Annex II

# AVAILABLE POWER, FIELD STRENGTH AND MULTIPATH COUPLING LOSS

## II. 1 Available Power from the Receiving Antenna

The definitions of system loss and transmission loss in volume 1 depend on the concept of available power, the power that would be delivered to the receiving antenna load if its impedance were conjugately matched to the receiving antenna impedance. For a given radio frequency $\nu$ in hertz, let $z_{\ell\nu}$, $z'_\nu$, and $z_\nu$ represent the impedances of the load, the actual lossy antenna in its actual environment, and an equivalent loss-free antenna, respectively:

$$z_{\ell\nu} = r_{\ell\nu} + i\,x_{\ell\nu} \tag{II. 1a}$$

$$z'_\nu = r'_\nu + i\,x'_\nu \tag{II. 1b}$$

$$z_\nu = r_\nu + i\,x_\nu \tag{II. 1c}$$

where $r$ and $x$ represent resistance and reactance, respectively. Let $w_{\ell\nu}$ represent the power delivered to the receiving antenna load and write $w'_{a\nu}$ and $w_{a\nu}$, respectively, for the available power at the terminals of the actual receiving antenna and at the terminals of the equivalent loss-free receiving antenna. If $v'_\nu$ is the actual open-circuit r.m.s. voltage at the antenna terminals, then

$$w_{\ell\nu} = \frac{v'^2_\nu r_{\ell\nu}}{|z'_\nu + z_{\ell\nu}|^2} \; . \tag{II. 2}$$

When the load impedance conjugately matches the antenna impedance, so that $z_{\ell\nu} = z'^*_\nu$ or $r_{\ell\nu} = r'_\nu$ and $x_{\ell\nu} = -x'_\nu$, (II. 2) shows that the power $w_{\ell\nu}$ delivered to the load is equal to the power $w'_{a\nu}$ available from the <u>actual</u> antenna:

$$w'_{a\nu} \equiv \frac{v'^2_\nu}{4\,r'_\nu} \; . \tag{II. 3}$$

Note that the available power from an antenna depends only upon the characteristics of the antenna, its open-circuit voltage $v'_\nu$, and the resistance $r'_\nu$, and is independent of the load impedance. Comparing (II. 2) and (II. 3), we define a mismatch loss factor

$$\ell_{m\nu} \equiv \frac{w'_{a\nu}}{w_{\ell\nu}} = \frac{\left(r'_\nu + r_{\ell\nu}\right)^2 + \left(x'_\nu + x_{\ell\nu}\right)^2}{4\, r'_\nu\, r_{\ell\nu}} \tag{II. 4}$$

such that the power delivered to a load equals $w'_{a\nu}/\ell_{m\nu}$. When the load impedance conjugately matches the antenna impedance, $\ell_{m\nu}$ has its minimum value of unity, and $w_{\ell\nu} = w'_{a\nu}$. For any other load impedance, somewhat less than the available power is delivered to the load. The power available from the equivalent loss-free antenna is

$$w_{a\nu} = \frac{v_\nu^2}{4\, r_\nu} \tag{II. 5}$$

where $v_\nu$ is the open circuit voltage for the equivalent loss-free antenna.

Comparing (II. 3) and (II. 5), it should be noted that the available power $w'_{a\nu}$ at the terminals of the actual lossy receiving antenna is less than the available power $w_{a\nu} \equiv \ell_{er\nu} w'_{a\nu}$ for a loss-free antenna at the same location as the actual antenna:

$$\ell_{er\nu} \equiv \frac{w_{a\nu}}{w'_{a\nu}} = \frac{r'_\nu\, v_\nu^2}{r_\nu\, v'^2_\nu} \geq 1\,. \tag{II. 6}$$

The open circuit voltage $v'_\nu$ for the actual lossy antenna will often be the same as the open circuit voltage $v_\nu$ for the equivalent loss-free antenna, but each receiving antenna circuit must be considered individually.

Similarly, for the transmitting antenna, the ratio of the total power $w'_{t\nu}$ delivered to the antenna at a frequency $\nu$ is $\ell_{et\nu}$ times the total power $w_{t\nu}$ radiated at the frequency $\nu$:

$$\ell_{et\nu} \equiv w'_{t\nu}/w_{t\nu}\,. \tag{II. 7}$$

The concept of available power from a transmitter is not a useful one, and $\ell_{et\nu}$ for the transmitting antenna is best defined as the above ratio. However, the magnitude of this ratio can be obtained by calculation or measurement by treating the transmitting antenna as a receiving antenna and then determining $\ell_{et\nu}$ to be the ratio of the available received powers from the equivalent loss-free and the actual antennas, respectively.

General discussions of $\ell_{er\nu}$ are given by Crichlow et al.[1955] and in a report prepared under CCIR Resolution No. 1 [Geneva 1963c]. The loss factor $\ell_{er\nu}$ was successfully determined in one case by measuring the power $w_{t\nu}$ radiated from a loss-free target transmitting antenna and calculating the transmission loss between the target transmitting antenna and the receiving antenna. There appears to be no way of directly measuring either $\ell_{er\nu}$ or $\ell_{et\nu}$ without calculating some quantity such as the radiation resistance or the transmission loss. In the case of reception with a unidirectional rhombic terminated in its characteristic impedance, $\ell_{er\nu}$ could theoretically be greater than 2 [Harper, 1941], since nearly half the received power is dissipated in the terminating impedance and some is dissipated in the ground. Measurements were made by Christiansen [1947] on single and multiple wire units and arrays of rhombics. The ratio of power lost in the termination to the input power varied with frequency and was typically less than 3 db.

For the frequency band $\nu_\ell$ to $\nu_m$ it is convenient to define the effective loss factors $L_{er}$ and $L_{et}$ as follows:

$$L_{er} = 10 \log \frac{\int_{\nu_\ell}^{\nu_m} (d\, w_{a\nu}/d\nu)\, d\nu}{\int_{\nu_\ell}^{\nu_m} (d\, w'_{a\nu}/d\nu)\, d\nu} \text{ db} \tag{II. 8}$$

$$L_{et} = 10 \log \frac{\int_{\nu_\ell}^{\nu_m} (d\, w'_{t\nu}/d\nu)\, d\nu}{\int_{\nu_\ell}^{\nu_m} (d\, w_{t\nu}/d\nu)\, d\nu} \text{ db} \tag{II. 9}$$

The limits $\nu_\ell$ and $\nu_m$ on the integrals (II. 8) and (II. 9) are chosen to include essentially all of the wanted signal modulation side bands, but $\nu_\ell$ is chosen to be sufficiently large and $\nu_m$ sufficiently small to exclude any appreciable harmonic or other unwanted radiation emanating from the wanted signal transmitting antenna.

II. 2 Propagation Loss and Field Strength

This subsection defines terms that are most useful at radio frequencies lower than those where tropospheric propagation effects are dominant.

Repeating the definitions of $r$ and $r'$ used in subsection II. 1, and introducing the new parameter $r_f$:

$r_{t,r}$ = antenna radiation resistance,

$r'_{t,r}$ = resistance component of antenna input impedance,

$r_{ft}, r_{fr}$ = antenna radiation resistance in free space,

where subscripts t and r refer to the transmitting antenna and receiving antenna, respectively. Next define

$$L_{et} = 10 \log (r'_t/r_t), \quad L_{er} = 10 \log (r'_r/r_r) \tag{II. 10}$$

$$L_{ft} = 10 \log (r'_t/r_{ft}), \quad L_{fr} = 10 \log (r'_r/r_{fr}) \tag{II. 11}$$

$$L_{rt} = 10 \log (r_t/r_{ft}) = L_{ft} - L_{et} \tag{II. 12}$$

$$L_{rr} = 10 \log (r_r/r_{fr}) = L_{fr} - L_{er} \tag{II. 13}$$

[Actually, (II. 8) and (II. 9) define $L_{et}$ and $L_{er}$ while (II. 10) defined $r_t$ and $r_r$, given $r'_t$ and $r'_r$].

Propagation loss first defined by Wait [1959] is defined by the CCIR [1963a] as

$$L_p = L_s - L_{ft} - L_{fr} = L - L_{rt} - L_{rr} \text{ db} . \tag{II. 14}$$

Basic propagation loss is

$$L_{pb} = L_p + G_p . \tag{II. 15}$$

Basic propagation loss in free space is the same as the basic transmission loss in free space, $L_{bf}$, defined by (II. 74).

The system loss $L_s$ defined by (2.1) is a measurable quantity, while transmission loss $L$, path loss $L_o$, basic transmission loss $L_b$, attenuation relative to free space $A$, propagation loss $L_p$, and the field strength $E$ are derived quantities, which in general require a theoretical calculation of $L_{et, er}$ and/or $L_{ft, fr}$ as well as a theoretical estimate of the loss in path antenna gain $L_{gp}$.

The following paragraphs explain why the concepts of effective power, and an equivalent plane wave field strength are not recommended for reporting propagation data.

