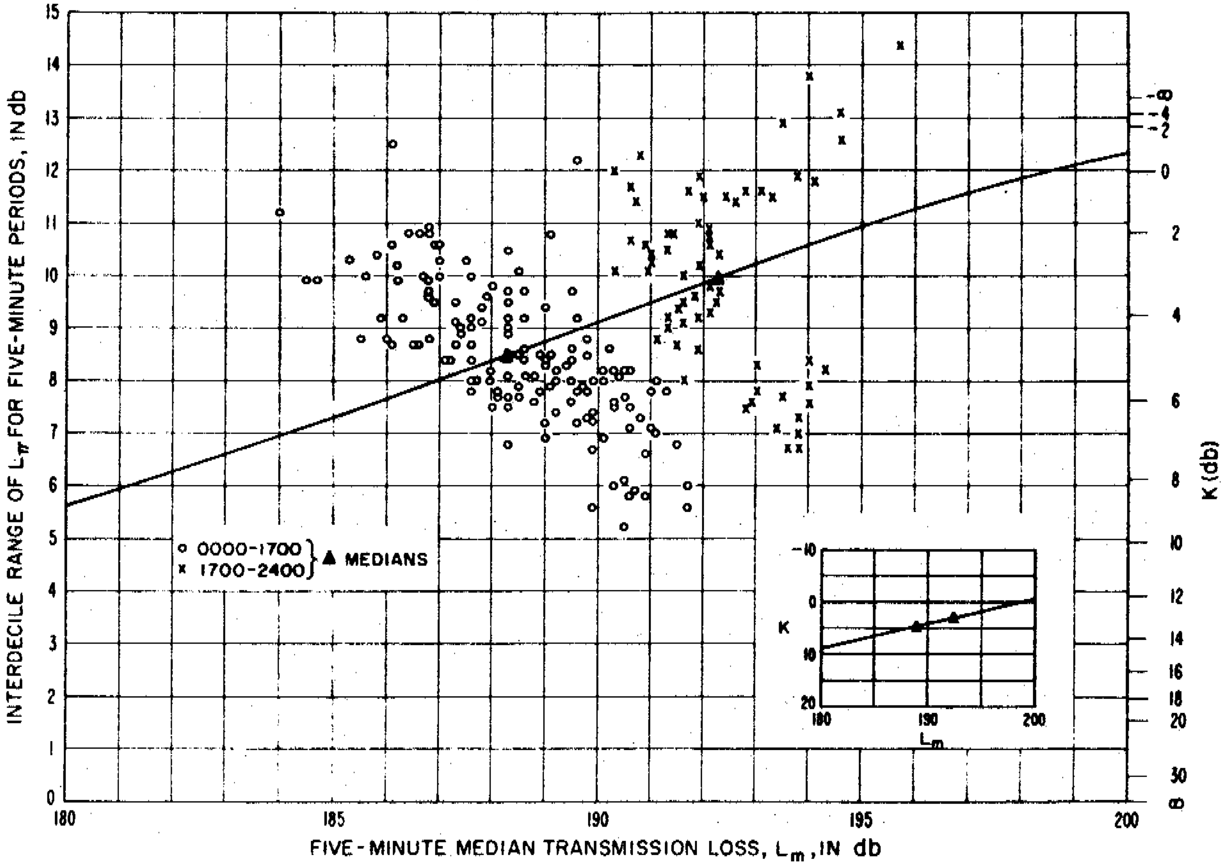
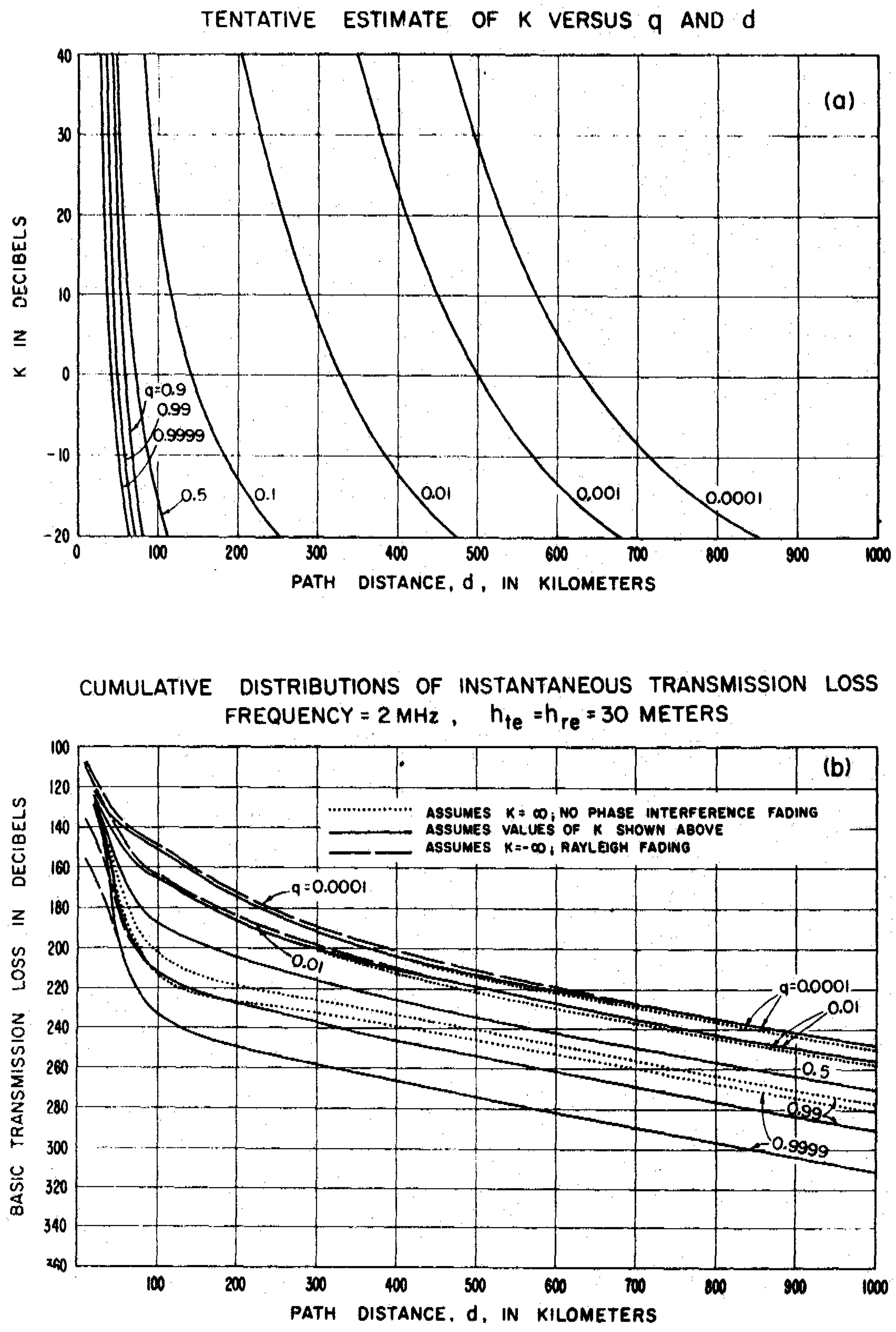


Figure V.4



TENTATIVE ESTIMATE OF K VERSUS q AND d
(a)
K IN DECIBELS
PATH DISTANCE, d, IN KILOMETERS
q=0.9
0.99
0.9999
0.5
0.1
0.01
0.001
0.0001
CUMULATIVE DISTRIBUTIONS OF INSTANTANEOUS TRANSMISSION LOSS
FREQUENCY = 2 MHz , h te = h re = 30 METERS
(b)
ASSUMES K = ∞; NO PHASE INTERFERENCE FADING
ASSUMES VALUES OF K SHOWN ABOVE
ASSUMES K = -∞; RAYLEIGH FADING
BASIC TRANSMISSION LOSS IN DECIBELS
PATH DISTANCE, d, IN KILOMETERS
q=0.0001
0.01
0.5
0.99
0.9999

Figure V.5

V.3 Noise-Limited Service

A detailed discussion of the effective noise bandwidth, the operating noise factor, and the operating sensitivity of a receiving system is presented in a recent report, "Optimum Use of the Radio Frequency Spectrum," prepared under Resolution 1 of the CCIR [Geneva, 1963c].

The median value of the total noise power w_{mn} watts in a bandwidth b cycles per second at the output load of the linear portion of a receiving system includes external noise accepted by the antenna as well as noise generated within the receiving system, including both principal and spurious responses of the antenna and transmission line as well as the receiver itself. This total noise power delivered to the pre-detection receiver output may be referred to the terminals of an equivalent loss-free antenna (as if there were only external noise sources) by dividing w_{mn} by g_o, the maximum value of the operating gain of the pre-detection receiving system.

