6. DETERMINATION OF ANGULAR DISTANCE FOR TRANSHORIZON PATHS

The angular distance, θ, is the angle between radio horizon rays in the great circle plane defined by the antenna locations. This important parameter is used in diffraction theory as well as in forward scatter theory. Angular distance depends upon the terrain profile, as illustrated in figure 6.1, and upon the bending of radio rays in the atmosphere. Figure 6.1 assumes a linear dependence on height of the atmospheric refractive index, n, which implies a nearly constant rate of ray refraction. If heights to be considered are less than one kilometer above the earth's surface, the assumption of a constant effective earth's radius, a, makes an adequate allowance for ray bending. Atmospheric refractivity $N = (n - 1) \times 10^6$ more than one kilometer above the earth's surface, however, is assumed to decay exponentially with height [Bean and Thayer, 1959]. This requires corrections to the effective earth's radius formulas, as indicated in subsection 6.4.

To calculate θ, one must first plot the great circle path and determine the radio horizons.

6.1 Plotting a Great Circle Path

For distances less than 70 kilometers, the great circle path can be approximated by a rhumb line, which is a line intersecting all meridians at the same angle. For greater distances, the organization of a map study is illustrated on figure 6.2. Here, a rhumb line is first plotted on an index map to show the boundaries of available detailed topographic sheets. Segments of the actual great circle path are later plotted on these detailed maps.

The spherical triangle used for the computation of points on a great circle path is shown on figure 6.3, where PAB is a spherical triangle, with A and B the antenna terminals, and P the north or south pole. B has a greater latitude than A, and P is in the same hemisphere. The triangle shown is for the northern hemisphere but may readily be inverted to apply to the southern hemisphere. B' is any point along the great circle path from A to B, and the triangle PAB' is the one actually solved. The latitudes of the points are denoted by Φ_A, Φ_B, and $\Phi_{B'}$, while C and C' are the differences in longitude between A and B and A and B', respectively. Z and Z' are the corresponding great circle path lengths. The following formulas are practical for hand computations as well as for automatic digital computers. Equations (6.1) to (6.4) have been taken, in this form, from a well-known reference book [I. T. and T., 1956], where they appear on pages 730-739.

The initial bearings (X from terminal A, and Y from terminal B) are measured from true north, and are calculated as follows:

$$\tan \frac{Y - X}{2} = \cot \frac{C}{2} \left[\left(\sin \frac{\Phi_B - \Phi_A}{2} \right) \Big/ \left(\cos \frac{\Phi_B + \Phi_A}{2} \right) \right] \tag{6.1}$$

$$\tan \frac{Y+X}{2} = \cot \frac{C}{2} \left[\left(\cos \frac{\Phi_B - \Phi_A}{2} \right) \Big/ \left(\sin \frac{\Phi_B + \Phi_A}{2} \right) \right] \tag{6.2}$$

$$\frac{Y+X}{2} + \frac{Y-X}{2} = Y, \quad \text{and} \quad \frac{Y+X}{2} - \frac{Y-X}{2} = X. \tag{6.3}$$

The great circle distance, Z, is given by

$$\tan \frac{Z}{2} = \tan \frac{\Phi_B - \Phi_A}{2} \left[\left(\sin \frac{Y+X}{2} \right) \Big/ \left(\sin \frac{Y-X}{2} \right) \right]. \tag{6.4}$$

To convert the angle Z obtained in degrees from (6.4) to units of length, the following is used, based on a mean sea level earth's radius of 6370 km:

$$d_{km} = 111.18\, Z^{\circ} \tag{6.5}$$

The following formulas show how to calculate either the latitude or the longitude of a point on the great circle path, when the other coordinate is given. The given coordinates correspond to the edges of detailed maps, and to intermediate points usually about 7.5 minutes apart, so that straight lines between points will adequately approximate a great circle path.

For predominantly east-west paths, calculate the latitude $\Phi_{B'}$ for a given longitude difference C':

$$\cos Y' = \sin X \sin C' \sin \Phi_A - \cos X \cos C' \tag{6.6}$$

$$\cos \Phi_{B'} = \sin X \cos \Phi_A / \sin Y' \tag{6.7}$$

For predominantly north-south paths, calculate the longitude difference C' for a given latitude $\Phi_{B'}$:

$$\sin Y' = \sin X \cos \Phi_A / \cos \Phi_{B'} \tag{6.8}$$

$$\cot \frac{C'}{2} = \tan \frac{Y'-X}{2} \left[\left(\cos \frac{\Phi_{B'} + \Phi_A}{2} \right) \Big/ \left(\sin \frac{\Phi_{B'} - \Phi_A}{2} \right) \right] \tag{6.9}$$

Where the bearing of a path is close to 45 degrees, either method may be used.

6.2 Plotting a Terrain Profile and Determining the Location of Radio Horizon Obstacles

This subsection explains how to determine the sea level arc distance, $d_{Lt,r}$ from an antenna to its radio horizon obstacle, and the height, $h_{Lt,r}$ of this obstacle above mean sea level. The horizon obstacles are represented by the points (d_{Lt}, h_{Lt}) and (d_{Lr}, h_{Lr}) in the great circle plane containing the antennas. These points may be determined by the tops of high buildings, woods, or hills, or may be entirely determined by the bulge of the earth itself. All of the predictions of this paper replace the earth by a cylinder whose elements are perpendicular to the great circle plane and whose cross-section is in general irregular and determined by the antenna and horizon locations in the great circle plane. When the difference in elevations of antenna and horizon greatly exceeds one kilometer, ray tracing is necessary to determine the location of radio horizons accurately [Bean and Thayer, 1959].

Elevations h_i of the terrain are read from topographic maps and tabulated versus their distances x_i from the transmitting antenna. The recorded elevations should include those of successive high and low points along the path. The terrain profile is plotted on linear graph paper by modifying the terrain elevations to include the effect of the average curvature of the radio ray path and of the earth's surface. The modified elevation y_i of any point h_i at a distance x_i from the transmitter along a great circle path is its height above a plane which is horizontal at the transmitting antenna location:

$$y_i = h_i - x_i^2/(2a) \tag{6.10}$$

where the effective earth's radius, a, in kilometers is calculated using (4.4), or is read from figure 4.2 as a function of N_s. The surface refractivity, N_s, is obtained from (4.3), where N_o is estimated from the map on figure 4.1.

A plot of y_i versus x_i on linear graph paper is the desired terrain profile. Figure 6.4 shows the profile for a line-of-sight path. The solid curve near the bottom of the figure indicates the shape of a surface of constant elevation ($h = 0$ km). Profiles for a path with one horizon common to both antennas and for a path with two radio horizons are shown in figures 6.5 and 6.6. The vertical scales of these three figures are exaggerated in order to provide a sufficiently detailed representation of terrain irregularities. Plotting terrain elevations vertically instead of radially from the earth's center leads to negligible errors where vertical changes are small relative to distances along the profile.