A half-wave antenna radiating a total of $w_t$ watts produces a free space field intensity equal to

$$s_o = 1.64\, w_t/(4\pi r^2) \text{ watts/km}^2 \tag{II.16}$$

at a distance $r$ kilometers in its equatorial plane, where the directive gain is equal to its maximum value 1.64, or 2.15 db. The field is linearly polarized in the direction of the antenna. In general, the field intensity $s_p$ at a point $\vec{r}$ in free space and associated with the principal polarization for an antenna is

$$s_p(\vec{r}) = w_t g_p(\hat{r})/(4\pi r^2) \text{ watts/km}^2 \tag{II.17}$$

as explained in a later subsection. In (II.17), $\vec{r} = r\,\hat{r}$ and $g_p(\hat{r})$ is the principal polarization directive gain in the direction $\hat{r}$. A similar relation holds for the field intensity $s_c(\vec{r})$ associated with the cross-polarized component of the field.

Effective radiated power is associated with a prescribed polarization for a test antenna and is determined by comparing $s_o$ as calculated using a field intensity meter or standard signal source with $s_p$ as measured using the test antenna:

$$\text{Effective Radiated Power} = W_t + 10 \log (s_p/s_o) = W_t + G_{pt}(\hat{r}_1) - 2.15 \text{ dbw} \tag{II.18}$$

where $G_{pt}(\hat{r}_1)$ is the principal polarization directive gain relative to a half-wave dipole in the direction $\hat{r}_1$ towards the receiving antenna in free space, and in general is the initial direction of the most important propagation path to the receiver.

These difficulties in definition, together with those which sometimes arise in attempting to separate characteristics of an antenna from those of its environment, make the effective radiated power an inferior parameter, compared with the total radiated power $W_t$, which can be more readily measured. The following equation, with $W_t$ determined from (II.18), may be used to convert reported values of Effective Radiated Power to estimates of the transmitter power output $W_{\ell t}$ when transmission line and mismatch losses $L_{\ell t}$ and the power radiation efficiency $1/\ell_{et}$ are known:

$$W_{\ell t} = W_t' + L_{\ell t} = W_t + L_{et} + L_{\ell t} \quad \text{dbw} \tag{II.19}$$

The electromagnetic field is a complex vector function in space and time, and information about amplitude, polarization, and phase is required to describe it. A real antenna responds to the total field surrounding it, rather than to E, which corresponds to the r.m.s. amplitude of the usual "equivalent" electromagnetic field, defined at a single point and for a specified polarization.

Consider the power averaged over each half cycle as the "instantaneous" available signal power, $w_\pi$

$$w_\pi = v^2/R_\nu \text{ watts}$$

where $v$ is the r.m.s. signal voltage and $R_\nu$ is the real part of the impedance of the receiving antenna, expressed in ohms. The signal power $w_\pi$ available from an actual receiving antenna is a directly measurable quantity.

The field strength and power flux density, on the other hand, cannot be measured directly, and both depend on the environment. In certain idealized situations the relationship of field strength $e$, and power flux density, $s$, to the available power may be expressed as

$$s = e^2/z = w_\pi 4\pi/(g\lambda^2) \text{ watts/m}^2$$

where $e$ is the r.m.s. electric field strength in volts/m, $z$ is the impedance in free space in ohms, $\lambda$ is the free space wavelength in meters and $g$ is the maximum gain of the receiving antenna.

The common practice of carefully calibrating a field strength measuring system in an idealized environment and then using it in some other environment may lead to appreciable errors, especially when high gain receiving antennas are used.

For converting reported values of $E$ in dbu to estimates of $W_{\ell t}$ or estimates of the available power $W_{\ell r}$ at the input to a receiver, the following relationships may be useful:

$$W_{\ell t} = E + L_{\ell t} + L_{ft} - G_t + L_{pb} - 20 \log f - 107.22 \quad \text{dbw} \tag{II. 20}$$

$$W_{\ell r} = E - L_{\ell r} - L_{fr} + G_r - L_{gp} - 20 \log f - 107.22 \quad \text{dbw} \tag{II. 21}$$

$$W_{\ell r} = W'_a - L_{\ell r} = W_a - L_{er} - L_{\ell r} \quad \text{dbw} \tag{II. 22}$$

In terms of reported values of field strength $E_{1kw}$ in dbu per kilowatt of effective radiated power, estimates of the system loss, $L_s$, basic propagation loss $L_{pb}$, or basic transmission loss $L_b$ may be derived from the following equations,

$$L_s = 139.37 + L_{et} + L_{fr} - G_p + G_t - G_{pt}(\hat{r}_1) + 20 \log f - E_{1kw} \quad \text{db} \tag{II. 23}$$

$$L_{pb} = 139.37 - L_{rt} + G_t - G_{pt}(\hat{r}_1) + 20 \log f - E_{1kw} \quad \text{db} \tag{II. 24}$$

$$L_b = 139.37 + L_{rr} + G_t - G_{pt}(\hat{r}_1) + 20 \log f - E_{1kw} \quad \text{db} \tag{II. 25}$$

provided that estimates are available for all of the terms in these equations.

For an antenna whose radiation resistance is unaffected by the proximity of its environment, $L_{rt} = L_{rr} = 0$ db, $L_{ft} = L_{et}$, and $L_{fr} = L_{er}$. In other cases, especially important for frequencies less than 30 MHz with antenna heights commonly used, it is often assumed that $L_{rt} = L_{rr} = 3.01$ db, $L_{ft} = L_{et} + 3.01$ db, and $L_{fr} = L_{er} + 3.01$ db, corresponding to the assumption of short vertical electric dipoles above a perfectly-conducting infinite plane. At low and very low frequencies, $L_{et}$, $L_{er}$, $L_{ft}$, and $L_{fr}$ may be very large. Propagation curves at HF and lower frequencies may be given in terms of $L_p$ or $L_{pb}$ so that it is not necessary to specify $L_{et}$ and $L_{er}$.

Naturally, it is better to measure $L_s$ directly than to calculate it using (II. 23). It may be seen that the careful definition of $L_s$, $L_p$, L, or $L_o$ is simpler and more direct than the definition of $L_b$, $L_{pb}$, A, or E.

The equivalent free-space field strength $E_o$ in dbu for one kilowatt of effective radiated power is obtained by substituting $W_{ft} = W_t$ = Effective Radiated Power = 30 dbw, $G_{pt}(\hat{r}_1) = G_t = 2.15$ db, $L_{lt} = L_{ft} = 0$ db, and $L_{pb} = L_{bf}$ in (II.18) - (II. 20), where $L_{bf}$ is given by (2. 16):

$$E_o = 106.92 - 20 \log d \quad \text{dbu/kw} \tag{II. 26}$$

where r in (2. 16) has been replaced by d in (II. 26). Thus $e_o$ is 222 millivolts per meter at one kilometer or 138 millivolts per meter at one mile. In free space, the "equivalent inverse distance field strength", $E_I$, is the same as $E_o$. If the antenna radiation resistances $r_t$ and $r_r$ are equal to the free space radiation resistances $r_{ft}$ and $r_{fr}$, then (II. 25) provides the following relationship between $E_{1kw}$ and $L_b$ with $G_{pt}(\hat{r}_1) = G_t$:

$$E_{1kw} = 139.37 + 20 \log f - L_b \quad \text{dbu/kw} \tag{II. 27}$$

Consider a short vertical electric dipole above a perfectly-conducting infinite plane, with an effective radiated power = 30 dbw, $G_t = 1.76$ db, and $L_{rr} = 3.01$ db. From (II. 18) $W_t = 30.39$ dbw, since $G_{pt}(\hat{r}_1) = 1.76$ db. Then from (II. 26) the equivalent inverse distance field is

$$E_I = E_o + L_{rt} + L_{rr} = 109.54 - 20 \log d \quad \text{dbu/kw} \tag{II. 28}$$

corresponding to $e_I = 300$ mv/m at one kilometer, or $e_I = 186.4$ mv/m at one mile. In this situation, the relationship between $E_{1kw}$ and $L_b$ is given by (II. 25) as

$$E_{1kw} = 142.38 + 20 \log f - L_b \quad \text{dbu/kw} \tag{II. 29}$$

The foregoing suggests the following general expressions for the equivalent free space field strength $E_o$ and the equivalent inverse distance field $E_I$:

$$E_o = (W_t - L_{rt} + G_t) - 20 \log d + 74.77 \quad \text{dbu} \tag{II.30}$$

$$E_I = E_o + L_{rt} + L_{rr} \quad \text{dbu} \tag{II.31}$$

Note that $L_{rt}$ in (II. 30) is not zero unless the radiation resistance of the transmitting antenna in its actual environment is equal to its free space radiation resistance. The definition of "attenuation relative to free space" given by (2. 20) as the basic transmission loss relative to that in free space, may be restated as

$$A = L_b - L_{bf} = L - L_f = E_I - E \quad \text{db} \tag{II.32}$$

Alternatively, attenuation relative to free space, $A_t$, might have been defined (as it sometimes is) as basic propagation loss relative to that in free space:

$$A_t = L_{pb} - L_{bf} = A - L_{rt} - L_{rr} = E_o - E \quad \text{db} \tag{II.33}$$

For frequencies and antenna heights where these definitions differ by as much as 6 db, caution should be used in reporting data. For most paths using frequencies above 50 MHz, $L_{rt} + L_{rr}$ is negligible, but caution should again be used if the loss in path antenna gain $L_{gp}$ is not negligible. It is then important not to confuse the "equivalent" free space loss $L_f$ given by (2. 19) with the loss in free space given by (2. 18).