The operating noise factor of the pre-detection receiving system, f_{op}, may be expressed as the ratio of the "equivalent available noise power" w_{mn}/g_o to the Johnson noise power $k T_o b$ that would be available in the band b from a resistance at a reference absolute temperature $T_o = 288.37$ degrees Kelvin, where $k = 1.38054 \times 10^{-23}$ joules per degree is Boltzmann's constant:

$$f_{op} = \frac{w_{mn}/g_o}{k\,T_o\,b} \tag{V.7}$$

or in decibels:

$$F_{op} = (W_{mn} - G_o) - (B - 204) \quad \text{db} . \tag{V.8}$$

The constant 204 in (V.8) is $-10 \log (k\,T_o)$.

Note that the available power from the antenna as defined in annex II has the desired property of being independent of the receiver input load impedance, making this concept especially useful for the definition and measurement of the operating noise factor f_{op} as defined under CCIR Resolution 1 [Geneva, 1963c].

At frequencies above 100 MHz, where receiver noise rather than external noise usually limits reception, f_{op} is essentially independent of external noise. In general, f_{op} is proportional to the total noise w_{mn} delivered to the pre-detection receiver output and so measures the degree to which the entire system, including the antenna, is able to discriminate against both external noise and receiver noise.

Let W_m represent the median available wanted signal power associated with phase interference fading at the terminals of an equivalent loss-free receiving antenna, and let the "operating threshold" $W_{mr}(g)$ represent the minimum value of W_m which will provide a grade g service in the presence of noise alone. The operating threshold W_{mr} assumes a specified type of wanted signal and a specified type of noise, but does not allow for other unwanted signals. Compared to the total range of their long-term variability, it is assumed that W_m and $W_{mr}(g)$ are hourly median values; i.e., that long-term power fading is negligible over such a short period of time. Let G_{ms} represent the hourly median operating signal gain of a pre-detection receiving system, expressed in db so that $W_m + G_{ms}$ very closely approximates the hourly median value of that component of available wanted signal power delivered to the pre-detection receiver output and associated with phase interference fading. The median wanted signal to median noise ratio available at the pre-detection receiver output is then

$$R_m = [W_m + G_{ms}] - W_{mn} \quad \text{db} \qquad \text{(V.9a)}$$

and the minimum value of R_m which will provide a desired grade of service in the presence of noise alone is

$$R_{mr}(g) = [W_{mr}(g) + G_{ms}] - W_{mn} \quad \text{db} . \qquad \text{(V.9b)}$$

V.4 Interference-Limited Service

Separation of the total fading into a phase interference component Y_π and the more slowly varying component Y as described in Section V.1 appears to be desirable for several reasons: (1) variations of Y_π associated with phase interference may be expected to occur completely independently for the wanted and unwanted signals, and this facilitates making a more precise determination of the required wanted-to-unwanted signal ratio, $R_{ur}(g)$, (2) the random variable Y_π follows the Nakagami-Rice distribution, as illustrated on figure V.1, while variations with time of Y are approximately normally distributed, (3) the variations with time of the median wanted and unwanted signal powers W_m and W_{um} tend to be correlated for most wanted and unwanted propagation paths, and an accurate allowance for this correlation is facilitated by separating the instantaneous fading into the two additive components Y and Y_π and (4) most of the contribution to the variance of W_m with time occurs at low fluctuation frequencies ranging from one cycle per year to about one cycle per hour, whereas most of the contribution to the variance of Y_π occurs at higher fluctuation frequencies, greater than one cycle per hour. Only the short-term variations of the wanted signal power w_π and the unwanted signal power $w_{u\pi}$ associated with phase interference fading are used in determining $r_{ur(g)}$, the ratio of the median wanted signal power w_m to the median unwanted signal power w_{um} required to provide a specified grade of service g. Let $R_{u\pi}$ denote the ratio between the instantaneous wanted signal power $W_m + Y_\pi$ and the instantaneous unwanted signal power $W_{um} + Y_{u\pi}$.

$$R_{u\pi} = W_m + Y_\pi - W_{um} - Y_{u\pi} = R_u + Z_\pi \tag{V.10}$$

where

$$Z_\pi \equiv Y_\pi - Y_{u\pi} \text{ and } R_u = W_m - W_{um} \; . \tag{V.11}$$

Note that the cumulative distribution function $Y_\pi(q, K)$ for Y_π will usually be different from the cumulative distribution function $Y_\pi(q, K_u)$ for $Y_{u\pi}$ since the wanted signal propagation path will differ from the propagation path for the unwanted signal. Let $Z_\pi > Z_\pi(q, K, K_u)$ with probability q; then the approximate cumulative distribution function of Z_i is given by:

$$Z_{\pi a}(q; K, K_u) = \pm\sqrt{Y_\pi^2(q, K) + Y_\pi^2(1-q, K_u)} \tag{V.12}$$

In the above, the plus sign is to be used when $q < 0.5$ and the minus sign when $q > 0.5$; note that $-Z_{\pi a}(1-q, K, K_u) = Z_{\pi a}(q, K, K_u)$. This method of approximation is suggested by two observations: (1) service may be limited for a fraction q of a short period of time either by downfades of the wanted signal corresponding to a level exceeded with a probability q or by uptades of an unwanted signal corresponding to a level exceeded with a probability $1-q$, and (2) the standard deviation of the difference of two uncorrelated random variables Y_π and $Y_{u\pi}$ equals the square root of the sum of their variances, and under fairly general conditions $Z_*^2(q)$ is very nearly equal to $Y_\pi^2(q) + Y_{u\pi}^2(1-q)$. Equation (V.11) is based on the reasonable assumption that Y_π and $Y_{u\pi}$ are independent random variables and, for this case, (V.11) would be exact if Y_π and $Y_{u\pi}$ were normally distributed. The departure from normality of the distribution of Y_π is greatest in the limiting case of a Rayleigh distribution, and for this special case, the following exact expression is available [Siddiqui, 1962]:

$$Z_\pi(q, \infty, \infty) = 10 \log \left(\frac{1}{q} - 1\right) \qquad \text{(V.13)}$$

Table V.2 compares the above exact expression $Z_\pi(q, \infty, \infty)$ with the approximate expression $Z_\pi(q, \infty, \infty)$. Note that the two expressions differ by less than 0.2 dB for any value of q and, since this difference may be expected to be even smaller for finite values of K, it appears that (V.12) should be a satisfactory approximation for most applications and for any values of K and q.

Table V.2

The Cumulative Distribution Function $Z_\pi(q, \infty, \infty)$ for the Special Case of the Ratio of Two Rayleigh-Distributed Variables

q	$Z_\pi(q, \infty, \infty)$	$Z_{\pi a}(q, \infty, \infty)$	$Z_\pi - Z_{\pi a}$
	db	db	db
0.0001	39.99957	40.0178	-0.01823
0.0002	36.98883	37.0362	-0.04737
0.0005	33.00813	33.0757	-0.06757
0.001	29.99566	30.1099	-0.11424
0.002	26.98101	27.1216	-0.14059
0.005	22.98853	23.1584	-0.16987
0.01	19.95635	20.1420	-0.18565
0.02	16.90196	17.0949	-0.19294
0.05	12.78754	12.9719	-0.18436
0.1	9.54243	9.7016	-0.15917
0.2	6.02060	6.1331	-0.11250
0.5	0	0	0

Let $R_{uro}(g)$ denote the required value of R_u for non-fading wanted and unwanted signals and it follows from (V.10) that the instantaneous ratio for fading signals will exceed $R_{uro}(g)$ with a probability at least equal to q provided that:

$$R_u > R_{ur}(g, q, K, K_u) = R_{uro}(g) - Z_{\pi}(q, K, K_u) \tag{V.14}$$

The use of (V.14) to determine an allowance for phase interference fading will almost always provide a larger allowance than will actually be necessary since (V.14) was derived on the assumption that $R_{uro}(g)$ is constant. For most services, $R_{uro}(g)$ will not have a fixed value for non-fading signals but will instead have either a probability distribution or a grade of service distribution; in such cases $R_{ur}(g)$ should be determined for a given q by a convolution of the distributions of $R_{uro}(g)$ and $-Z_{\pi}$. In still other cases the mean duration of the fading below the level $Z_{\pi}(q, K, K_u)$ will be comparable to the mean duration of the individual message elements and a different allowance should then be made. In some cases it may be practical to determine R_{ur} as a function of g, q, K, and K_u in the laboratory by generating wanted and unwanted signals that vary with time the same as Y_{π} and $Y_{u\pi}$. This latter procedure will be successful only to the extent that the fading signal generators properly simulate natural phase interference fading both as regards their amplitude distributions and their fading duration distributions. As this annex is intended to deal only with general definitions and procedures, functions applicable to particular kinds of wanted and unwanted signals which include an appropriate phase interference fading allowance are not developed here.