On a cartesian plot of y_i versus x_i, as illustrated in figures 6.4, 6.5, and 6.6, the ray from each antenna to its horizon is a straight line, provided the difference in antenna and horizon elevations is less than one kilometer. Procedures to be followed where this is not the case are indicated in the next subsection.

6.3 Calculation of Effective Antenna Heights for Transhorizon Paths

If an antenna is located on another structure, or on a steep cliff or mountainside, the height of this structure, cliff, or mountain above the surrounding terrain should be included as part of the antenna height. To obtain the effective height of the transmitting antenna, the average height above sea level $\bar{h}_t$ of the central 80 per cent of the terrain between the transmitter and its horizon is determined. The following formula may be used to compute $\bar{h}_t$ for 31 evenly spaced terrain elevations h_{ti} for $i = 0, 1, 2, \cdots 30$, where h_{t0} is the height above sea level of the ground below the transmitting antenna, and, $h_{t30} = h_{Lt}$:

$$\bar{h}_t = \frac{1}{25} \sum_{i=3}^{27} h_{ti}, \quad h_t = h_{ts} - \bar{h}_t \text{ for } \bar{h}_t < h_{t0}; \tag{6.11a}$$

otherwise

$$h_t = h_{ts} - h_{t0} \tag{6.11b}$$

where h_{ts} is the height of the transmitting antenna above mean sea level. The height h_r is similarly defined.

If h_t or h_r as defined above is less than one kilometer, $h_{te} = h_t$ or $h_{re} = h_r$. For antennas higher than one kilometer, a correction Δh_e, read from figure 6.7, is used to reduce h_t or h_r to the value h_{te} or h_{re}:

$$h_{te} = h_t - \Delta h_e(h_t, N_s), \quad h_{re} = h_r - \Delta h_e(h_r, N_s). \tag{6.12}$$

The correction Δh was obtained by ray tracing methods described by Bean and Thayer [1959]. For a given effective earth's radius, the effective antenna height h_{te} corresponding to a given horizon distance d_{Lt} is smaller than the actual antenna height, h_t. Over a smooth spherical earth with $h_{te} < 1$ km and $h_{re} < 1$ km, the following approximate relationship exists between effective antenna heights and horizon distances:

$$h_{te} = d_{Lt}^2/(2a), \quad h_{re} = d_{Lr}^2/(2a) \tag{6.13a}$$

If the straight line distance r between antennas is substantially different from the sea level arc distance d, as in communication between an earth terminal and a satellite, the effective antenna heights must satisfy the exact relation:

$$h_{te} = a[\sec(d_{Lt}/a) - 1], \quad h_{re} = a[\sec(d_{Lr}/a) - 1] \tag{6.13b}$$

6.4 Calculation of the Angular Distance, θ

The angular distance, θ, is the angle between horizon rays in the great circle plane, and is the minimum diffraction angle or scattering angle unless antenna beams are elevated. Calculations for cases where the antenna beams are elevated are given in annex III.

In calculating the angular distance, one first calculates the angles θ_{et} and θ_{er} by which horizon rays are elevated or depressed relative to the horizontal at each antenna, as shown on figure 6.1. In this report, all heights and distances are measured in kilometers, and angles are in radians unless otherwise specified. When the product θd is less than 2,

$$\theta = \theta_{oo} = d/a + \theta_{et} + \theta_{er} \tag{6.14}$$

where a in (6.14) is the effective earth's radius defined in section 4. The horizon ray elevation angles θ_{et} and θ_{er} may be measured with surveying instruments in the field, or determined directly from a terrain profile plot such as that of figure 6.5 or 6.6, but are usually computed using the following equations:

$$\theta_{et} = \frac{h_{Lt} - h_{ts}}{d_{Lt}} - \frac{d_{Lt}}{2a}, \quad \theta_{er} = \frac{h_{Lr} - h_{rs}}{d_{Lr}} - \frac{d_{Lr}}{2a} \tag{6.15}$$

where h_{Lt}, h_{Lr} are heights of horizon obstacles, and h_{ts}, h_{rs} are antenna elevations, all above mean sea level. As a general rule, the location (h_{Lt}, d_{Lt}) or (h_{Lr}, d_{Lr}) of a horizon obstacle is determined from the terrain profile by using (6.15) to test all possible horizon locations. The correct horizon point is the one for which the horizon elevation angle θ_{et} or θ_{er} is a maximum. When the trial values are negative, the maximum is the value nearest zero. For a smooth earth,

$$\theta_{et,er} = -\sqrt{2 h_{te,re}/a} \quad \text{for} \quad h_{te,re} < 1 \text{ km} .$$

At the horizon location, the angular elevation of a horizon ray, θ_{ot} or θ_{or}, is greater than the horizon elevation angle θ_{et} or θ_{er}:

$$\theta_{ot} = \theta_{et} + d_{Lt}/a , \quad \theta_{or} = \theta_{er} + d_{Lr}/a . \tag{6.16}$$

If the earth is smooth, θ_{ot} and θ_{or} are zero, and $\theta \cong D_s/a$, where

$$D_s = d - d_{Lt} - d_{Lr} \tag{6.17}$$

Figure 6.8, valid only for $\theta_{ot} + \theta_{or} = 0$, is a graph of θ versus D_s for various values of surface refractivity, N_s.

In the general case of irregular terrain, the angles α_{oo} and β_{oo} shown in figure 6.1 are calculated using the following formulas:

$$\alpha_{oo} = \frac{d}{2a} + \theta_{et} + \frac{h_{ts} - h_{rs}}{d} \tag{6.18a}$$

$$\beta_{oo} = \frac{d}{2a} + \theta_{er} + \frac{h_{rs} - h_{ts}}{d} \tag{6.18b}$$

These angles are positive for beyond-horizon paths. To allow for the effects of a non-linear refractivity gradient, α_{oo} and β_{oo} are modified by corrections $\Delta\alpha_o$ and $\Delta\beta_o$ to give the angles α_o and β_o whose sum is the angular distance, θ, and whose ratio defines a path asymmetry factor s:

$$\alpha_o = \alpha_{oo} + \Delta\alpha_o \tag{6.19a}$$

$$\beta_o = \beta_{oo} + \Delta\beta_o \tag{6.19b}$$

$$\theta = \alpha_o + \beta_o, \qquad s = \alpha_o/\beta_o . \tag{6.19c}$$

The corrections $\Delta\alpha_o$ and $\Delta\beta_o$ are functions of the angles θ_{ot} and θ_{or}, (6.16), and of the distances d_{st} and d_{sr} from each horizon obstacle to the crossover of horizon rays. These distances are approximated as

$$d_{st} = d\beta_{oo}/\theta_{oo} - d_{Lt}, \qquad d_{sr} = d\alpha_{oo}/\theta_{oo} - d_{Lr} . \tag{6.20}$$

The sum of d_{st} and d_{sr} is the distance D_s between horizon obstacles, defined by (6.17). Over a smooth earth $d_{st} = d_{sr} = D_s/2$.