## II. 3 MULTIPATH COUPLING LOSS

Ordinarily, to minimize the transmission loss between two antennas, they are oriented to take advantage of maximum directive gains (directivity) and the polarizations are matched. This maximizes the path antenna gain. With a single uniform plane wave incident upon a receiving antenna, there will be a reduction in the power transferred if the antenna beam is not oriented for maximum free space gain. If the polarization of the receiving antenna is matched to that of the incident wave, this loss in path antenna gain is due to "orientation coupling loss", and if there is a polarization mismatch, there will be an additional "polarization coupling loss". In general, more than one plane wave will be incident upon a receiving antenna from a single source because of reflection, diffraction, or scattering by terrain or atmospheric inhomogeneities. Mismatch between the relative phases of these waves and the relative phases of the receiving antenna response in different directions will contribute to a "multipath coupling loss" which will include orientation, polarization, and phase mismatch effects. If multipath propagation involves non-uniform waves whose amplitudes, polarizations, and phases can only be described statistically, the corresponding loss in path antenna gain will include "antenna-to-medium coupling loss", a statistical average of phase incoherence effects.

This part of the annex indicates how multipath coupling loss may be calculated when incident waves are plane and uniform with known phases, and when the directivity, polarization, and phase response of the receiving antenna are known for every direction. It is assumed that the radiation resistance of the receiving antenna is unaffected by its environment, and that the electric and magnetic field vectors of every incident wave are perpendicular to each other and perpendicular to the direction of propagation.

### II. 3. 1 Representation of Complex Vector Fields

Studying the response of a receiving antenna to coherently phased plane waves with several different directions of arrival, it is convenient to locate the receiving antenna at the center of a coordinate system. A radio ray traveling a distance $r$ from a transmitter to the receiver may be refracted or reflected so that its initial and final directions are different. If $-\hat{r}$ is the direction of propagation at the receiver, $\vec{r} = \hat{r}\, r$ is the vector distance from the receiver to the transmitter if the ray path is a straight line, but not otherwise.

A paper by Kales [1951] shows how the amplitude, phase, and polarization of a uniform, monochromatic, elliptically polarized and locally plane wave may be expressed with the aid of complex vectors. For instance, such a wave may be expressed as the real part of the sum of two linearly polarized complex plane waves $\sqrt{2}\,\vec{e}_r \exp(i\tau)$ and $i\sqrt{2}\,\vec{e}_i \exp(i\tau)$. These components are in time phase quadrature and travel in the same direction $-\hat{r}$, where $i = \sqrt{-1}$ and $\vec{e}_r$ and $\vec{e}_i$ are real vectors perpendicular to $\hat{r}$. The vector $\vec{e}_r + i\vec{e}_i$ is then a complex vector. Field strengths are denoted in volts/km ($10^3$ microvolts per meter) and field intensities in watts/$km^2$ ($10^{-3}$ milliwatts per square meter), since all lengths are in kilometers.

The time-varying phase

$$\tau = k(ct - r) \qquad \text{(II. 34)}$$

is a function of the free-space wavelength $\lambda$, the propagation constant $k = 2\pi/\lambda$, the free-space velocity of radio waves $c = 299792.5 \pm 0.3$ km/sec, the time $t$ at the radio source, and the length of a radio ray between the receiver and the source.

Figure II-1 illustrates three sets of coordinates which are useful in studying the phase and polarization characteristics associated with the radiation pattern or response pattern of an antenna. Let $\vec{r} = \hat{r}\,r$ represent the vector distance between the antenna and a distant point, specified either in terms of a right-handed cartesian unit vector coordinate system $\hat{x}_o$, $\hat{x}_1$, $\hat{x}_2$ or in terms of polar coordinates $r$, $\theta$, $\phi$:

$$\vec{r} = \hat{r}\,r = \hat{x}_o x_0 + \hat{x}_1 x_1 + \hat{x}_2 x_2, \qquad r^2 = x_0^2 + x_1^2 + x_2^2 \qquad \text{(II. 35a)}$$

$$x_0 = r\cos\theta, \qquad x_1 = r\sin\theta\cos\phi, \qquad x_2 = r\sin\theta\sin\phi \qquad \text{(II. 35b)}$$

$$\hat{r} = \hat{r}(\theta, \phi) = \hat{x}_o\cos\theta + (\hat{x}_1\cos\phi + \hat{x}_2\sin\phi)\sin\theta. \qquad \text{(II. 35c)}$$

As a general rule, either of two antennas separated by a distance $r$ is in the far field or radiation field of the other antenna if $r > 2D^2/\lambda$, where $D$ is the largest linear dimension of either antenna.

The amplitude and polarization of electric field vectors $\vec{e}_\theta$ and $\vec{e}_\phi$, perpendicular to each other and to $\hat{r}$, is often calculated or measured to correspond to the right-handed cartesian unit vector coordinate system $\hat{r}$, $\hat{e}_\theta$, $\hat{e}_\phi$ illustrated in figure II-1. The unit vector $\hat{e}_\phi$ is perpendicular to $\hat{r}$ and $\hat{x}_o$, and $\hat{e}_\theta$ is perpendicular to $\hat{e}_\phi$ and $\hat{r}$. In terms of vector cross-products:

$$\hat{e}_\phi = (\hat{r} \times \hat{x}_o)/\sin\theta = \hat{x}_1\sin\phi - \hat{x}_2\cos\phi \qquad \text{(II. 36a)}$$

$$\hat{e}_\theta = \hat{e}_\phi \times \hat{r} = (\hat{x}_o - \hat{r}\cos\theta)/\sin\theta. \qquad \text{(II. 36b)}$$

The directive gain $g$, a scalar, may be expressed as the sum of directive gains $g_\theta$ and $g_\phi$ associated with polarization components $\vec{e}_\theta \equiv \hat{e}_\theta e_\theta$ and $\vec{e}_\phi \equiv \hat{e}_\phi e_\phi$, where the coefficients $e_\theta$ and $e_\phi$ are expressed in volts/km:

$$g = g_\theta + g_\phi. \qquad \text{(II. 37)}$$

Subscripts $t$ and $r$ are used to refer to the gains $g_t$ and $g_r$ of transmitting and receiving antennas, while $g$ is the ratio of the available mean power flux density and $e_o^2/\eta_o$, where

$e_o$ as defined by (II. 38) is the free space field strength at a distance $r$ in kilometers from an isotropic antenna radiating $w_t$ watts:

$$e_o = [\eta_o w_t/(4\pi r^2)]^{1/2} \text{ volts/km}. \tag{II. 38}$$

Here, $\eta_o = 4\pi c \cdot 10^{-7} = 376.7304 \pm 0.0004$ ohms is the characteristic impedance of free space. The maximum amplitudes of the $\theta$ and $\phi$ components of a radiated or incident field are $|\vec{e}_\theta|\sqrt{2}$ and $|\vec{e}_\phi|\sqrt{2}$, where

$$|\vec{e}_\theta| = e_\theta = e_o g_\theta^{1/2} \text{ volts/km}, \qquad |\vec{e}_\phi| = e_\phi = e_o g_\phi^{1/2} \text{ volts/km}. \tag{II. 39}$$

If phases $\tau_\theta$ and $\tau_\phi$ are associated with the electric field components $\vec{e}_\theta$ and $\vec{e}_\phi$, which are in phase quadrature in space but not necessarily in time, the total complex wave at any point $\vec{r}$ is

$$\sqrt{2}\,(\vec{e}_r + i\,\vec{e}_i)\exp(i\tau) = \sqrt{2}\,[\vec{e}_\theta \exp(i\tau_\theta) + \vec{e}_\phi \exp(i\,\tau_\phi)]\,\exp(i\tau). \tag{II. 40}$$

From this expression and a knowledge of $\vec{e}_{\theta,\phi}$, $\tau_{\theta,\phi}$, we may determine the real and imaginary components $\vec{e}_r$ and $\vec{e}_i$, which are in phase quadrature in time but not necessarily in space:

$$\vec{e}_r \equiv \hat{e}_r e_r = \vec{e}_\theta \cos\tau_\theta + \vec{e}_\phi \cos\tau_\phi, \tag{II. 41a}$$

$$\vec{e}_i \equiv \hat{e}_i e_i = \vec{e}_\theta \sin\tau_\theta + \vec{e}_\phi \sin\tau_\phi. \tag{II. 41b}$$

The next section of this annex introduces components of this wave which are in phase quadrature in both time and space.