The ratio R_u defined as an hourly median value equal to the difference between $W_m(0.5) + Y$ and $W_{um}(0.5) + Y_u$ will also vary with time:

$$R_u \equiv W_m - W_{um} = W_m(0.5) - W_{um}(0.5) + Z \tag{V.15}$$

$$Z \equiv Y - Y_u . \tag{V.16}$$

The random variables Y and Y_u tend to be approximately normally distributed with a positive correlation coefficient ρ which will vary considerably with the propagation paths and the particular time block involved. For the usual period of all hours of the day for several years preliminary analyses of data indicate that ρ will usually exceed 0.4 even for propagation paths in opposite directions from the receiving point. Z will exceed a value $Z_a(q)$ for a fraction of time q where the approximate cumulative distribution function $Z_a(q)$ of Z is given by

$$Z_a(q) = \pm\sqrt{Y^2(q) + Y_u^2(1-q) + 2\rho\, Y(q)\, Y_u(1-q)} \qquad \text{(V. 17)}$$

where the plus sign is to be used for $q < 0.5$ and the minus sign for $q > 0.5$ while $Z_a(0.5) \equiv 0$. It follows from (V.15) and (V.17) that R_u will exceed $R_{ur}(g)$ for at least a fraction of time q provided that

$$W_m(0.5) - W_{um}(0.5) > R_{ur}(g) - Z_a(q) \;. \qquad \text{(V. 18)}$$

In some cases it may be considered impractical to determine the function $R_{ur}(g)$ by adding an appropriate phase interference fading allowance to $R_{uro}(g)$; in such cases it may be useful to use the following approximate relation which will ensure that the instantaneous ratio $R_{u\pi} > R_{uro}(g)$ for at least a fraction of time q :

$$W_m(0.5) - W_{um}(0.5) > R_{uro}(g) \pm\sqrt{Z_a^2(q) + Z_\pi^2(q, K, K_u)} \;. \qquad \text{(V. 19)}$$

In the above, the minus sign is to be used for $q < 0.5$ and the plus sign for $q > 0.5$. Although (V.19), or its equivalent, has often been used in past allocation studies, this usage is deprecated since it does not provide a solution which is as well adapted to the actual nature of the problem as the separation of the fading into its two components Y_π and Y and the separate use of (V.14) and (V.18). Note that (V.19) provides a fading allowance which is too small compared with that estimated using (V.14) and (V.18) separately. The latter formulas are recommended. They make more appropriate allowance for the fact that communications at particular times of the day or for particular seasons of the year are especially difficult.

V.5 The Joint Effect of Several Sources of Interference Present Simultaneously

The effects of interference from unwanted signals and from noise have so far been considered in this report as though each affected the fidelity of reception of the wanted signal independently. Let $w_{mr}(g)$ and $r_{ur}(g)\, w_{um}$ denote power levels which the wanted median signal power w_m must exceed in order to achieve a specified grade of service when each source of interference is present alone. To the extent that the various sources of interference have a character approximating that of white noise, this same grade of service may be expected from a wanted signal with median level

$$w_m = w_{mr}(g) + \sum r_{ur}(g)\, w_{um}$$

when these sources are present simultaneously.

An approximate method has been developed [Norton, Staras, and Blum, 1952] for determining for a broadcasting service the distribution with time and receiving location of the ratio

$$w_m / \left[w_{mr}(g) + \sum r_{ur}(g)\, w_{um} \right]$$

Although this approach to the problem of adding the effects of interference will probably always provide a good upper bound to the interference, this assumption that the interference power is additive is often not strictly valid. For example, when intelligible cross-talk from another channel is present in the receiver output circuit, the addition of some white noise will actually reduce the nuisance value of this cross-talk.

Frequently, however, both $w_{mr}(g)$ and w_{um} will be found to vary more or less independently over wide ranges with time and a good approximation to the percentage of time that objectionable interference is present at a particular receiving location may then be obtained [Barsis, et al, 1961] by adding the percentage of time that w_m is less than $w_{mr}(g)$ to the percentages of time that w_m is less than each of the values of $r_{ur}(g)\, w_{um}$. When this total time of interference is small, say less than 10%, this will represent a satisfactory estimate of the joint influence of several sources of interference which are present simultaneously. Thus, when the fading ranges of the various sources of interference are sufficiently large so that this latter method of analysis is applicable, the various values of $w_{mr}(g)$ and of $r_{ur}(g)\, w_{um}$ will have comparable magnitudes for negligible percentages of the time so that one may, in effect, assume that the various sources of interference occur essentially independently in time.

Minimum acceptable wanted-to-unwanted signal ratios r_{ur} may sometimes be a function of r_m, the available wanted signal-to-noise ratio. When r_{ur} is within 3 db of r_{mr}, an unwanted signal may be treated the same as external noise, and, in a similar fashion, long-term distributions of available wanted-to-unwanted signal ratios may be determined for each class of unwanted signals for which r_{ur} is nearly the same.

V. 6 The System Equation for Noise-Limited Service

Essential elements of a noise-limited communication circuit are summarized in the following system equation. The transmitter output $W_{\ell t}$ dbw which will provide W_t dbw of total radiated power in the presence of transmission line and matching network losses $L_{\ell t}$ db, and which will provide a median delivered signal at the pre-detection receiver output which is R_m db above the median noise power W_{mn} delivered to the pre-detection receiver output is given by

$$W_{\ell t} = L_{\ell t} + L_m + R_m + (W_{mn} - G_{ms}) \quad \text{dbw} \tag{V.20}$$

in the presence of a median transmission loss L_m and a median operating receiving system signal gain G_{ms}. The operating signal gain is the ratio of the power delivered to the pre-detection receiver output to the power available at the terminals of an equivalent loss-free antenna. Let G_o be the maximum of all values of operating signal gain in the receiver pass band, and G_{ms} the median value for all signal frequencies in the pass band. $(W_{mn} - G_{ms})$ in (V. 20) is the equivalent median noise power at the antenna terminals, as defined in section V. 3.

It is appropriate to express the system equation (V.20) in terms of the operating noise factor F_{op} defined by (V.8), rather than in terms of W_{mn} or $(W_{mn} - G_{ms})$ in order to separate studies of receiving system characteristics from studies of propagation. For this reason all predicted power levels are referred to the terminals of an equivalent loss-free antenna, and receiving system characteristics such as F_{op}, G_o, G_{ms}, and $B = 10 \log b$ are separated from transmission loss and available power in the formulas.

Rearranging terms of (V.8), the equivalent median noise power $(W_{mn} - G_{ms})$ delivered to the antenna terminals may be expressed as

$$W_{mn} - G_{ms} = F_{op} + (G_o - G_{ms}) + (B - 204) \tag{V. 21}$$

where G_o and G_{ms} are usually nearly equal. Assuming that $L_{\ell t}$, G_o, G_{ms}, and B are constant, it is convenient to combine these parameters into an arbitrary constant K_o:

$$K_o = L_{\ell t} + G_o - G_{ms} + B - 204 \quad \text{dbw} \tag{V. 22}$$

and rewrite the system equation as:

$$W_{\ell t} = K_o + L_m + R_m + F_{op} \quad \text{dbw} \tag{V. 23}$$

In general, if unwanted signals other than noise may be disregarded, service exists whenever $R_m(q)$ exceeds $R_{mr}(g)$, where $R_m(q)$ is the value of R_m exceeded a fraction q of all hours. With G_{ms} and W_{mn} assumed constant, so that

$$R_m(q) = W_m(q) + G_{ms} - W_{mn} \tag{V. 24}$$

service exists whenever $W_m(q)$ exceeds W_{mr}, or whenever $L_m(q)$ is less than the maximum allowable transmission loss $L_{mo}(g)$. An equivalent statement may be made in terms of the system equation. The transmitter power $W_{\ell t}(q)$ which will provide for a fraction q of all hours at least the grade g service defined by the required signal-to-noise ratio $R_{mr}(g)$ is

$$W_{\ell t}(q) = K_o + L_m(q) + F_{op} + R_{mr}(g) \qquad \text{(V.25)}$$

where $L_m(q)$ is the hourly median transmission loss not exceeded for a fraction q of all hours.

For a fixed transmitter power W_o dbw, the signal-to-noise ratio exceeded q percent of all hours is

$$R_m(q) = W_o - K_o - F_{op} - L_m(q) \quad \text{db} \qquad \text{(V.26)}$$

for a "median" propagation path for which the service probability, Q, is by definition equal to 0.5.

The maximum allowable transmission loss

$$L_{mo}(g) = W_o - K_o - F_{op} - R_{mr}(g) \qquad \text{(V.27)}$$

is set equal to the loss $L_m(q, Q)$ exceeded during a fraction $(1 - q)$ of all hours with a probability Q. This value is fixed when P_o, K_o, and $R_{mr}(g)$ have been determined, and for each time availability q there is a corresponding service probability, Q(q). Section V.9 will explain how to calculate Q(q).