Figure 6.9, drawn for $N_s = 301$, shows $\Delta\alpha_o$ as a function of θ_{ot} and d_{st}. Similarly, $\Delta\beta_o$ is read from the figure as a function of θ_{or} and d_{sr}. For values of N_s other than 301, the values as read from the figure are multiplied by $C(N_s)$:

$$\Delta\alpha_o(N_s) = C(N_s)\,\Delta\alpha_o(301), \quad \Delta\beta_o(N_s) = C(N_s)\,\Delta\beta_o(301) \tag{6.21a}$$

$$C(N_s) = (1.3\, N_s^2 - 60\, N_s) \times 10^{-5} \tag{6.21b}$$

For instance, $C(250) = 0.66$, $C(301) = 1.0$, $C(350) = 1.38$, and $C(400) = 1.84$. Figure 6.10 shows $C(N_s)$ plotted versus N_s.

For small $\theta_{ot,r}$ no correction $\Delta\alpha_o$ or $\Delta\beta_o$ is required for values of $d_{st,r}'$ less than 100 Km. When both $\Delta\alpha_o$ and $\Delta\beta_o$ are negligible:

$$\theta = \theta_{oo} = \alpha_{oo} + \beta_{oo} \tag{6.22}$$

which is the same as (6.14).

If θ_{ot} or θ_{or} is negative, compute

$$d_{st}' = d_{st} - |a\,\theta_{ot}| \quad \text{or} \quad d_{sr}' = d_{sr} - |a\,\theta_{or}| \;, \tag{6.23}$$

substitute d_{st}' for d_{st} or d_{sr}' for d_{sr}, and read figure 6.9, using $\theta_{ot,r} = 0$.

If either θ_{ot} or θ_{or} is greater than 0.1 radian and less than 0.9 radian, determine $\Delta\alpha_o$ or $\Delta\beta_o$ for $\theta_{ot} = 0.1$ radian and add the additional correction term

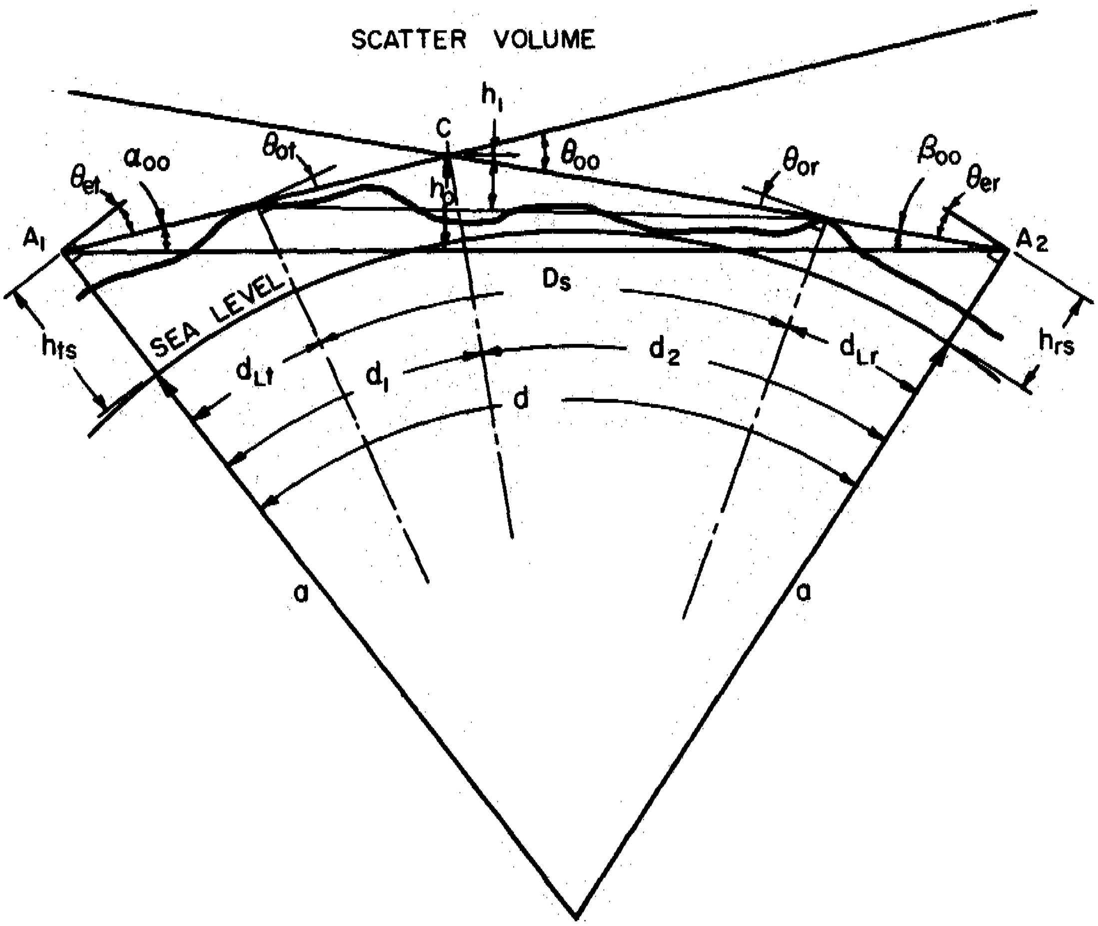
$$N_s(9.97 - \cot\theta_{ot,r})\,[1 - \exp(-0.05\,d_{st,r})] \times 10^{-6} \text{ radians} \;.$$

The bending of radio rays elevated more than 0.9 radian above the horizon and passing all the way through the atmosphere is less than 0.0004 radian, and may be neglected.

Other geometrical parameters required for the calculation of expected transmission loss are defined in the sections where they are used.

Many of the graphs in this and subsequent sections assume that $s = \alpha_o/\beta_o \leq 1$, where α_o and β_o are defined by (6.19a) and (6.19b). It is therefore convenient, since the transmission loss is independent of the actual direction of transmission, to denote as the transmitting antenna whichever antenna will make s less than or equal to unity. Alternatively, s may be replaced by $1/s$ and the subscripts t and r may be interchanged in some of the formulas and graphs, as noted in later sections.