II.3.2 Principal and Cross-Polarization Components

Principal and cross-polarization components of an incident complex wave $\sqrt{2}\,(\vec{e}_r + i\vec{e}_i)\exp(i\tau)$ may be defined in terms of a time-independent phase $\tau_i$ which is a function of $\vec{r}$ [Kales, 1951]. If we write

$$\vec{e}_r + i\vec{e}_i = (\vec{e}_1 + i\vec{e}_2)\exp(i\tau_i) \tag{II.42}$$

and solve for the real and imaginary components of the complex vector $\vec{e}_1 + i\vec{e}_2$, we find that

$$\vec{e}_1 \equiv \hat{e}_1 e_1 = \vec{e}_r \cos\tau_i + \vec{e}_i \sin\tau_i \tag{II.43a}$$

$$\vec{e}_2 \equiv \hat{e}_2 e_2 = \vec{e}_i \cos\tau_i - e_r \sin\tau_i . \tag{II.43b}$$

Whichever of these vectors has the greater magnitude is the principal polarization component $\vec{e}_p$, and the other is the orthogonal cross-polarization component $\vec{e}_c$:

$$e_1^2 = e_r^2 \cos^2\tau_i + e_i^2 \sin^2\tau_i + \vec{e}_r \cdot \vec{e}_i \sin(2\tau_i) \tag{II.44a}$$

$$e_2^2 = e_r^2 \sin^2\tau_i + e_i^2 \cos^2\tau_i - \vec{e}_r \cdot \vec{e}_i \sin(2\tau_i), \tag{II.44b}$$

The phase angle $\tau_i$ is determined from the condition that $\vec{e}_1 \cdot \vec{e}_2 = 0$:

$$\tan(2\tau_i) = 2\vec{e}_r \cdot \vec{e}_i/(e_r^2 - e_i^2). \tag{II.45}$$

Any incident plane wave, traveling in a direction $-\hat{r}$ is then represented as the real part of the complex wave given by

$$\sqrt{2}\,\underline{\vec{e}}\exp[i(\tau + \tau_i)] \equiv \sqrt{2}\,(\vec{e}_p + i\vec{e}_c)\exp[i(\tau + \tau_i)]. \tag{II.46}$$

The principal and cross-polarization directions $\hat{e}_p$ and $\hat{e}_c$ are chosen so that their vector product is a unit vector in the direction of propagation:

$$\hat{e}_p \times \hat{e}_c = -\hat{r}. \tag{II.47}$$

A bar is used under the symbol for the complex vector $\underline{\vec{e}} = \vec{e}_p + i\vec{e}_c$ in (II.46) to distinguish it from real vectors such as $\vec{e}_\theta$, $\vec{e}_\phi$, $\vec{e}_r$, $\vec{e}_i$, $\vec{e}_p$, and $\vec{e}_c$. The absolute values of the vector coefficients $e_p$ and $e_c$ may be found using (II.44).

As the time t at the transmitter or the time $\tau$ at the receiver increases, the real vector component of (II.46), or "polarization vector",

$$\sqrt{2}\,[\vec{e}_p \cos(\tau + \tau_i) - \vec{e}_c \sin(\tau + \tau_i)]$$

describes an ellipse in the plane of the orthogonal unit vectors $\hat{e}_p = \vec{e}_p/e_p$ and $\hat{e}_c = \vec{e}_c/e_c$. Looking in the direction of propagation $-r(\theta, \phi)$ with $e_p$ and $e_c$ both positive or both negative, we see a clockwise rotation of the polarization vector as $\tau$ increases.

Right-handed polarization is defined by the IRE or IEEE and in CCIR Report 321 [1963m] to correspond to a clockwise rotation of a polarization ellipse, looking in the direction of propagation with r fixed and t or $\tau$ increasing. This is opposite to the definition used in classical physics.

The "axial ratio" $e_c/e_p$ of the polarization ellipse of an incident plane wave $\sqrt{2}\,\underline{\vec{e}} \exp[i(\tau + \tau_o)]$ is denoted here as

$$a_x \equiv e_c/e_p \qquad \text{(II.48)}$$

and may be either positive or negative depending on whether the polarization of the incident wave is right-handed or left-handed. The range of possible values for $a_x$ is -1 to +1.

### II.3.3 Unit Complex Polarization Vectors

If the receiving antenna were a point source of radio waves, it would produce a plane wave $\sqrt{2}\,\underline{\vec{e}}_r \exp[i(\tau + \tau_r)]$ at a point $\vec{r}$ in free space. The receiving pattern of such an antenna as it responds to an incident plane wave $\sqrt{2}\,\underline{\vec{e}} \exp[i(\tau + \tau_r)]$ traveling in the opposite direction $-\hat{r}$ is proportional to the complex conjugate of $\underline{\vec{e}}_r \exp(i\tau_r)$ [S. A. Schelkunoff and H. T. Friis, 1952]:

$$[\underline{\vec{e}}_r \exp(i\tau_r)]^* = (\vec{e}_{pr} - i\vec{e}_{cr}) \exp(-i\tau_r). \qquad (II.49)$$

The axial ratio $e_{cr}/e_{pr}$ of the type of wave that would be radiated by a receiving antenna is defined for propagation in the direction $\hat{r}$. An incident plane wave, however, is propagating in the direction $-\hat{r}$, and by definition the sense of polarization of an antenna used for reception is opposite to the sense of polarization when the antenna is used as a radiator. The polarization associated with a receiving pattern is right-handed or left-handed depending on whether $a_{xr}$ is positive or negative, where

$$a_{xr} \equiv -e_{cr}/e_{pr}, \qquad e_{cr} = -e_{pr}\,a_{xr}. \qquad (II.50)$$

The amplitudes $|e_{pr}|$ and $|e_{cr}|$ of the principal and cross-polarization field components $\vec{e}_{pr}$ and $\vec{e}_{cr}$ are proportional to the square roots of principal and cross-polarization directive gains $g_{pr}$ and $g_{cr}$, respectively. It is convenient to define a unit complex polarization vector $\underline{\hat{p}}_r$ which contains all the information about the polarization response associated with a receiving pattern:

$$\underline{\hat{p}}_r = (\hat{e}_{pr} + i\,\hat{e}_{cr}\,a_{xr})(1 + a_{xr}^2)^{-1/2} \qquad (II.51)$$

$$a_{xr}^2 = g_{cr}/g_{pr}. \qquad (II.52)$$

The directions $\hat{e}_{pr}$ and $\hat{e}_{cr}$ are chosen so that

$$\hat{e}_{pr} \times \hat{e}_{cr} = \hat{r}. \qquad (II.53)$$

In a similar fashion, the axial ratio $a_x$ defined by (II.48) and the orientations $\hat{e}_p$ and $\hat{e}_c$ of the principal and cross-polarization axes of the polarization ellipse completely describe the state of polarization of an incident wave $\sqrt{2}\,\underline{\vec{e}} \exp[i(\tau + \tau_i)]$, and its direction of propagation $-\hat{r} = e_p \times e_c$. The unit complex polarization vector for the incident wave is

$$\underline{\hat{p}} = \vec{\underline{e}}/|\vec{\underline{e}}| = (\hat{e}_p + i\hat{e}_c a_x)(1 + a_x^2)^{-1/2}. \tag{II. 54}$$

The magnitude of a complex vector $\vec{\underline{e}} = \vec{e}_p + i\vec{e}_c$ is the square root of the product of $\vec{\underline{e}}$ and its complex conjugate $\vec{e}_p - i\vec{e}_c$:

$$|\vec{\underline{e}}| = (\vec{\underline{e}} \cdot \vec{\underline{e}}^*)^{1/2} = (e_p^2 + e_c^2)^{1/2} \quad \text{volts/km}. \tag{II. 55}$$

### II.3.4 Power Flux Densities

The coefficients $e_p$ and $e_c$ of the unit vectors $\hat{e}_p$ and $\hat{e}_c$ are chosen to be r.m.s. values of field strength, expressed in volts/km, and the mean power flux densities $s_p$ and $s_c$ associated with these components are

$$s_p = e_p^2/\eta_o \quad \text{watts/km}^2, \qquad s_c = e_c^2/\eta_o \quad \text{watts/km}^2, \tag{II.56}$$

The corresponding principal and cross-polarization directive gains $g_p$ and $g_c$ are

(II.57)

where $w_t$ is the total power radiated from the transmitting antenna. This is the same relation as that expressed by (II.39) between the gains $g_\theta$, $g_\phi$, and the orthogonal polarization components $\vec{e}_\theta$ and $\vec{e}_\phi$.

The total mean power flux density $s$ at any point where $\vec{e}$ is known to be in the radiation field of the transmitting antenna and any reradiating sources is

$$s = |\vec{e}|^2/\eta_o = g\, e_o^2/\eta_o = s_p + s_c = (e_p^2 + e_c^2)/\eta_o$$

$$= (e_r^2 + e_i^2)/\eta_o = (e_\theta^2 + e_\phi^2)/\eta_o \quad \text{watts/km}^2 \tag{II.58a}$$

$$g = g_p + g_c = g_\theta + g_\phi = 4\pi r^2 s/p_t = s\,\eta_o/e_o^2 \tag{II.58b}$$

where $e_o$ is given by (II.38). The power flux density $s$ is proportional to the transmitting antenna gain $g_t$, but in general $g$ is not equal to $g_t$ as there may be a fraction $a_p$ of energy absorbed along a ray path or scattered out of the path. We therefore write

$$g \equiv g_p(1 + a_x^2) = a_p\, g_{pt}(1 + a_x^2) \equiv a_p\, g_t. \tag{II.59}$$

The path absorption factor $a_p$ can also be useful in approximating propagation mechanisms which are more readily described as a sum of modes than by using geometric optics. For instance, in the case of tropospheric ducting a single dominant TEM mode may correspond theoretically to an infinite number of ray paths, and yet be satisfactorily approximated by a single great-circle ray path if $a_p$ is appropriately defined. In such a case, $a_p$ will occasionally be greater than unity rather than less.