When external noise is both variable and not negligible, the long-term variability of F_{op} must be considered, and the following relationships may be used to satisfy the condition

$$R_m(q) > R_{mr}(g) \qquad \text{(V.28)}$$

$$R_m(q) \cong R_m(0.5) + Y_m(q) \qquad \text{(V.29)}$$

$$R_m(0.5) \cong W_o - K_o - F_{op}(0.5) - L_m(0.5) \qquad \text{(V.30)}$$

$$Y_m^2(q) \cong Y^2(q) + Y_n^2(1 - q) - 2\rho_{tn} Y(q) Y_n(1 - q) \qquad \text{(V.31)}$$

$$Y(q) \equiv L_m(0.5) - L_m(q), \qquad Y_n(q) \equiv F_{op}(q) - F_{op}(0.5) \qquad \text{(V.32)}$$

where ρ_{tn} is the long-term correlation between W_m and F_{op}. Though ρ_{tn} could theoretically have any value between -1 and 1, it is usually zero.

V.7 The Time Availability of Interference-Limited Service

Let ρ_{tu} denote the long-term correlation between W_m and W_{um}, the power expected to be available at least q percent of all hours at the terminals of an equivalent loss-free receiving antenna from wanted and unwanted stations radiating w_o and w_u watts, respectively:

$$W_m(q) = W_o - L_m(q) \quad \text{dbw}, \qquad W_{um}(q) = W_u - L_{um}(q) \quad \text{dbw} \tag{V.33}$$

$$W_o = 10 \log w_o \quad \text{dbw}, \qquad W_u = 10 \log w_u \quad \text{dbw} \tag{V.34}$$

The criterion for service of at least grade g in the presence of a single unwanted signal and in the absence of other unwanted signals or appreciable noise is

$$R_u(q) > R_{ur}(g, q) \tag{V.35}$$

where

$$R_u(q) = R_u(0.5) + Y_R(q) \tag{V.36}$$

$$R_u(0.5) \cong W_m(0.5) - W_{um}(0.5) \tag{V.37}$$

$$Y_R^2(q) \cong Y^2(q) + Y_u^2(1-q) - 2\rho_{tu} Y(q) Y_u(1-q) \tag{V.38}$$

$$Y_u(q) = W_{um}(q) - W_{um}(0.5) = L_{um}(0.5) - L_{um}(q) \tag{V.39}$$

If W_m, W_{um}, and F_{op} were exactly normally distributed, (V.31) and (V.38) would be exact; they represent excellent approximations in practice.

V.8 The Estimation of Prediction Errors

Consider the calculation of the power $W_m(q)$ available at the terminals of an equivalent loss-free receiving antenna during a fraction q of all hours. $W_m(q)$ refers to hourly median values expressed in dbw. For a specific propagation path it is calculated in accordance with the methods given in sections 2-10 using a given set of path parameters $(d, f, \theta, h_{te}$, etc.). Denote by $W_{mo}(q)$ observations made over a large number of randomly different propagation paths, which, however, can all be characterized by the same set of prediction parameters. Values of $W_{mo}(q)$ will be very nearly normally distributed with a mean (and median) equal to $W_m(q)$, and a variance denoted by $\sigma_c^2(q)$. This path-to-path variability is illustrated in Fig. V.6 for a hypothetical situation which assumes a random distribution of all parameters which are not taken into account in the prediction method.

The variance σ_c^2 of deviations of observation from prediction depends on available data and the prediction method itself. The most sophisticated of the methods given in this report for predicting transmission loss as a function of carrier frequency, climate, time block, antenna gains, and path geometry have been adjusted to show no bias, on the average, for the data discussed in section 10 and in annex I.

Most of these data are concentrated in the 40-1000 MHz frequency range, and were obtained primarily for transhorizon paths in climates 1, 2, and 3. Normally, one antenna was on the order of 10 meters above ground and the other one was higher, near 200 meters. Even the low receiving antennas were located on high ground or in clear areas well removed from hills and terrain clutter. Few of the data were obtained with narrow-beam antennas. An attempt has been made to estimate cumulative distributions of hourly transmission loss medians for accurately specified time blocks, including estimates of year-to-year variability.

A prediction for some situation that is adequately characterized by the prediction parameters chosen here requires only interpolation between values of these parameters for which data are available. In such a case, $\sigma_c(q)$ represents the standard error of prediction. The mean square error of prediction, referred to a situation for which data are not available, is $\sigma_c^2(q)$ plus the square of the bias of the prediction method relative to the new situation.

Based on an analysis of presently available transhorizon transmission loss data, the variance $\sigma_c^2(q)$ is estimated as

$$\sigma_c^2(q) = 12.73 + 0.12\,Y^2(q) \quad \mathrm{db}^2 \tag{V.40}$$

where $Y(q)$ is defined in section 10. Since $Y(0.5) \equiv 0$, the variance $\sigma_c^2(0.5)$ of the difference between observed and predicted long-term medians is $12.73\ \mathrm{db}^2$, with a corresponding standard deviation $\sigma_c(0.5) = 3.57$ db.

It is occasionally very difficult to estimate the prediction error $\sigma_c(q)$ and the service probability Q. Where only a small amount of data is available there is no adequate way of estimating the bias of a prediction. One may, however, assign weights to the curves of $V(0.5, d_e)$ in figure 10.13 for climates 1-7 based on the amount of supporting data available:

Climate Number	Weight
1	300
2	120
3	60
4	2
5	(deleted)
6	5
7	5

As an example, for $d_e = 600$ km, the average $V(0.5, d_e)$ weighted in accordance with the above is 0.1 db, and the corresponding climate-to-climate variance of $V(0.5)$ is $3.1\ \mathrm{db}^2$. If a random sampling of these climates is desired the predicted median value $L(0.5)$ is L_{cr} - $V(0.5) = L_{cr} - 0.1$ db, with a standard error of prediction equal to $(12.7 + 3.1)^{\frac{1}{2}} = 4$ db, where $12.7\ \mathrm{db}^2$ is the variance of $V(0.5)$ within any given climate.

If there is doubt as to which of two particular climates i and j should be chosen, the best prediction of $L_b(q)$ might depend on the average of $V_i(0.5, d_e)$ and $V_j(0.5, d_e)$ and the root-mean square of $Y_i(q, d_e)$ and $Y_j(q, d_e)$:

$$L(q) = L_{cr} - 0.5\left[V_i(0.5, d_e) + V_j(0.5, d_e)\right] - Y_{ij}(q, d_e) \quad \mathrm{db}, \tag{V.41}$$

$$Y_{ij}(q, d_e) = \left[0.5\, Y_i^2(q, d_e) + 0.5\, Y_j^2(q, d_e)\right]^{\frac{1}{2}} \quad \mathrm{db} \tag{V.42}$$

The bias of this prediction may be as large as $\left[0.5\ V_i(0.5, d_e) - V_j(0.5, d_e)\right]$ db. The root-mean square prediction error may therefore be estimated as the square root of the sum of the variance, $\sigma_c^2(0.5)$ and the square of the bias, or

$$\left\{12.73 + 0.12\, Y_{ij}^2(q, d_e) + 0.25\left[V_i(0.5, d_e) - V_j(0.5, d_e)\right]\right\}^{\frac{1}{2}} \quad \mathrm{db}.$$

According to figure 10.13, $V(0.5, d_e)$ is expected to be the same for climates 1 and 8. This conclusion and the estimate for $Y(q, d_e)$ shown in figure III.29 for climate 8 are based solely on meteorological data. In order to obtain these estimates, the percentages of time for which surface-based ducts existed in the two regions were matched with the same value of $Y(q, d_e)$ for both climates. In this way, $Y_8(q, d_e)$ was derived from $Y_1(q, d_e)$ by relating q_8 to q_1 for a given Y instead of relating Y_8 to Y_1 for a given q.

V.9 The Calculation of Service Probability Q for a Given Time Availability q

For noise-limited service of at least grade g and time availability q, the service probability Q is the probability that

$$L_{mo}(g) - L_m(q) > 0 \tag{V.43}$$

if external noise is negligible. $L_{mo}(g)$ is defined by (V.27). The criterion for service limited by variable external noise is

$$R_m(q) - R_{mr}(g) > 0 \quad \text{(from equation V.28)}.$$

For service limited only by interference from a single unwanted signal,

$$R_u(q) - R_{ur}(g, q) > 0 \quad \text{(from equation V.35)}.$$

Combining (V.22) and (V.27), (V.43) may be rewritten as

$$W_o - L_{\ell t} - G_o + G_{ms} - B + 204 - F_{op} - R_{mr}(g) - L_m(q) > 0 \tag{V.44}$$

where the terms are defined in (V.8) and section V.6. Assuming that the error of estimation of these terms from system to system is negligible except for the path-to-path variance $\sigma_c^2(q)$ of $L_m(q)$ it is convenient to represent the service probability Q as a function of the standard normal deviate z_{mo}:

$$z_{mo} = \frac{L_{mo} - L_m(q)}{\sigma_c(q)} \tag{V.45}$$

which has a mean of zero and a variance of unity. L_{mo} is identified as the transmission loss exceeded a fraction (1-q) of the time with a probability Q, which is expressed in terms of the error function as

$$Q(z_{mo}) = \tfrac{1}{2} + \tfrac{1}{2}\,\mathrm{erf}(z_{mo}/\sqrt{2}) . \tag{V.46}$$

Figure V.7 is a graph of Q versus z_{mo}.