PATH GEOMETRY



DISTANCES ARE MEASURED IN KILOMETERS ALONG A GREAT CIRCLE ARC.

$$\theta_{oo} = \frac{D_s}{a} + \theta_{ot} + \theta_{or} = \frac{d}{a} + \theta_{et} + \theta_{er}$$

Figure 6.1

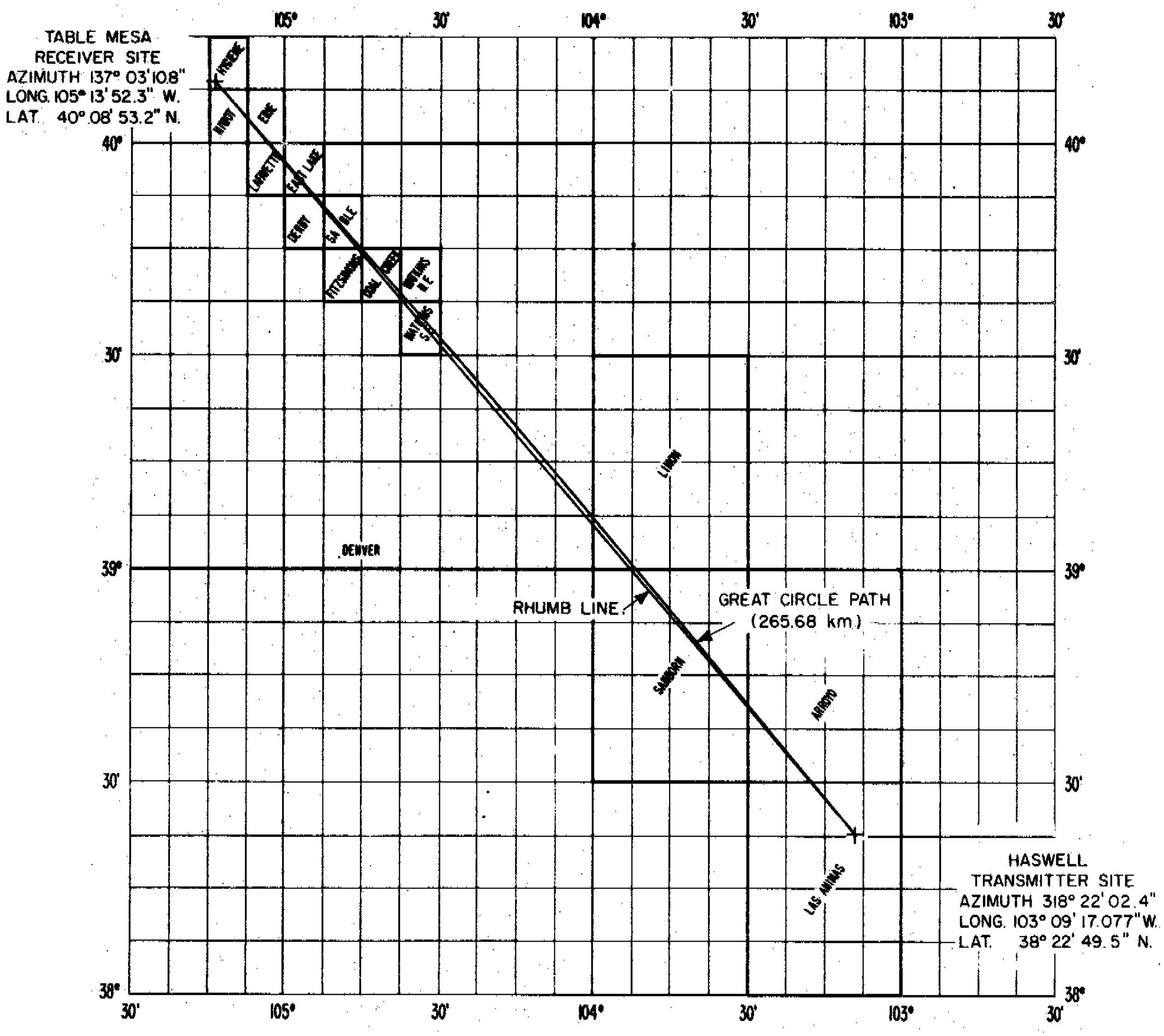
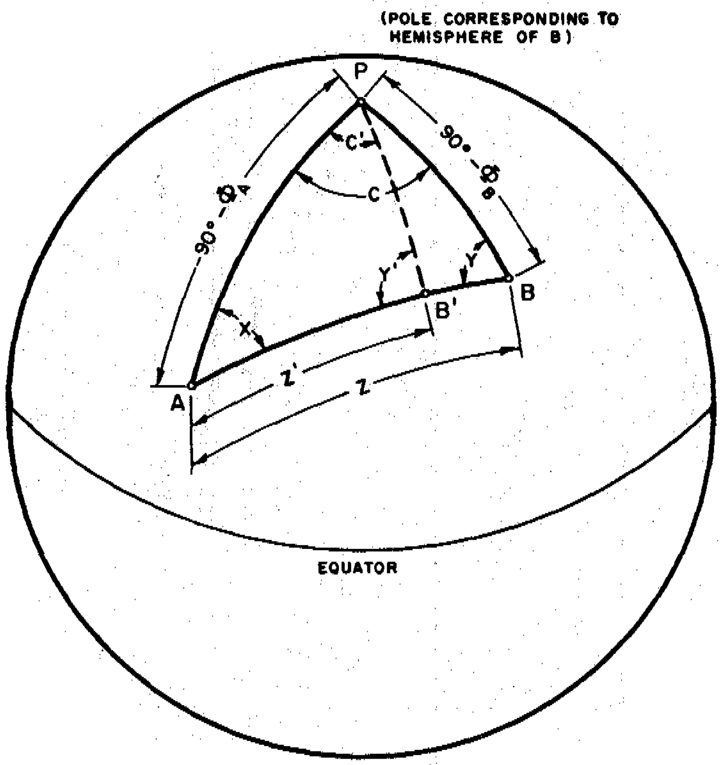


TABLE MESA
RECEIVER SITE
AZIMUTH 137° 03'10.8"
LONG. 105° 13' 52.3" W.
LAT. 40° 08' 53.2" N.
105°
30'
104°
30'
103°
30'
40°
30'
39°
30'
38°
DENVER
LIMON
RHUMB LINE
GREAT CIRCLE PATH
(265.68 km)
SANBORN
ARROYO
LAS ANIMAS
HASWELL
TRANSMITTER SITE
AZIMUTH 318° 22' 02.4"
LONG. 103° 09' 17.077" W.
LAT. 38° 22' 49.5" N.