Orienting a receiving dipole for maximum reception to determine $s_p$ and for minimum reception to determine $s_c$ will also determine $\hat{\varepsilon}_p$ and $\hat{\varepsilon}_c$, except in the case of circular polarization, where the direction of $\hat{\varepsilon}_p$ in the plane normal to $\vec{r}$ is arbitrary. In the general case where $|a_x| < 1$, either of two opposite directions along the line of principal polarization is equally suitable for $\hat{\varepsilon}_p$.

Reception with a dipole will not show the sense of polarization. Right-handed and left-handed circularly polarized receiving antennas will in theory furnish this information, since $\vec{\underline{e}}$ may also be written to correspond to the difference of right-handed and left-handed circularly polarized waves which are in phase quadrature in time and space:

$$\vec{\underline{e}} = (\hat{e}_p + i\hat{e}_c)\left(\frac{e_p + e_c}{2}\right) - i(\hat{e}_c - i\hat{e}_p)\left(\frac{e_p - e_c}{2}\right). \qquad \text{(II. 60)}$$

$$s_r = (e_p + e_c)^2/(2\eta_o) \ \text{watts/km}^2 \qquad \text{(II. 61a)}$$

$$s_\ell = (e_p - e_c)^2/(2\eta_o) \ \text{watts/km}^2 \qquad \text{(II. 61b)}$$

so the sense of polarization may be determined by whether $s_r/s_\ell$ is greater than or less than unity. The flux densities $s_r$ and $s_\ell$ are equal only for linear polarization, where $e_c = 0$.

II. 3. 5 Polarization Efficiency

The polarization efficiency for a transfer of energy from a single plane wave to the terminals of a receiving antenna at a given radio frequency may be expressed as a function of the unit complex polarization vectors defined by (II.51) and (II.54) and the angle $\psi_p$ between principal polarization directions associated with $\vec{\underline{e}}$ and $\vec{\underline{e}}_r$. This polarization efficiency is

$$|\hat{\underline{p}} \cdot \hat{\underline{p}}_r|^2 = \frac{\cos^2 \psi_p (a_x a_{xr} + 1)^2 + \sin^2 \psi_p (a_x + a_{xr})^2}{(a_x^2 + 1)(a_{xr}^2 + 1)} \tag{II. 62}$$

where

$$\hat{e}_p \cdot \hat{e}_{pr} = -\hat{e}_c \cdot \hat{e}_{cr} = \cos \psi_p, \quad \hat{e}_p \cdot \hat{e}_{cr} = \hat{e}_{pr} \cdot \hat{e}_c = \sin \psi_p. \tag{II.63}$$

As noted in section 2 following (2.11), any receiving antenna is completely "blind" to an incoming plane wave $\sqrt{2}\,\vec{\underline{e}} \exp[i(\tau + \tau_i)]$ which has a sense of polarization opposite to that of the receiving antenna if the eccentricities of the polarization ellipses are the same $(|a_x| = |a_{xr}|)$ and if the principal polarization direction $\hat{e}_p$ of the incident wave is perpendicular to $\hat{e}_{pr}$. In such a case, $\cos \psi_p = 0$, $a_x = -a_{xr}$, and (II.62) shows that the polarization efficiency $|\hat{\underline{p}} \cdot \hat{\underline{p}}_r|^2$ is zero. As an interesting special case, reflection of a circularly polarized wave incident normally on a perfectly conducting sheet will change the sense of polarization so that the antenna which radiates such a wave cannot receive the reflected wave. In such a case $a_x = -a_{xr} = \pm 1$, so that $|\hat{\underline{p}} \cdot \hat{\underline{p}}_r|^2 = 0$ for any value of $\psi_p$.

On the other hand, the polarization efficiency given by (II.62) is unity and a maximum transfer of power will occur if $a_x = a_{xr}$ and $\psi_p = 0$, that is, if the sense, eccentricity, and principal polarization direction of the receiving antenna match the sense, eccentricity, and principal polarization direction of the incident wave.

For transmission in free space, antenna radiation efficiencies, their directive gains, and the polarization coupling efficiency are independent quantities, and all five must be maximized for a maximum transfer of power between the antennas. A reduction in either one of the directive gains $g(-\hat{r})$ and $g_r(\hat{r})$ or a reduction in the polarization efficiency $|\hat{\underline{p}} \cdot \hat{\underline{p}}_r|^2$ will reduce the transfer of power between two antennas.

With each plane wave incident on the receiving antenna there is associated a ray of length $r$ from the transmitter, an initial direction of radiation, and the radiated wave $\vec{\underline{e}}_t \exp[i(\tau + \tau_t)]$ which would be found in free space at this distance and in this direction. When it is practical to separate antenna characteristics from environmental and path characteristics, it is assumed that the antenna phase response $\tau_t$, like $\tau_r$, is a characteristic of the antenna and its environment and that

$$\tau_i = \tau_t + \tau_p \qquad \text{(II. 64)}$$

where $\tau_p$ is a function of the ray path and includes allowances for path length differences and diffraction or reflection phase shifts.

Random phase changes in either antenna, absorption and reradiation by the environment, or random fluctuations of refractive index in the atmosphere will all tend to fill in any sharp nulls in a theoretical free-space radiation pattern $\underline{\vec{e}}$ or $\underline{\vec{e}}_r$. Also, it is not possible to have a complex vector pattern $\underline{\vec{e}}/r$ which is independent of $r$ in the vicinity of antenna nulls unless the radiation field, proportional to $1/r$, dominates over the induction field, which is approximately proportional to $1/r^2$.

II. 3. 6 Multipath Coupling Loss

Coherently phased multipath components from a single source may arrive at a receiving antenna from directions sufficiently different so that $\tau_i$ and $\tau_r$ vary significantly. It is then important to be able to add complex signal voltages at the antenna terminals. Let n = 1, 2, ---, N and assume N discrete plane waves incident on an antenna from a single source. The following expressions represent the complex open-circuit r.m.s. signal voltage $v_n$ corresponding to a radio frequency $\nu$ cycles per second, a single incident plane wave $\sqrt{2}\,\underline{\vec{e}}_n \exp[i(\tau + \tau_{in})]$, a loss-free receiving antenna with a directivity gain $g_{rn}$ and an effective absorbing area $a_{en}$, matched antenna and load impedances, and an input resistance $r_\nu$ which is the same for the antenna and its load:

$$v_n = (4r_\nu s_n a_{en})^{1/2} (\underline{\hat{\rho}}_n \cdot \underline{\hat{\rho}}_{rn}) \exp[i(\tau + \tau_{pn} + \tau_{tn} - \tau_{rn})] \quad \text{volts} \tag{II. 65}$$

$$s_n = |\underline{\vec{e}}_n|^2/\eta_o = w_r a_{pn} g_{tn} /(4\pi r_n^2) \quad \text{watts/km}^2 \tag{II. 66}$$

$$a_{en} = g_{rn} \lambda^2/(4\pi) \quad \text{km}^2 \tag{II. 67}$$

$$\underline{\hat{\rho}}_n \cdot \underline{\hat{\rho}}_{rn} = [(1 + a_{xn}^2)(1 + a_{xrn}^2)]^{-1/2} [(1 + a_{xn} a_{xrn}) \cos\psi_{pn} + i(a_{xn} + a_{xrn}) \sin\psi_{pn}]. \tag{II. 68}$$

If the polarization of the receiving antenna is matched to that of the incident plane wave, then $a_{xn} = a_{xrn}$, $\psi_{pn} = 0$, $\underline{\hat{\rho}}_n \cdot \underline{\hat{\rho}}_{rn} = 1$, and

$$v_n = [4r_\nu w_r a_{pn} g_{tn} g_{rn} \lambda^2/(4\pi r_n)^2]^{1/2} \exp[i(\tau + \tau_{pn} + \tau_{tn} - \tau_{rn})] \quad \text{volts.} \tag{II. 69}$$

If the coefficient of the phasor in (II. 69) has the same value for two incident plane waves, but the values of $\tau_{in} - \tau_{rn}$ differ by $\pi$ radians, the sum of the corresponding complex voltages is zero. This shows that the multipath coupling efficiency can theoretically be zero even when the beam orientation and polarization coupling are maximized. Adjacent lobes in a receiving antenna directivity pattern, for instance, may be 180° out of phase and thus cancel two discrete in-phase plane-wave components.