For the method described here, the condition

$$0.12\, Y(q)\, z_{mo}(Q) < -\sigma_c(q) \tag{V.47}$$

is sufficient to insure that the service probability Q increases as the time availability q is decreased. A less restrictive condition is

$$Y(q)\,[L_{mo} - L_m(0.5)] < 106 \quad db^2 \tag{V.48}$$

An example is shown in figure V.8, with q versus Q for radiated powers $W_o = 30$ dbw and $W_o = 40$ dbw, and $L_m(q, Q) = W_o + 140$ db. Here,

$$q = 0.5 + 0.5 \operatorname{erf}\left[\frac{L_m(q) - 140}{10\sqrt{2}}\right], \tag{V.49}$$

corresponding to a normal distribution with a mean $L_m(0.5) = 140$ db and a standard deviation $Y(0.158) = 10$ db. [Note that $L_m(q)$ versus q as estimated by the methods of section 10 is usually not normally distributed].

To obtain the time availability versus service probability curves on figure V.8, $L_m(q)$ was obtained from q, $Y(q)$ from (V.4), $\sigma_c^2(q)$ from (V.40), z_{mo} from (V.45), and Q from figure V.7. This same method of calculation may be used when there are additional sources of prediction error by adding variances to $\sigma_c^2(q)$. Examining possible trade-offs between time availability and service probability shown in figure V.8, note the increase from $q = 0.965$ to $q = 0.993$ for $Q = 0.95$, or the increase from $Q = 0.78$ to $Q = 0.97$ for $q = 0.99$, as the radiated power is increased from one to ten kilowatts.

For the case of service limited by external noise (V.28) to (V.30) may be rewritten as

$$W_o - K_o - F_{op}(0.5) - L_m(0.5) + Y_m(q) - R_{mr}(g) > 0 \cdot \tag{V.50}$$

One may ignore any error of estimation of W_o, K_o, and $R_{mr}(g)$ as negligible and assume no path-to-path correlation between $F_{op}(0.5)$ and $L_m(0.5)$. The variance $\sigma_{op}^2(q)$ of $F_{op}(0.5) + L_m(0.5) - Y_m(q)$ in (V.50) may then be written as a sum of component variances σ_F^2 and $\sigma_c^2(q)$:

$$\sigma_{op}^2(q) = \sigma_F^2 + 12.73 + 0.12\, Y_m^2(q) \quad \text{db}^2 \cdot \tag{V.51}$$

Very little is known about values for the variance σ_F^2 of $F_{op}(0.5)$, but is is probably on the order of 20 db^2.

The corresponding standard normal deviate z_{op} is:

$$z_{op} = \frac{R_m(q) - R_{mr}(g)}{\sigma_{op}(q)} \tag{V.52}$$

and the service probability $Q(q)$ is given by (V.46) with z_{mo} replaced by z_{op}. The restriction (V.47) still holds with z_{mo} and σ_c replaced by z_{op} and σ_{op}. A less restriction condition equivalent to (V.48) can be stated only if a specific value of σ_F is assumed.

For the case of service limited only by interference from a single unwanted radio signal (V. 35) to (V. 39) may be rewritten as

$$L_{um}(0.5) - L_m(0.5) + Y_R(q) - R_{ur}(g, q) > 0 \cdot \qquad (V.53)$$

Let $\rho_{\ell u}$ denote the normalized correlation or covariance between path-to-path variations of $W_m(0.5)$ and $W_{um}(0.5)$. Then assuming a variance of $25.5(1 - \rho_{\ell u}) + 0.12\, Y_R^2(q)\ db^2$ for $R_u(q)$, given by the first three terms of (V. 53) and a variance σ_{ur}^2 for the estimate of $R_{ur}(g, q)$, the total variance $\sigma_{uc}^2(q)$ of any estimate of the service criterion given by (V. 53) may be written as

$$\sigma_{uc}^2(q) = 25.5(1 - \rho_{\ell u}) + 0.12\, Y_R^2(q) + \sigma_{ur}^2 \qquad (V.54)$$

where $Y_R^2(q)$ is given by (V. 38). The corresponding standard normal deviate z_{uc} is:

$$z_{uc} = \frac{R_u(q) - R_{ur}(g, q)}{\sigma_{uc}(q)} \qquad (V.55)$$

and the service probability Q(q) is given by (V. 46) with z_{mo} replaced by z_{uc}. The variance σ_{ur}^2 may range from 10 db^2 to very much higher values. The restrictions (V. 47) and (V. 48) apply with z_{mo} and σ_c replaced by z_{uc} and σ_{uc} and with 106 db in (V. 48) replaced by $(212 + \sigma_{ur}^2)\ db^2$.

TYPICAL PATH - TO - PATH VARIATION OF INFINITE - TIME DISTRIBUTIONS
FOR A SINGLE SET OF VALUES OF THE PREDICTION PARAMETERS