Figure 6.2



SPHERICAL TRIANGLE FOR
GREAT CIRCLE PATH COMPUTATIONS

Figure 6.3

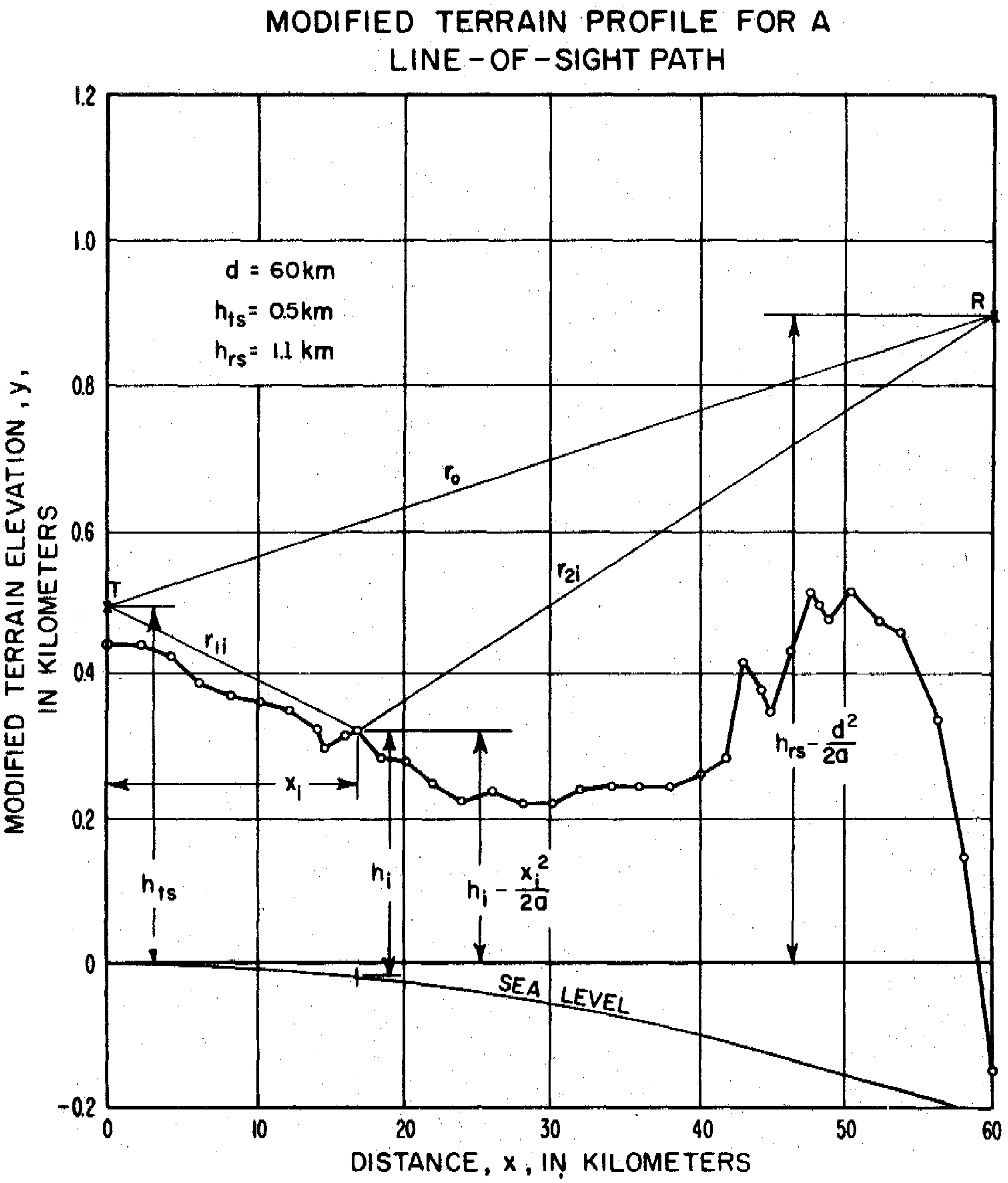

MODIFIED TERRAIN PROFILE FOR A LINE-OF-SIGHT PATH
d = 60km
h_{ts} = 0.5km
h_{rs} = 1.1 km
MODIFIED TERRAIN ELEVATION, y, IN KILOMETERS
DISTANCE, x, IN KILOMETERS
T
R
r_o
r_{1i}
r_{2i}
x_i
h_{ts}
h_i
$h_i - \frac{x_i^2}{2a}$
$h_{rs} - \frac{d^2}{2a}$
SEA LEVEL
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
60