Equation (II. 3) shows the relation between the total open-circuit r.m.s. voltage

$$v_\nu = \left[ \sum_{\substack{n=1 \\ m=1}}^{N} v_n v_m^* \right]^{1/2} \quad \text{volts} \tag{II. 70}$$

and the power $w_a$ available at the terminals of a loss-free receiving antenna:

$$w_a = v_\nu^2/(4 r_\nu) \text{ watts} \tag{II. 71}$$

In writing $w_a$ for $w_{a\nu}$ in (II. 71), the subscript $\nu$ has been suppressed, as with almost all of the symbols in this annex. Studying (II. 65) - (II. 68), (II. 70), and (II.71), it is seen that the expression for $w_a$ is symmetrical in the antenna gains $g_p$, $g_{pr}$, and $g_c = a_x^2 g_p$, $g_{cr} = a_{xr}^2 g_{pr}$, and that $w_a$ is a linear function of these parameters, though $v_\nu$ is not. From this follows a theorem of reciprocity, that the transmission loss $L = -10 \log (w_a/w_t)$ is the same if the roles of the transmitting and receiving antennas are reversed.

The basic transmission loss $L_b$ is the system loss that would be expected if the actual antennas were replaced at the same locations by hypothetical antennas which are:

(a) loss-free, so that $L_{et} = L_{er} = 0$ db. See (2.3).

(b) isotropic, so that $g_t = g_r = 1$ in every direction important to propagation between the actual antennas.

(c) free of polarization coupling loss, so that $|\underline{\hat{p}} \cdot \underline{\hat{p}}_r|^2 = 1$ for every locally plane wave incident at the receiving antenna.

(d) isotropic in their phase response, so that $\tau_t = \tau_r = 0$ in every direction.

The available power $w_{ab}$ corresponding to propagation between hypothetical isotropic antennas is then

$$w_{ab} = \frac{w_t \lambda^2}{(4\pi)^2} \sum_{\substack{n=1 \\ m=1}}^{N} \frac{(a_{pn} a_{pm})^{1/2} \cos(\tau_{pn} - \tau_{pm})}{r_n r_m} \tag{II. 72}$$

The basic transmission loss $L_b$ corresponding to these assumptions is

$$L_b = -10 \log(w_{ab}/w_t) = W_t - W_{ab} \text{ db} \tag{II. 73}$$

The basic transmission loss in free space, $L_{bf}$, corresponds to $N = 1$, $a_{p1} = 1$, $\tau_{p1} = 0$, $r_1 = r$:

$$L_{bf} = -10 \log [\lambda/(4\pi r)]^2 = 32.45 + 20 \log f + 20 \log r \text{ db} \tag{II. 74}$$

where f is in megacycles per second and r is in kilometers. Compare with (2.16).

As may be seen from the above relations, only a fraction $s_e$ of the total flux density $s_n$ per unit radiated power $w_t$ contributes to the available received power $w_a$ from N plane waves. While $s_n$ is expressed in watts/km$^2$, $s_e$ is expressed in watts/km$^2$ for each watt of the power $w_t$ radiated by a single source:

$$s_e = 4\pi w_a/(\lambda^2 w_t) \tag{II. 75}$$

For each plane wave from a given source, $\vec{\underline{e}}_n \exp(i\tau_{in})$ or $\hat{\underline{p}}_{rn} \exp(-i\tau_{rn})$ may sometimes be regarded as a statistical variable chosen at random from a uniform distribution, with all phases from $-\pi$ to $\pi$ equally likely. Then real power proportional to $|\vec{\underline{e}}_n \cdot \vec{\underline{e}}^*_{rn}|^2$ may be added at the antenna terminals, rather than the complex voltages defined by (II. 65)-(II. 68). For this case, the statistical "expected value" $<s_e>$ of $s_e$ is

$$<s_e> = \sum_{n=1}^{N} a_{pn} g_{tn} g_{rn} |\hat{\underline{p}}_n \cdot \hat{\underline{p}}_{rn}|^2/(4\pi r_n^2). \tag{II. 76}$$

In terms of $s_e$, the transmission loss L is

$$L = 21.46 + 20 \log f - 10 \log s_e \quad \text{db}. \tag{II. 77}$$

Substituting $<s_e>$ for $s_e$ in (II. 77), we would not in general obtain the statistical expected value $<L>$ of L, since $<L>$ is an ensemble average of logarithms, which may be quite different from the logarithm of the corresponding ensemble average $<s_e>$. For this reason, median values are often a more practical measure of central tendency than "expected" values. With $w_t$ and $\lambda$ fixed, median values of $s_e$ and L always obey the relation (II. 77), while average values of $s_e$ and L often do not.

The remainder of this annex is concerned with a few artificial problems designed to show how these formulas are used and to demonstrate some of the properties of radiation and response patterns. In general, information is needed about antenna patterns only in the few directions which are important in determining the amplitude and fading of a tropospheric signal. Although section II. 3. 7 shows how a complex vector radiation or reception pattern may be derived from an integral over all directions, it is proposed that the power radiation efficiencies and the gains $g_r(\hat{r})$ or $g_t(-\hat{r})$ for actual antennas should be determined by measurements in a few critical directions using standard methods and a minimum of calculations.

## II.3.7 Idealized Theoretical Antenna Patterns

Consider a point source of plane waves, represented by complex dipole moments in three mutually perpendicular directions, $\hat{x}_0$, $\hat{x}_1$, and $\hat{x}_2$. These three unit vectors, illustrated in figure II.1, define a right-handed system, and it is assumed that the corresponding elementary dipoles support r.m.s. currents of $I_0$, $I_1$, and $I_2$ amperes, respectively. The corresponding peak scalar current dipole moments are $\sqrt{2}\, I_m \ell$ ampere-kilometers, where m = 0, 1, 2, and the sum of the complex vector dipole moments $\hat{x}_m \sqrt{2}\, I_m \ell \exp(i\tau_m)$ may be expressed as follows:

$$\underline{\vec{a}} = \vec{a}_1 + i\vec{a}_2 \tag{II.78a}$$

$$\vec{a}_1 = \sqrt{2}\, I\ell(\hat{x}_0 c_0 + \hat{x}_1 c_1 + \hat{x}_2 c_2), \qquad \vec{a}_2 = \sqrt{2}\, I\ell(\hat{x}_0 s_0 + \hat{x}_1 s_1 + \hat{x}_2 s_2) \tag{II.78b}$$

$$I^2 = I_0^2 + I_1^2 + I_2^2, \qquad c_m = (I_m/I)\cos\tau_m, \qquad s_m = (I_m/I)\sin\tau_m, \qquad m = 0, 1, 2. \tag{II.79}$$

Here, $\tau_0$, $\tau_1$, and $\tau_2$ represent initial phases of the currents supported by the elementary dipoles. The time phase factor is assumed to be $\exp(ikct)$.

Using the same unit vector coordinate system to represent the vector distance $\vec{r}$ from this idealized point source to a distant point:

$$\vec{r} = \hat{x}_0 x_0 + \hat{x}_1 x_1 + \hat{x}_2 x_2 = \hat{r}\, r \tag{II.80}$$

where $x_0$, $x_1$, and $x_2$ are given by (II.35b) as functions of r, θ, ϕ. The complex wave at $\vec{r}$ due to any one of the elementary dipoles is polarized in a direction

$$\hat{r} \times (\hat{x}_m \times \hat{r}) = \hat{x}_m - \hat{r}\, x_m/r \tag{II.81}$$

which is perpendicular to the propagation direction $\hat{r}$ and in the plane of $\hat{x}_m$ and $\hat{r}$. The total complex wave at $\vec{r}$ may be represented in the form given by (II.41):

$$\sqrt{2}\,\vec{e}(\vec{r})\exp(i\tau) = \sqrt{2}\,(\vec{e}_r + i\vec{e}_i)\exp(i\tau) = \sqrt{2}\,(\vec{e}_p + i\vec{e}_c)\exp[i(\tau + \tau_t)]$$

$$= [\hat{r} \times (\underline{\vec{a}} \times \hat{r})]\,[\eta_o/(2\lambda r)]\exp(i\tau) \tag{II.82}$$

$$\tau = k(ct - r) + \pi/4 \tag{II.83}$$

$$\sqrt{2}\,\vec{e}_r = [\vec{a}_1 - \hat{r}(\vec{a}_1 \cdot \hat{r})]\,\eta_o/(2\lambda r) \quad \text{volts/km} \tag{II.84a}$$

$$\sqrt{2}\,\vec{e}_i = [\vec{a}_2 - \hat{r}(\vec{a}_2\cdot\hat{r})]\,\eta_o/(2\lambda r)\ \text{volts/km.} \tag{II.84b}$$

The total mean power flux density $s(\vec{r})$ at $\vec{r}$ is given by (II. 58a):

$$s(\vec{r}) = (e_r^2 + e_i^2)/\eta_o = [a_1^2 - (\vec{a}_1\cdot\hat{r})^2 + a_2^2 - (\vec{a}_2\cdot\hat{r})^2]/\eta_o$$

$$= \frac{\eta_o (I\ell)^2}{4\lambda^2 r^2}\,[1 - (I_0^2 x_0^2 + I_1^2 x_1^2 + I_2^2 x_2^2)/(Ir)^2 - 2(c_{01} x_0 x_1 + c_{02} x_0 x_2 + c_{12} x_1 x_2)/r^2] \tag{II.85}$$

$$c_{mn} = (I_m I_n / I^2)\cos(\tau_m - \tau_n)\ . \tag{II.86}$$

The total radiated power $w_t$ is obtained by integrating $s(r)$ over the surface of a sphere of radius $r$, using the spherical coordinates $r$, $\theta$, $\phi$ illustrated in figure II. 1:

$$w_t = \int_o^{2\pi} d\phi \int_o^{\pi} d\theta\, r^2\, s(\vec{r})\sin\theta = \frac{2\pi\,\eta_o (I\ell)^2}{3\lambda^2}\ \text{watts}\ . \tag{II.87}$$