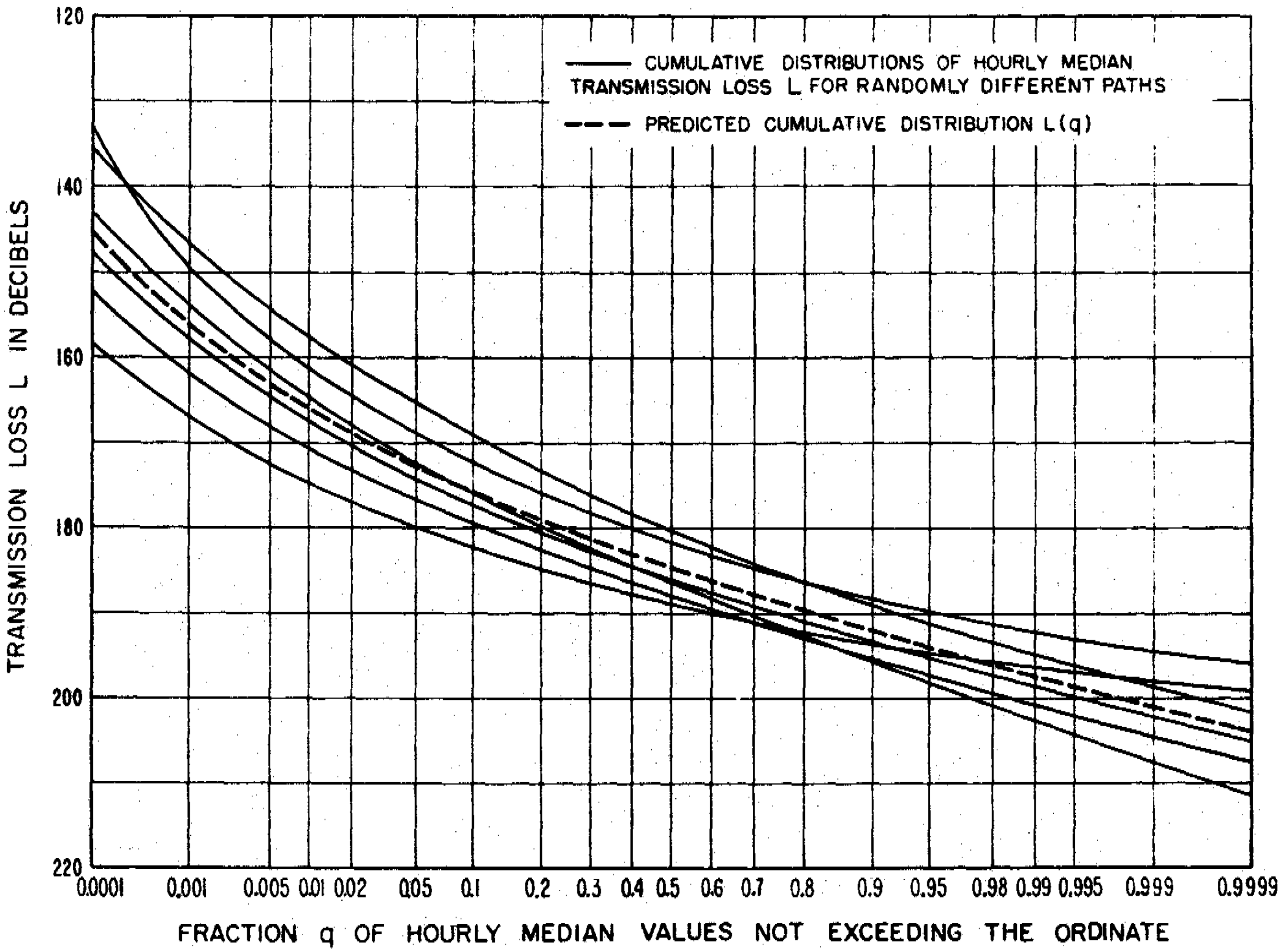


Figure V.6

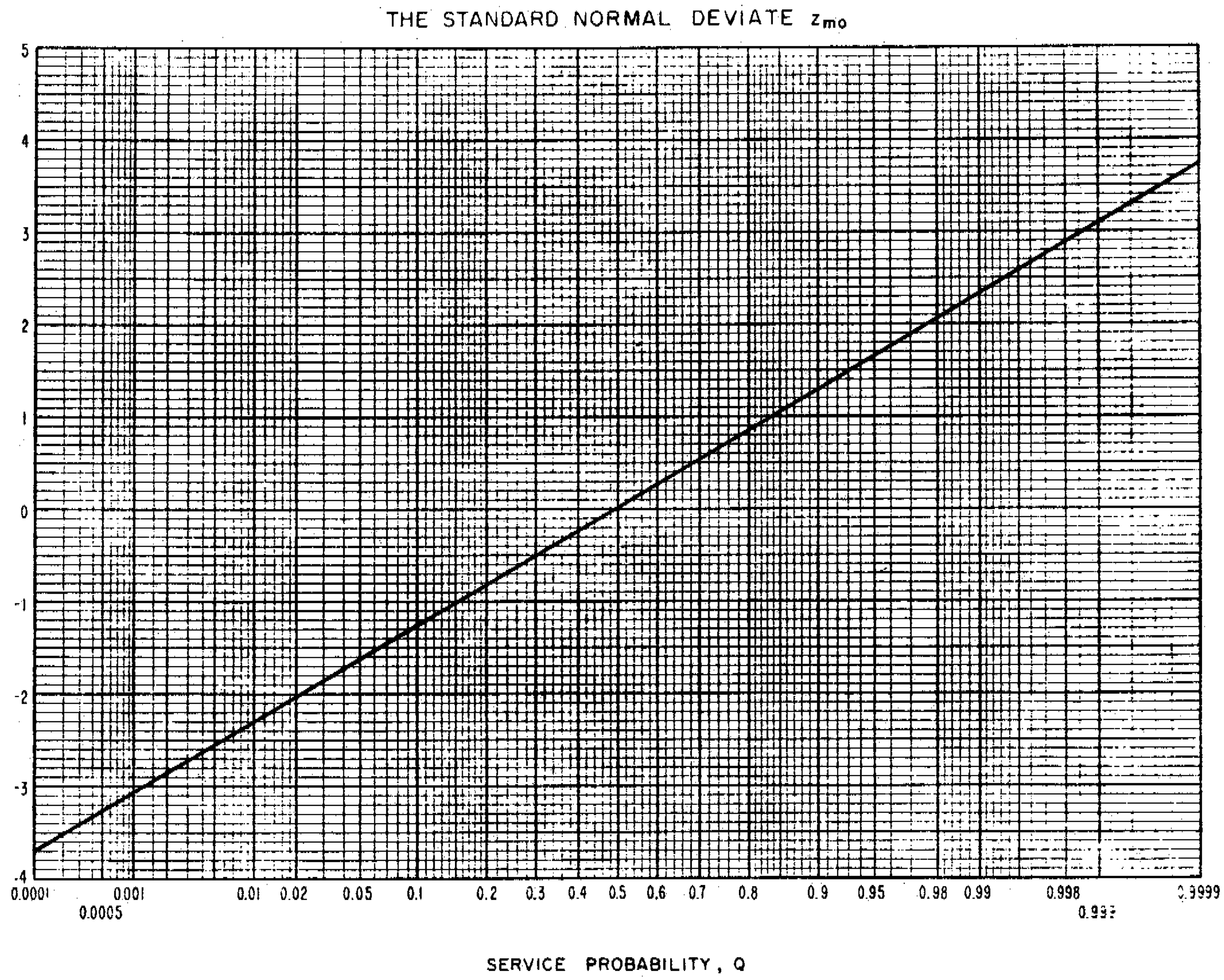

THE STANDARD NORMAL DEVIATE z_{mo}
z_{mo}
5
4
3
2
1
0
-1
-2
-3
-4
0.0001
0.0005
0.001
0.01
0.02
0.05
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.95
0.98
0.99
0.998
0.999
0.9999
SERVICE PROBABILITY, Q

Figure V.7

TIME AVAILABILITY VERSUS SERVICE PROBABILITY

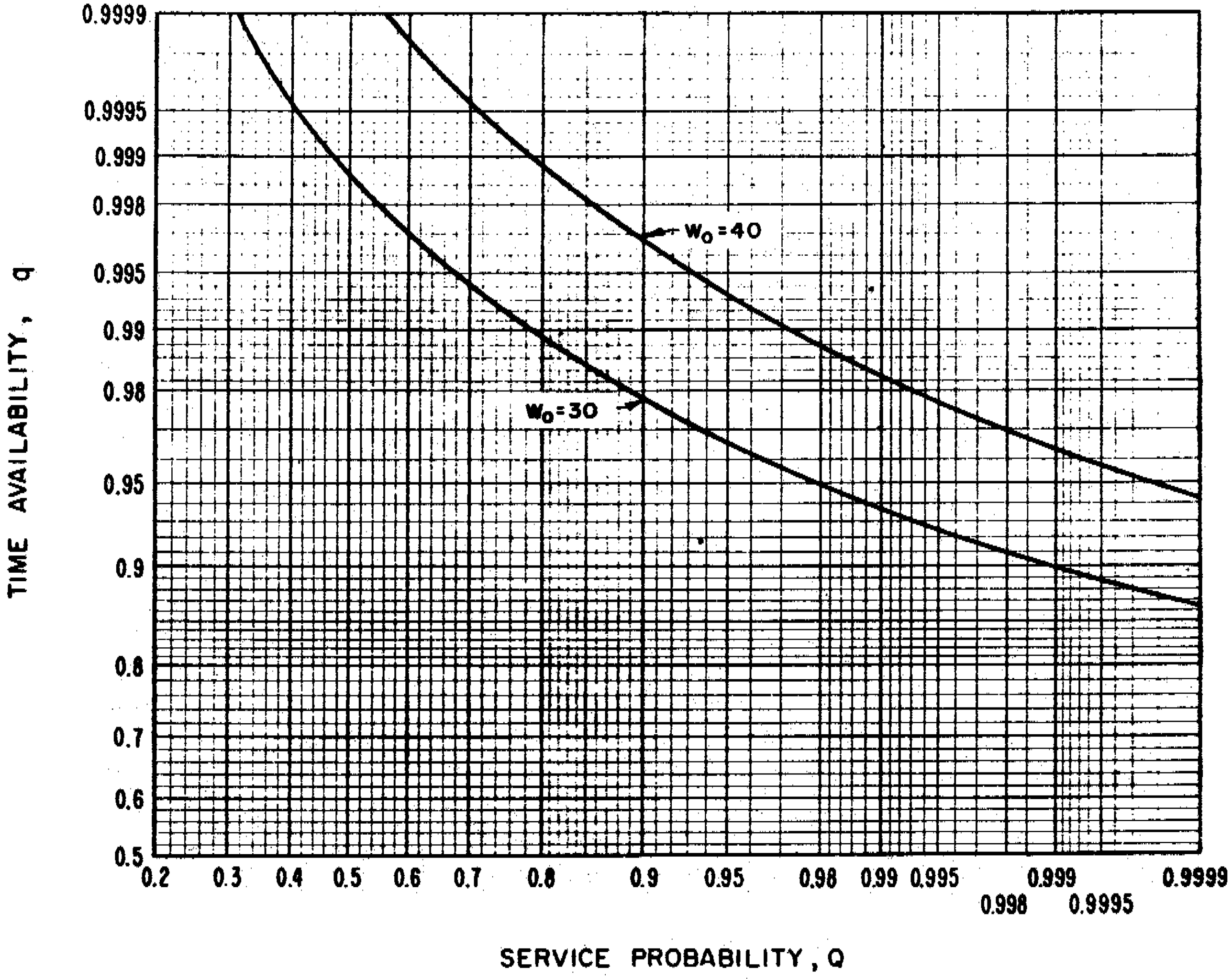


Figure V.8

V.10 Optimum Use of the Radio Frequency Spectrum

The business of the telecommunications engineer is to develop efficient radio systems, and the principal tool for improving efficiency is to adjust the various parameters to their optimum values. For example, it is usually more economical to use lower effective radiated powers from the transmitting systems by reducing the operating sensitivities of the receiving systems. Receiving system sensitivities can be reduced by (a) reducing the level of internally generated noise, (b) using antenna directivity to reduce the effects of external noise, (c) reducing man-made noise levels by using suppressors on noise generators such as ignition systems, relays, power transmission systems, etc., and (d) using space or time diversity and coding. The use of more spectrum in a wide band FM system or in a frequency diversity system can also reduce the receiving system operating sensitivity as well as reduce the acceptance ratios against unwanted signals other than noise.