Figure 6.4

MODIFIED TERRAIN PROFILE FOR A COMMON-HORIZON PATH

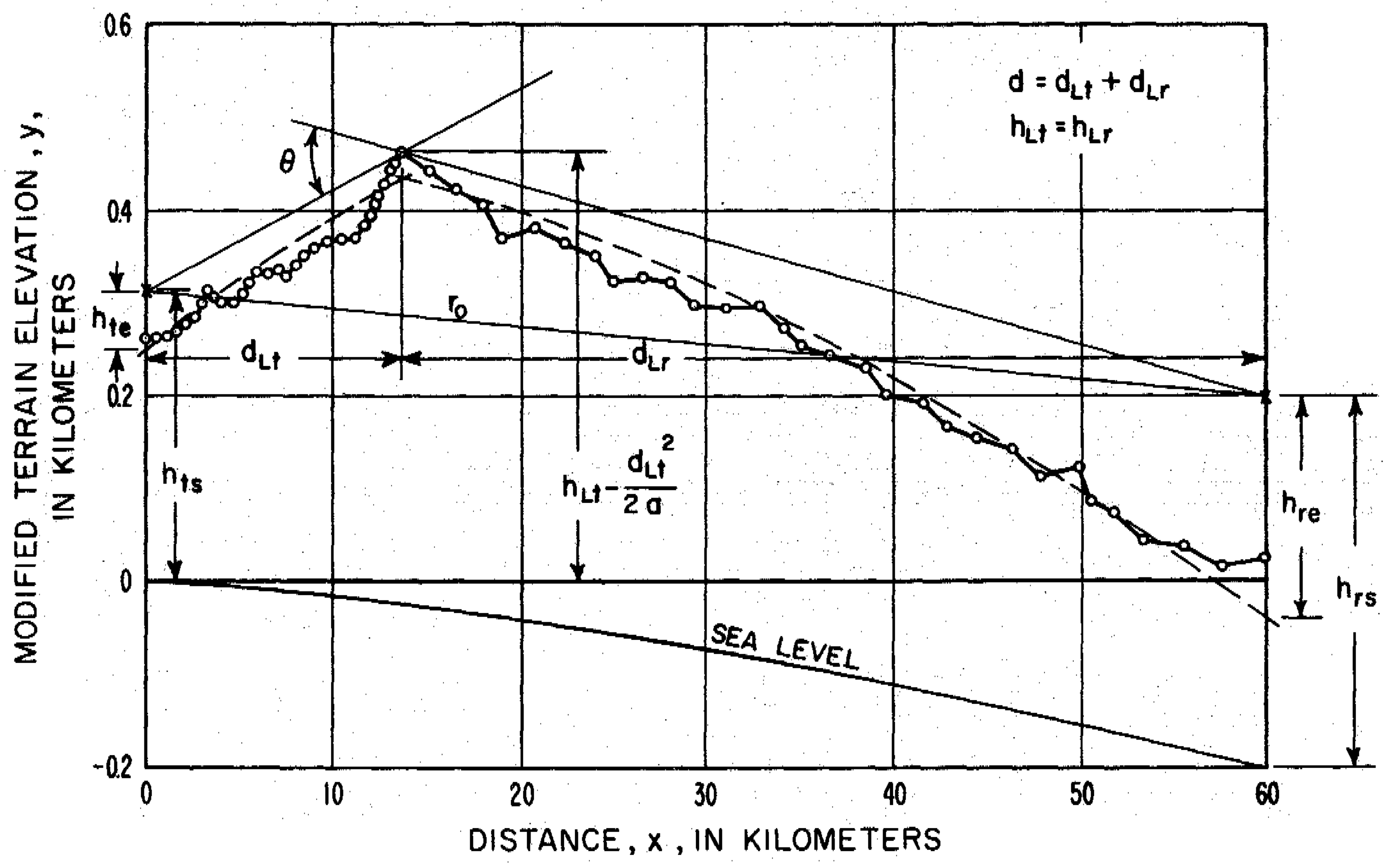


Figure 6.5

MODIFIED TERRAIN PROFILE FOR A DOUBLE-HORIZON PATH

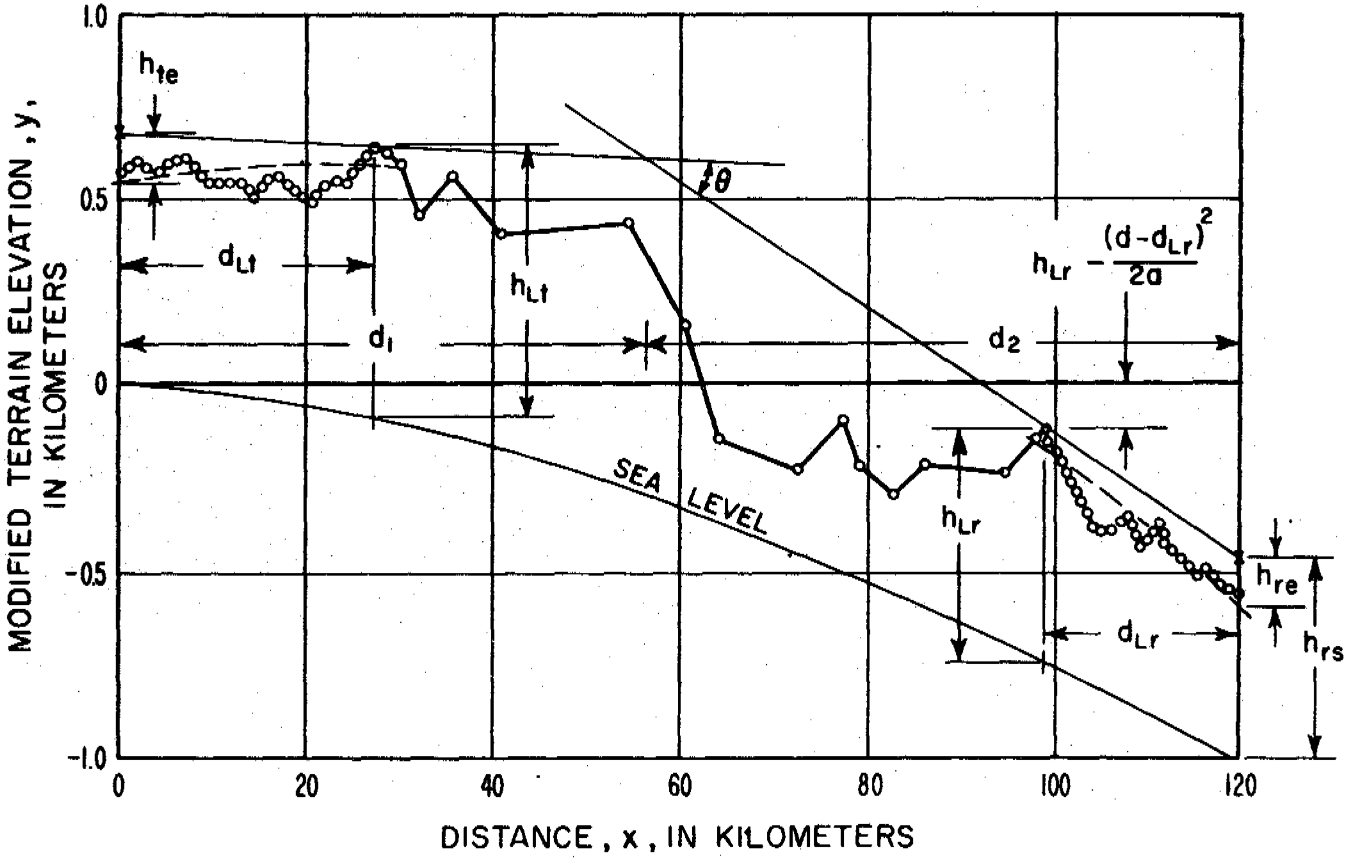


Figure 6.6