From (II. 87) it is seen that the peak scalar dipole moment $\sqrt{2}\,I\ell$ used to define $\vec{a}_1$ and $\vec{a}_2$ in (II. 78) may be expressed in terms of the total radiated power:

$$\sqrt{2}\,I\ell = \lambda\sqrt{3w_t/(\pi\eta_o)}\ \text{ampere-kilometers}\ . \tag{II.88}$$

The directive gain $g(\hat{r})$ is

$$g(\hat{r}) = 4\pi r^2 s(\vec{r})/w_t = \frac{3}{2}\left[1 - \left(\frac{I_0}{I}\right)^2 \cos^2\theta - \left(\frac{I_1}{I}\right)^2 \sin^2\theta\cos^2\phi\right.$$

$$\left. - \left(\frac{I_2}{I}\right)^2 \sin^2\theta\sin^2\phi - \left(c_{01}\cos\phi + c_{02}\sin\phi\right)\sin(2\theta) - c_{12}\sin^2\theta\,\sin(2\phi)\right]\ . \tag{II.89}$$

This is the most general expression possible for the directive gain of any combination of elementary electric dipoles centered at a point. Studying (II. 89), it may be shown that no combination of values for $I_0$, $I_1$, $I_2$, $\tau_0$, $\tau_1$, $\tau_2$ will provide an isotropic radiator. As defined in this annex, an isotropic antenna radiates or receives waves of any phase and polarization equally in every direction.

For the special case where $I_0 = I_1 = I_2 = I/\sqrt{3}$, $\tau_0 = \pi/2$, $\tau_1 = 0$, and $\tau_2 = \pi$, (II.89) shows that

$$g(\hat{r}) = 1 + \sin^2\theta \ \sin\phi \ \cos\phi \ . \qquad \text{(II.90)}$$

With these specifications, (II.78) shows that $c_0 = 0$, $c_1 = -c_2 = 1/\sqrt{3}$, $s_0 = 1/\sqrt{3}$, $s_1 = s_2 = 0$, and (II.78) with (II.88) shows that

$$\vec{a}_1 = (\hat{x}_1 - \hat{x}_2)b\ , \quad \vec{a}_2 = \hat{x}_o b \qquad \text{(II.91a)}$$

$$b = \lambda\left[w_r/(\pi\eta_o)\right]^{1/2} . \qquad \text{(II.91b)}$$

Substituting next in (II.84) with the aid of (II.2):

$$\sqrt{2}\,\vec{e}_r = e_o(\hat{x}_1 - \hat{x}_2 - \hat{r}\,b_2)\ , \quad \sqrt{2}\,\vec{e}_i = e_o(\hat{x}_o - \hat{r}\cos\theta) \qquad \text{(II.92)}$$

$$e_o = \left[\eta_o w_t/(4\pi r^2)\right]^{1/2} , \quad b_2 = \sin\theta(\cos\phi - \sin\phi)\ . \qquad \text{(II.93)}$$

The principal and cross-polarization gains determined using (II.57) and (II.58) are

$$g_p(\hat{r}) = 1 + \sin^2\theta(\sin\phi\ \cos\phi - 1/2)\ , \quad g_c(\hat{r}) = 1/2 \sin^2\theta\ . \qquad \text{(II.94)}$$

The subscripts p and c in (II.94) should be reversed whenever $g(\theta, \phi)$ is less than $\sin^2\theta$. Minimum and maximum values of g are 1/2 and 3/2 while $g_p$ ranges from 1/3 to 1 and $g_c$ from 0 to 1/2.

The importance of phases to multipath coupling is more readily demonstrated using a somewhat more complicated antenna. The following paragraphs derive an expression for a wave which is approximately plane at a distance r exceeding 200 wavelengths, radiated by an antenna composed of two three-dimensional complex dipoles located at $-5\lambda\hat{x}_o$ and $+5\lambda\hat{x}_o$ and thus spaced 10 wavelengths apart. When the radiation pattern has been determined, it will be assumed that this is the receiving antenna. Its response to known plane waves from two given directions will then be calculated.

With the radiated power $w_t$ divided equally between two three-dimensional complex dipoles, $\underline{\vec{a}}$ is redefined as

$$\underline{\vec{a}} = (b/\sqrt{2})\,\underline{\vec{a}}_o\ , \quad \underline{\vec{a}}_o = \hat{x}_1 - \hat{x}_2 + i\hat{x}_o . \qquad \text{(II.95)}$$

Since $5\lambda$ is negligible compared to $r$ except in phase factors critically depending on $r_1 - r_2$, the exact expressions

$$\vec{r}_1 = \vec{r} - 5\lambda\,\hat{x}_o, \quad \vec{r}_2 = \vec{r} + 5\lambda\,\hat{x}_o \tag{II.96}$$

lead to the following approximations and definitions:

$$r_1 = r(1-\epsilon), \quad r_2 = r(1+\epsilon), \quad \epsilon = 5(\lambda/r)\cos\theta \tag{II.97}$$

$$\hat{r}_1 = \hat{r}(1+\epsilon) - \hat{x}_o\epsilon\,\sec\theta, \quad \hat{r}_2 = \hat{r}(1-\epsilon) + \hat{x}_o\epsilon\,\sec\theta \tag{II.98}$$

$$\hat{r} = \hat{x}_o\cos\theta + (\hat{x}_1\cos\phi + \hat{x}_2\sin\phi)\sin\theta\ . \tag{II.99}$$

For distances $r$ exceeding 200 wavelengths, $|\epsilon| < 0.025$ and $\epsilon^2$ is neglected entirely, so that

$$\hat{r}_1 r_1 = \vec{r} - 5\lambda\,\hat{x}_o, \quad \hat{r}_2 r_2 = \vec{r} + 5\lambda\,\hat{x}_o\ . \tag{II.100}$$

At a point $\vec{r}$, the complex wave radiated by this antenna is approximately plane and may be represented as

$$\sqrt{2}\left(\vec{e}_r + i\,\vec{e}_i\right)\exp(i\tau) = \sqrt{2}\left[\vec{\underline{c}}_1\exp(i\tau_1) + \vec{\underline{c}}_2\exp(i\tau_2)\right] \tag{II.101}$$

where

$$\tau = k(ct - r) + \pi/4 \tag{II.102}$$

$$\tau_1 = \tau + \tau_a, \quad \tau_2 = \tau - \tau_a, \quad \tau_a = 10\pi\cos\theta\ . \tag{II.103}$$

As in (II.82), the waves radiated by the two main elements of this antenna are represented in (II.101) as the product of phasors $\exp(i\tau_1)$ and $\exp(i\tau_2)$ multiplied by the complex vectors $\sqrt{2}\,\vec{\underline{e}}_1$ and $\sqrt{2}\,\vec{\underline{c}}_2$, respectively:

$$\sqrt{2}\,\vec{\underline{c}}_1 = \left[\hat{r}_1\times\left(\vec{\underline{a}}\times\hat{r}_1\right)\right]\eta_o/(2\lambda r) = (e_o/2)\left[\vec{\underline{a}}_o - \hat{r}_1\left(\vec{\underline{a}}_o\cdot\hat{r}_1\right)\right] \tag{II.104a}$$

$$\sqrt{2}\,\vec{\underline{e}}_2 = \left[\hat{r}_2\times\left(\vec{\underline{a}}\times\hat{r}_2\right)\right]\eta_o/(2\lambda r) = (e_o/2)\left[\vec{\underline{a}}_o - \hat{r}_2\left(\vec{\underline{a}}_o\cdot\hat{r}_2\right)\right] \tag{II.104b}$$

Evaluating $\vec{a}_o \cdot \hat{r}_1$, $\vec{a}_o \cdot \hat{r}_2$, $\vec{e}_1$, and $\vec{e}_2$ with the aid of (II.95), (II.98), (II.99) and (II.104):

$$\vec{a}_o \cdot \hat{r}_1 = b_2(1+\epsilon) + i\left[\cos\theta - \epsilon(\sec\theta - \cos\theta)\right] \tag{II.105a}$$

$$\vec{a}_o \cdot \hat{r}_2 = b_2(1-\epsilon) + i\left[\cos\theta + \epsilon(\sec\theta - \cos\theta)\right] \tag{II.105b}$$

$$\vec{e}_1 = (e_o/2)\left\{\left[\hat{x}_1 - \hat{x}_2 - \hat{r}b_2(1+2\epsilon) + \hat{x}_o b_2\,\epsilon\sec\theta\right] + i\left[\hat{x}_o(1+\epsilon) - \hat{r}\cos\theta + \hat{r}\epsilon(\sec\theta - 2\cos\theta)\right]\right\} \tag{II.106a}$$

$$\vec{e}_2 = (e_o/2)\left\{\left[\hat{x}_1 - \hat{x}_2 - \hat{r}b_2(1-2\epsilon) - \hat{x}_o b_2\,\epsilon\sec\theta\right] + i\left[\hat{x}_o(1-\epsilon) - \hat{r}\cos\theta - \hat{r}\epsilon(\sec\theta - 2\cos\theta)\right]\right\}\ . \tag{II.106b}$$