Unfortunately, unlike other natural resources such as land, minerals, oil, and water, there is currently no valid method for placing a monetary value on each hertz of the radio spectrum. Thus, in the absence of a common unit of exchange, these tradeoffs are often made unrealistically at the present time. It is now generally recognized that the use of large capacity computers is essential for optimizing the assignment of frequencies to various classes of service including the development of optimum channelization schemes. Typical inputs to such computers are:

1. Nominal frequency assignments.
2. Transmitting system locations, including the antenna heights.
3. Transmitting system signatures; i.e., the radiated emission spectrum characteristics including any spurious emission spectrums.
4. Transmitting antenna characteristics.
5. Receiving system locations, including the antenna heights.
6. Spurious emission spectrums of the receiving systems.
7. Operating sensitivities of the receiving systems in their actual environments which thus make appropriate allowance for the effects of both man-made and natural noise.
8. Required values of wanted-to-unwanted phase interference median signal powers for all unwanted signals which could potentially cause harmful interference to the wanted signal; these acceptance ratios include appropriate allowances for reductions in the effects of fading achieved by the use of diversity reception and coding.

9. Long-term median reference values of basic transmission loss and path antenna gain for the wanted path and all of the unwanted signal propagation paths; these path antenna gains include allowances for antenna orientation, polarization, and multipath phase mismatch coupling losses.
10. Distributions with time of the transmission loss for the wanted signal path and all of the unwanted signal paths.
11. Correlations between the transmission losses on the wanted and on each of the unwanted propagation paths.
12. Transmission line and antenna circuit losses.
13. The spurious emission spectrum of any unwanted signals arising from unlicensed sources such as diathermy machines, electronic heaters, welders, garage door openers, etc.
14. Assigned hours of operation of each wanted and each unwanted emission.

The output of the computer indicates simply the identity and nature of the cases of harmful interference encountered. Harmful interference is defined as a failure to achieve the specified grade of service for more than the required fraction of time during the assigned hours of operation. Changing some of the inputs to the computer, an iterative process can be defined which may lead to an assignment plan with no cases of harmful interference.

It is assumed that a given band of radio frequencies has been assigned to the kind of radio service under consideration and that the nature of the services occupying the adjacent frequency bands is also known. Furthermore, it is assumed that the geographical locations of each of the transmitting and receiving antennas are specified in advance, together with the <u>relative</u> values of the radiated powers from each transmitting antenna and the widths and spacings of the radio frequency channels. In the case of a broadcasting service the specification of the intended receiving locations can be in terms of proposed service areas. With this information given, use may be made of the following procedures in order to achieve optimum use of the spectrum by this particular service:

(a) The system loss for each of the wanted signal propagation paths should be minimized and for each of the unwanted signal propagation paths should be maximized; this may be accomplished by maximizing the path antenna power gains for each of the wanted signal propagation paths, minimizing the path antenna gains for each of the unwanted propagation paths, and in exceptional cases, by appropriate antenna siting. The path antenna gains for the unwanted signal propagation paths may be minimized by the use of high-gain transmitting and receiving antennas with optimum side lobe suppression and front-to-back ratios and, in some cases, by the use of alternate polarizations for geographically adjacent stations or by appropriate shielding.

(b) The required protection ratios $r_{ur}(g)$ should be minimized by (1) appropriate radio system design, (2) the use of stable transmitting and receiving oscillators, (3) the use of linear transmitting and receiving equipment, (4) the use of wanted and unwanted signal propagation paths having the minimum practicable phase interference fading ranges; from band 6 to band 9 (0.3 to 3000 MHz), minimum phase interference fading may be achieved by the use of the maximum practicable transmitting and receiving antenna heights, and (5) the use of space diversity, time diversity, and coding.

(c) Wanted signal propagation paths should be employed having the minimum practicable long-term power fading ranges. In bands 8 and 9, minimum fading may be achieved by the use of the maximum practicable transmitting and receiving antenna heights.

The above procedures should be carried out with various choices of transmitting and receiving locations, relative transmitter powers, and channel spacings until a plan is developed which provides the required service with a minimum total spectrum usage. After the unwanted signal interference has been suppressed to the maximum practicable extent by the above methods so that, at each receiving location each of the values of r_u exceeds the corresponding protection ratio $r_{ur}(g)$ for a sufficiently large percentage of the time, then the following additional procedures should be adopted in order to essentially eliminate interference from noise:

(d) The system loss on each of the wanted signal propagation paths should be minimized; this may be accomplished by (1) the use of the highest practicable transmitting and receiving antenna heights in bands 8 and 9, and (2) maximizing the path antenna power gains for each of the wanted signal propagation paths. The path antenna power gains of the wanted signal propagation paths may be maximized by using the maximum practicable transmitting and receiving antenna gains and by minimizing the antenna circuit and polarization coupling losses. The minimization of the system loss on each of the wanted signal propagation paths will already have been achieved to a large extent in connection with procedures (a), (b), and (c) above.

(e) In general, receiving systems should be employed which have the lowest practicable values of operating sensitivity $w_{mr}(g)$.

(f) Finally, sufficiently high transmitter powers should be used [keeping the relative powers at the optimum relative values determined by procedures (a), (b), and (c)] so that the wanted signal power w_m will exceed the operating sensitivity $w_{mr}(g)$ for a sufficiently large fraction of the time during the intended period of operation at every receiving location.

Although it might at first seem impracticable, serious consideration should be given to the use of auxiliary channels from wanted receivers to wanted transmitters. The provision of such channels might well be feasible in those cases where two-way transmissions are involved and might lead to important economies in both power and spectrum occupancy [Hitchcock and Morris, 1961].

Ultimately, when optimum use of the spectrum has been achieved, it will not be possible to find a single receiving location at which radio noise rather than either wanted or unwanted signals can be observed for a large percentage of the time throughout the usable portions of the radio spectrum not devoted to the study of radio noise sources, as is the radio astronomy service. Although everyone will agree that the attainment of this ideal goal of interference-free spectrum usage by the maximum number of simultaneous users can be achieved only over a very long period of time because of the large investments in radio systems currently in operation, nevertheless it seems desirable to have a clear statement of the procedures which should be employed in the future in order to move in the direction of meeting this ultimately desirable goal whenever appropriate opportunities arise.

b, B	Effective bandwidth, b, of a receiver in cycles per second, $B = 10 \log b$ decibels, (V.7) and (V.8).
f_{op}, F_{op}	Operating noise factor of the pre-detection receiving system, $F_{op} = 10 \log f_{op}$ db, (V.7) and (V.8)
g	Grade of service. A specified grade of service provided by a given signal will guarantee a corresponding degree of fidelity of the information delivered to the receiver output.
g_o, G_o	The maximum value of the operating gain of a pre-detection receiving system, $G_o = 10 \log g_o$ db, (V.7) and (V.8).
g_{ms}, G_{ms}	The hourly median operating signal gain of a pre-detection receiving system, $G_{ms} = 10 \log g_{ms}$ db, (V.9).
k	Boltzmann's constant, $k = 1.38054 \times 10^{-23}$ joules per degree, (V.7).
$k T_o b$	Johnson's noise power that would be available in the bandwidth b cycles per second at a reference absolute temperature $T_o = 288.37$ degrees Kelvin, (V.7).
K	The decibel ratio of the amplitude of the constant or power-fading component of a received signal relative to the root-sum-square value of the amplitudes of the Rayleigh components, figure V.1.
K_o	An arbitrary constant that combines several parameters in the systems equation, (V.22).
$L_{\ell t}$	Transmission line and matching network losses at the transmitter, (V.20).
L_m	Hourly median transmission loss, (V.20).
$L_m(q)$	Hourly median transmission loss not exceeded for a fraction q of all hours, or exceeded $(1-q)$ of all hours, (V.25).
$L_m(q, Q)$	Hourly median transmission loss exceeded for a fraction $(1-q)$ of all hours with a probability Q, section V.6.
$L_m(0.5)$	Median value of $L_m(q)$, (V.2).
$L_{mo}(g)$	Maximum allowable hourly median transmission loss for a grade g of service, (V.27).
$L_o(q)$	Observed values of transmission loss not exceeded a fraction q of the recording period, (V.5).
$L_{um}(q)$	Hourly median transmission loss of unwanted signal not exceeded for a fraction q of all hours, (V.33).
$L_{um}(0.5)$	Long-term median value of $L_{um}(q)$, (V.39).