REDUCTION OF ANTENNA HEIGHT FOR VERY HIGH ANTENNAS

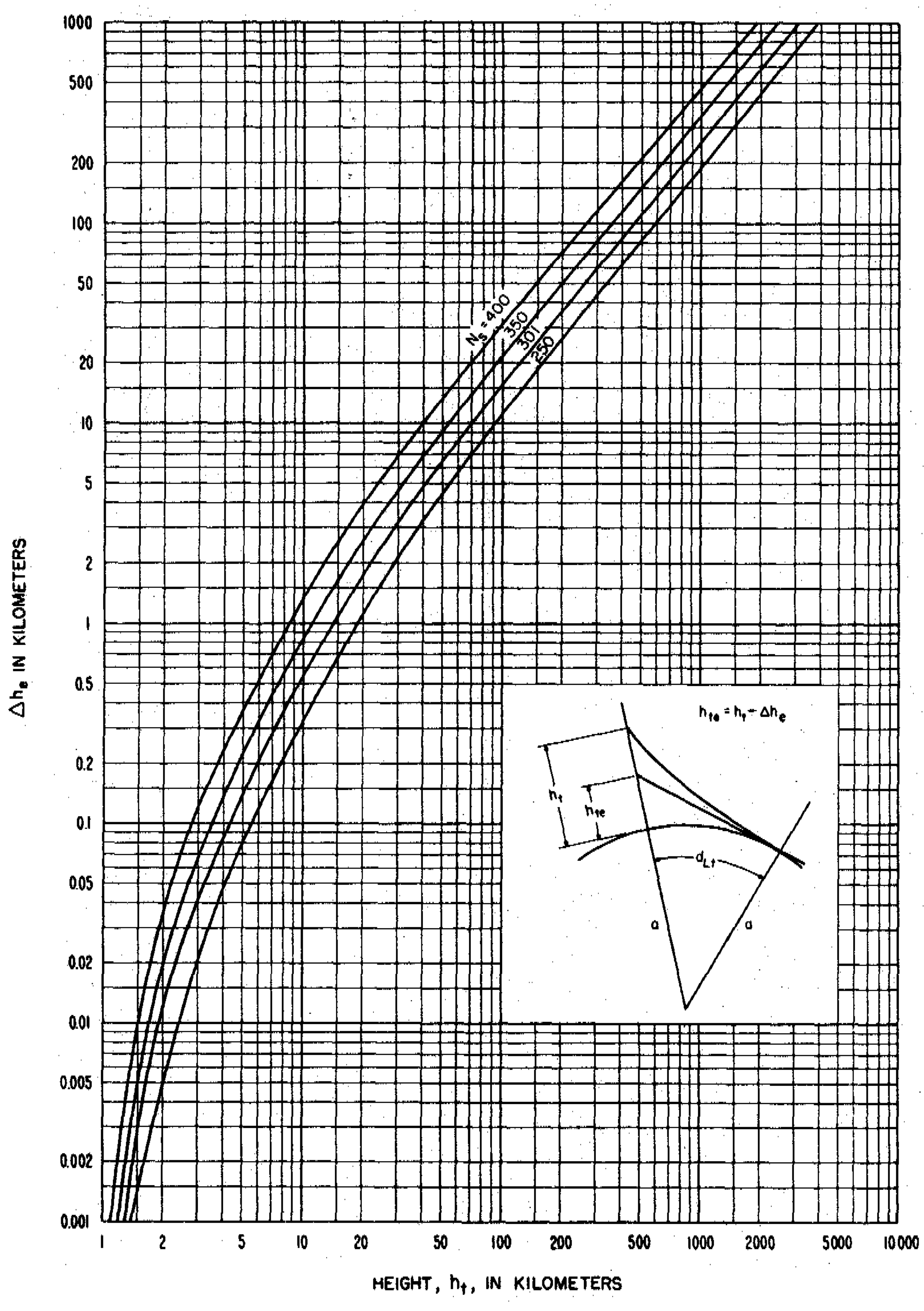


Figure 6.7

ANGULAR DISTANCE, θ, OVER A SMOOTH EARTH
VS DISTANCE BETWEEN HORIZONS, D_S

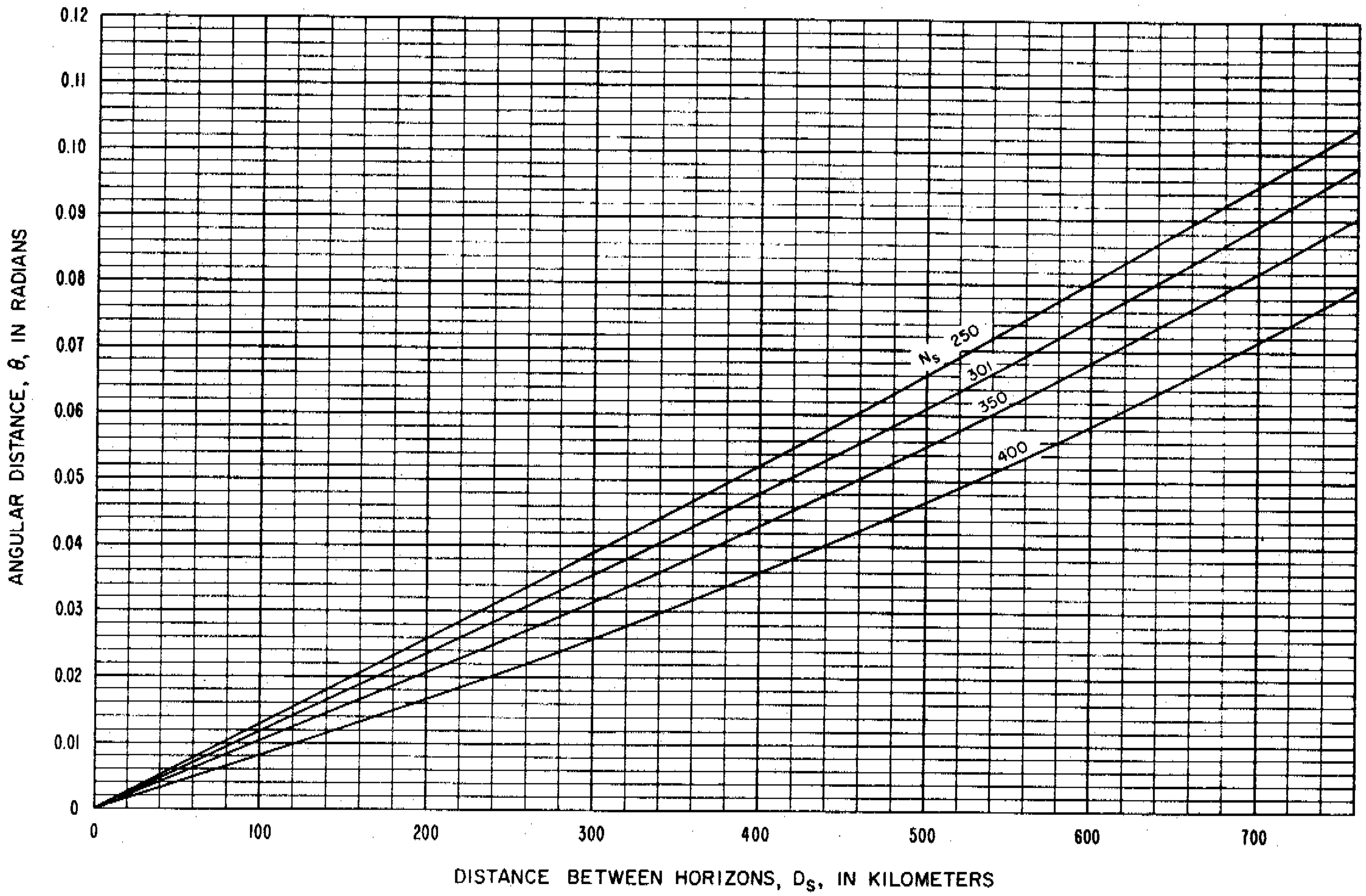


Figure 6.8

CORRECTION TERMS $\Delta\alpha_o$, $\Delta\beta_o$ FOR $N_s = 301$

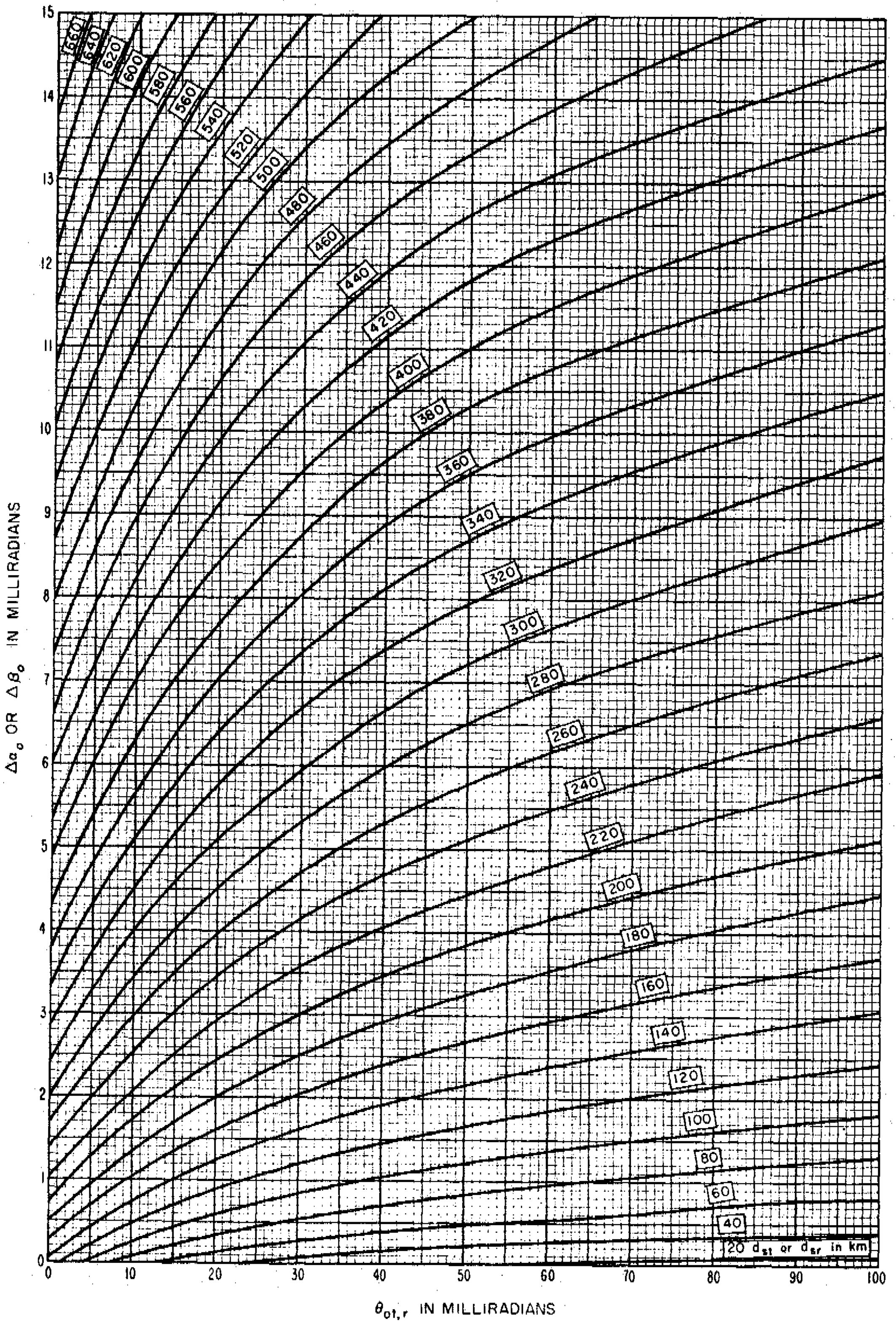


Figure 6.9

THE COEFFICIENT $C(N_S)$

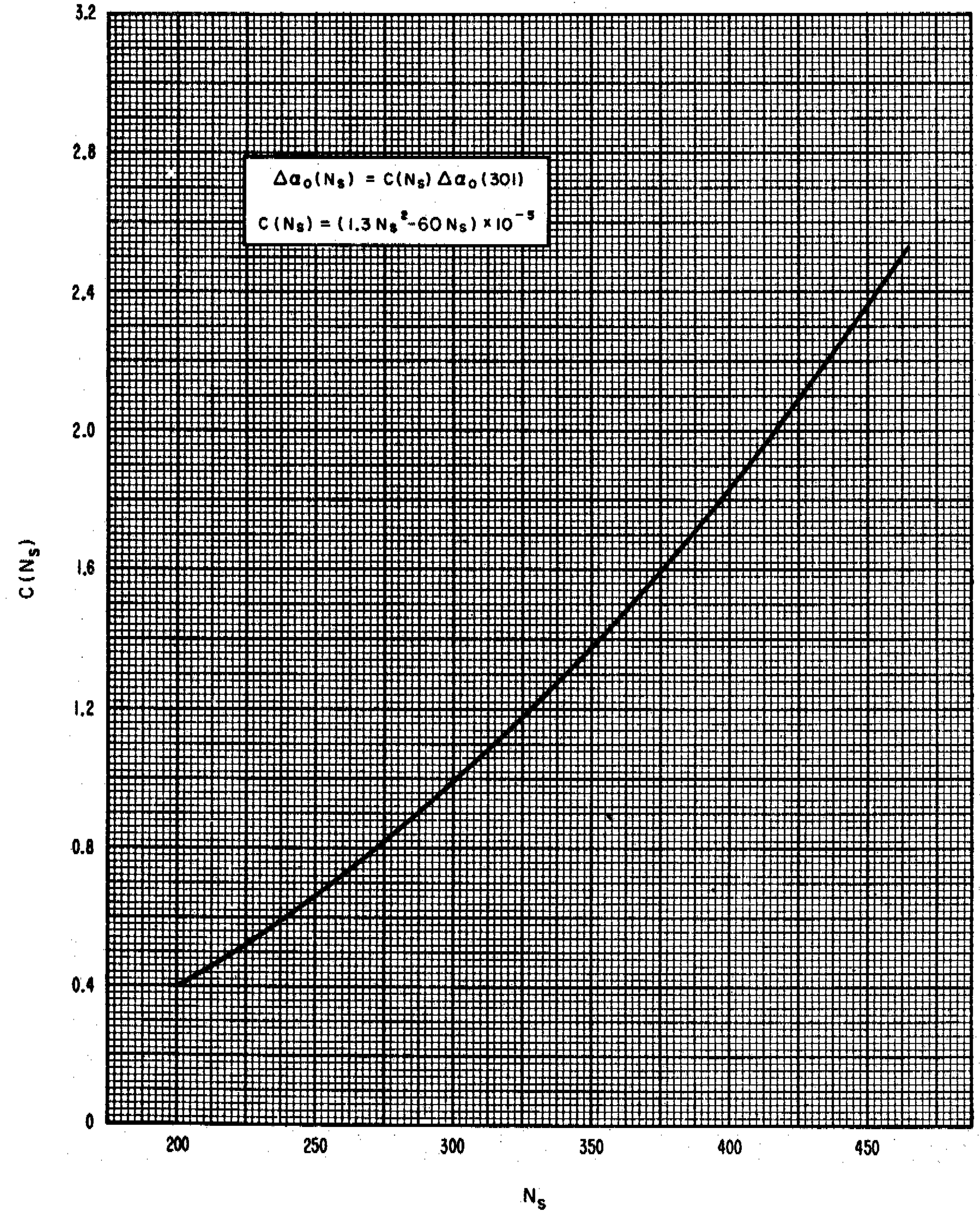


Figure 6.10