L_π	Transmission loss associated with the "instantaneous" power W_π, (V.6) and figure V.2.
$L_\pi(q)$	Transmission loss L_π not exceeded a fraction q of the time.
$L_\pi(0.1)$	The interdecile range $L_\pi(0.9) - L_\pi(0.1)$ of values of transmission loss associated with the "instantaneous" power W_π, figure V.2.
q	Time availability.
q_1, q_8	Time availability in climates 1 and 8, section V.8.
Q	Service probability, discussed in section V.8.
$Q(q)$	The probability Q of obtaining satisfactory service for a fraction of time q, figure V.6.
$Q(z_{mo})$	Service probability Q expressed in terms of the error function of z_{mo}, (V.46), figure V.5.
r_m, R_m	Ratio of the hourly median wanted signal power to the hourly median operating noise power, $R_m = 10 \log r_m$ db, (V.9).
r_{mr}, R_{mr}	A specified value of r_m which must be exceeded for at least a specified fraction of time to provide satisfactory service in the presence of noise alone, $R_{mr} = 10 \log r_{mr}$ db, (V.9).
r_u, R_u	Ratio of hourly median wanted to unwanted signal power available at the receiver, $R_u = 10 \log r_u$ db, (V.14).
r_{ur}, R_{ur}	A specified value of r_u which must be exceeded for at least a specified fraction of time to provide satisfactory service in the presence of a single unwanted signal, $R_{ur} = 10 \log r_{ur}$ db, (V.14).
$r_{mr}(g)$, $R_{mr}(g)$	The minimum acceptable signal to noise ratio which will provide service of a given grade g in the absence of unwanted signals other than noise, $R_{mr}(g) = 10 \log r_{mr}(g)$ db, (V.9).
$r_{ur}(g)$, $R_{ur}(g)$	The protection ratio r_{ur} required to provide a specified grade of service g, $R_{ur}(g) = 10 \log r_{ur}(g)$ db, sections V.4 and V.5.
$R_m(q)$	The value of R_m exceeded at least a fraction q of the time, (V.24).
$R_m(0.5)$	The median value of R_m, (V.29).
$R_u(q)$	A specified value of R_u exceeded at least a fraction q of the time, (V.36).
$R_u(0.5)$	The median value of R_u, (V.36).
$R_{ur}(g, q)$	The required ratio R_{ur} to provide service of grade g for at least a fraction q of the time, (V.35).
$R_{uro}(g)$	The required ratio R_{ur} for non-fading wanted and unwanted signals, (V.14).
$R_{u\pi}$	The ratio between the instantaneous wanted and unwanted signal powers, (V.10).
T_o	Reference absolute temperature $T_o = 288.37$ degrees Kelvin, (V.7).
$V(0.5)$, d_e)	A parameter used to adjust the predicted reference median for various climatic regions or periods of time, section V.8 and section 10, volume 1.

$V_i(0.5, d_e)$, $V_j(0.5, d_e)$ — The parameter $V(0.5, d_e)$ for each of two climates represented by the subscripts i and j, (V.41).

w_m, W_m — The median wanted signal power available at a receiver, $W_m = 10 \log w_m$ dbw, (V.1).

w_{mn}, W_{mn} — The median value of the total noise power is w_{mn} watts, $W_{mn} = 10 \log w_{mn}$ dbw, (V.7) and (V.8).

w_{mr}, W_{mr} — Operating threshold, the median wanted signal power required for satisfactory service in the presence of noise, $W_{mr} = 10 \log w_{mr}$ dbw, (V.9).

w_o, W_o — A fixed value of transmitter output power w_o in watts, $W_o = 10 \log w_o$ dbw, (V.26).

w_t, W_t — Total radiated power in watts and in dbw, section V.6.

w_u, W_u — Power radiated from an unwanted or interfering station, w_u watts, $W_u = 10 \log w_u$ dbw, (V.33).

w_{um}, W_{um} — Median unwanted signal power w_{um} in watts, $W_{um} = 10 \log w_{um}$ dbw, (V.33) and (V.34).

$w_{u\pi}$, $W_{u\pi}$ — Unwanted signal power associated with phase interference fading, $w_{u\pi}$ in watts, $W_{u\pi} = 10 \log w_{u\pi}$ dbw, section V.4.

w_π, W_π — Wanted signal power associated with phase interference fading, w_π is defined as the average power for a single cycle of the radio frequency, $W_\pi = 10 \log w_\pi$ dbw, (V.1).

$W_{\ell t}$ — Transmitter output power, (V.20).

$W_{\ell t}(q)$ — Transmitter power that will provide at least grade g service for a fraction q of all hours, (V.25).

$W_m(q)$ — The hourly median wanted signal power exceeded for a fraction q of all hours, (V.24).

$W_m(0.5)$ — Long-term median value of W_m, (V.2).

$W_{mo}(q)$ — Observed values of $W_m(q)$ made over a large number of paths which can be characterized by the same set of prediction parameters, section V.8.

$W_{mr}(g)$ — The operating threshold of a receiving system, defined as the minimum value of W_m required to provide a grade of service g in the presence of noise alone, (V.9).

$W_{um}(q)$ — The hourly median unwanted signal power W_{um} expected to be available at least a fraction q of all hours, (V.33).

$W_{um}(0.5)$ — The median value of $W_{um}(q)$, (V.37).

$W_\pi(q)$ — The "instantaneous" power W_π exceeded for a fraction of time q, (V.6).

$W_\pi(0.1)$, $W_\pi(0.9)$ — The interdecile range $W_\pi(0.1) - W_\pi(0.9)$ of the power $W_\pi(q)$, equivalent to the interdecile range of short term transmission loss shown on figure V.2.

Y — A symbol used to describe long-term fading, (V.1) and (V.3).

Y_u — Long-term fading of an unwanted signal, (V.16).

$Y_{u\pi}$ Phase interference component of the total fading of an unwanted signal, (V. 10).

Y_{π} Phase interference fading component for a wanted signal, (V. 10).

$Y(q)$ Long-term variability Y for a given fraction of hourly medians q, defined by (V.4).

$Y(0.5$ The median value of Y, which by definition is zero.

$Y_i(q, d_e)$, $Y_j(q, d_e)$ Values of Y for climates i and j, (V.41) and (V.42).

$Y_{ij}(q, d_e)$ The root-mean-square value of the variability for two climates, (V.42).

$Y_m(q)$ Long-term variability in the presence of variable external noise, (V.31).

$Y_n(q)$ Variability of the operating noise factor, F_{op}, (V.31) and (V.32).

$Y_R(q)$ Long-term variability of the wanted to unwanted signal ratio, (V.38).

$Y_u(q)$ Long-term variability of an unwanted signal, (V.39).

$Y_{u\pi}(q)$ The phase interference fading component of the total variability of an unwanted signal, section V.4.

$Y_{\pi}(q)$ The phase interference fading component of the total variability of a wanted signal, section V.4.

Y_1, Y_8 Values of Y for climates 1 and 8, section V.9.

z_{mo}, z_{op}, z_{uc} Standard normal deviates defined by (V.45), (V.52) and (V.55).

Z The decibel ratio of the long-term fading, Y, of a wanted signal and the long-term fading, Y_u, of an unwanted signal, (V.16).

$Z_a(q)$ The approximate cumulative distribution function of the variable ratio Z, (V.17).

$Z_a(0.5)$ Median value of the variable ratio Z, $Z_a(0.5) \approx 0$.

Z_{π} The decibel ratio of the phase interference fading component Y_{π} for a wanted signal and the phase interference fading component $Y_{u\pi}$ for an unwanted signal, (V.11).

$Z_{\pi a}(q, K, K_u)$ The approximate cumulative distribution function of Z_{π}, (V.12).

$\rho_{\ell u}$ The normalized correlation or covariance between path-to-path variations of $W_m(0.5)$ and $W_{um}(0.5)$, (V.54).

ρ_{tn} The long-term correlation between W_m and F_{op}, (V.31).

ρ_{tu} The long-term correlation between W_m and W_{um}, (V.38).

σ_c^2 The path-to-path variance of deviations of observed from predicted transmission loss, section V.8.

$\sigma_c^2(q)$ The path-to-path variance of the difference between observed and predicted values of transmission loss expected for a fraction q of all hours.

$\sigma_c^2(0.5)$ The path-to-path variance of the difference between observed and predicted long-term median values of transmission loss, (V.40) and the following paragraph.

σ_F^2 The variance of the operating noise factor F_{op}, (V.51).

$\sigma_{op}^2(q)$ Total variance of any estimate of the service criterion for service limited only by external noise, (V.51).

$\sigma^2_{uc}(q)$ Total variance of any estimate of the service criterion for service limited only by interference from a single unwanted source, (V. 54).

σ^2_{ur} Variance of the estimate of $R_{ur}(g, q)$, (V. 54).

GPO 857-642