Since the sum and difference of $\exp(i\tau_1)$ and $\exp(i\tau_2)$ are $2\cos\tau_a\ \exp(i\tau)$ and $2i\sin\tau_a\exp(i\tau)$, respectively, $\vec{e}_r$ and $\vec{e}_i$ as defined by (II.101) are

$$\vec{e}_r = e_o\left\{\left[\hat{x}_1 - \hat{x}_2 - \hat{r}b_2\right]\cos\tau_a - \epsilon\left[\hat{x}_o + \hat{r}(\sec\theta - 2\cos\theta)\right]\sin\tau_a\right\} \tag{II.107a}$$

$$\vec{e}_i = e_o\left[\hat{x}_o - \hat{r}\cos\theta\right]\cos\tau_a - b_2\left[2\hat{r} - \hat{x}_o\sec\theta\right]\sin\tau_a\Big\}\ . \tag{II.107b}$$

The complex wave $\sqrt{2}\left(\vec{e}_r + i\vec{e}_i\right)$ is a plane wave only when $\vec{e}_r$ and $\vec{e}_i$ are both perpendicular to the direction of propagation, $\hat{r}$, or when

$$\hat{r}\cdot\left(\vec{e}_r + i\vec{e}_i\right) = \epsilon\sin\tau_a\left[(\cos\theta - \sec\theta) + i\sin\theta(\cos\phi - \sin\phi)\right] = 0 \tag{II.108}$$

which requires that $\epsilon = 0$, $\sin\tau_a = 0$, or $\theta = 0$. If $\epsilon$ is negligible, the total mean power flux density in terms of the directive gain $g(\hat{r})$ is given by

$$s(\vec{r}) = \left(e_r^2 + e_i^2\right)/\eta_o = g(\hat{r})\,e_o^2/\eta_o \tag{II.109}$$

$$g(\hat{r}) = 2(1 + \sin^2\theta \sin\phi \cos\phi) \cos^2 \tau_a . \qquad \text{(II.110)}$$

That $w_t$ is the corresponding total radiated power may be verified by substituting (II.108) and (II.110) in (II.87), with the aid of (II.93) and (II.103).

Now let this antenna be a receiving antenna, and suppose that direct and ground-reflected waves arrive from directions $\hat{r}(\theta, \phi)$ equal to

$$\hat{r}_1(0.32, \pi/4) = 0.9492\, \hat{x}_o + 0.2224(\hat{x}_1 + \hat{x}_2) \qquad \text{(II.111a)}$$

$$\hat{r}_2(0.28, 0.75) = 0.9611\, \hat{x}_o + 0.1430\, \hat{x}_1 + 0.1332\, \hat{x}_2 . \qquad \text{(II.111b)}$$

Note that $\hat{r}_1$ and $\hat{r}_2$ in (II.111) are not related to $\hat{r}_1$ and $\hat{r}_2$ in (II.98) but are two particular values of $\hat{r}$. Corresponding values of $\tau_a$, $\cos\tau_a$, and $\sin\tau_a$ are

$$\tau_{a1} = 29.82111 , \quad \cos\tau_{a1} = -0.0240 , \quad \sin\tau_{a1} = -0.9997 \qquad \text{(II.112a)}$$

$$\tau_{a2} = 30.19245 , \quad \cos\tau_{a2} = 0.3404 , \quad \sin\tau_{a2} = -0.9403 . \qquad \text{(II.112b)}$$

The incoming waves in the two directions $\hat{r}_1$ and $\hat{r}_2$ are assumed to be plane, and the distances $r_1$ and $r_2$ to their source are assumed large enough so that $\epsilon_1 \sin\tau_{a1}$ and $\epsilon_2 \sin\tau_{a2}$ are negligible compared to $\cos\tau_{a1}$ and $\cos\tau_{a2}$, respectively. The plane wave response of the receiving antenna in these directions may be expressed in terms of the complex vectors associated with $\hat{r}_1$ and $\hat{r}_2$:

$$\vec{e}_{r1} + i\vec{e}_{i1} = -0.024\, e_o \left[ (\hat{x}_1 - \hat{x}_2) + i(0.099\, \hat{x}_o - 0.211\, \hat{x}_1 - 0.211\, \hat{x}_2) \right] \qquad \text{(II.113a)}$$

$$\vec{e}_{r2} + i\,\vec{e}_{i2} = 0.340\, e_o \left[ (-0.013\, \hat{x}_o + 0.998\, \hat{x}_1 - 1.002\, \hat{x}_2) + i(0.076\, \hat{x}_o - 0.137\, \hat{x}_1 - 0.128\, \hat{x}_2) \right] . \qquad \text{(II.113b)}$$

Since $\vec{e}_{r1} \cdot \vec{e}_{r2} = 0$, (II.44) with (II.42) and (II.43) shows that $\tau_{r1} = 0$, so that $\vec{e}_{r1}$ and $\vec{e}_{i1}$ are principal and cross-polarization components of the complex vector receiving pattern:

$$\vec{e}_{pr1} + i\,\vec{e}_{cr1} = \vec{e}_{r1} + i\,\vec{e}_{i1} . \qquad \text{(II.114a)}$$

These same equations show that $\tau_{r2} = -0.005$ and that

$$\vec{e}_{pr2} + i\,\vec{e}_{cr2} = 0.340\, e_o \left[ (-0.013\,\hat{x}_o + 0.999\,\hat{x}_1 - 1.001\,\hat{x}_2) + i(0.076\,\hat{x}_o - 0.132\,\hat{x}_1 - 0.113\,\hat{x}_2) \right] \tag{II.114b}$$

differing only slightly from (II.113b), since $\tau_{r2}$ is almost zero.

The axial ratios of the two polarization ellipses, defined by (II.50), are

$$a_{xr1} = -0.222\,, \qquad a_{xr2} = -0.143 \tag{II.115}$$

and the unit complex polarization vectors $\hat{\underline{p}}_{r1}$ and $\hat{\underline{p}}_{r2}$ defined by (II.51) are therefore

$$\hat{\underline{p}}_{r_1} = 0.674(\hat{x}_1 - \hat{x}_2) + i(0.067\,\hat{x}_o - 0.142\,\hat{x}_1 - 0.142\,\hat{x}_2) \tag{II.116a}$$

$$\hat{\underline{p}}_{r2} = (-0.009\,\hat{x}_o + 0.692\,\hat{x}_1 - 0.694\,\hat{x}_2) + i(0.053\,\hat{x}_o - 0.095\,\hat{x}_1 - 0.089\,\hat{x}_2)\;. \tag{II.116b}$$

The antenna gains $g_r(\hat{r}_1)$ and $g_r(\hat{r}_2)$ are given by (II.110):

$$g_r(\hat{r}_1) = 0.0021, \quad g_r(\hat{r}_2) = 0.241 \tag{II.117}$$

which shows that the gain $G_r(\hat{r}_1) = 10 \log g_r(\hat{r}_1)$ associated with the direct ray is 29.2 db below that of an isotropic antenna, while the gain $G_r(\hat{r}_2)$ associated with the ground-reflected ray is -6.2 db. It might be expected that only the incident wave propagating in the direction $-\hat{r}_2$ would need to be considered in determining the complex voltage at the receiving antenna terminals. Suppose, however, that the ground-reflected ray has been attenuated considerably more than the direct ray, so that the path attenuation factor $a_{p2}$ is 0.01, while $a_{p1} = 1$. Suppose further that the transmitting antenna gain associated with the ground-reflected ray is 6 db less than that associated with the direct ray. Then the mean incident flux density $s_2$ associated with the ground-reflected ray will be 26 db less than the flux density $s_1$ associated with the direct ray.

In order to calculate the complex received voltage $v$ given by (II.70) then, the following is assumed:

$$r_v = 52 \text{ ohms}\,, \qquad s_1 = 1 \text{ watt/km}^2 \,(= -30 \text{ dbm/m}^2)$$

$$s_2 = 0.0025 \text{ watts/km}^2 , \quad \lambda = 0.0003 \text{ km} (f = 1000 \text{ MHz})$$

$$a_{x1} = 0.2 , \quad a_{x2} = 0.4 , \quad \psi_{p1} = \pi/2 , \quad \psi_{p2} = 1.5$$

$$\tau_{i1} = \tau_{p1} + \tau_{t1} = 0 , \quad \tau_{i2} = \tau_{p2} + \tau_{t2} = \pi . \qquad \text{(II. 118)}$$

It will be seen that these assumptions imply a more nearly complete polarization coupling loss between the direct wave and the receiving antenna than between the ground-reflected wave and the receiving antenna. The effective absorbing area of the receiving antenna for each wave, as given by (II. 67) is

$$a_{e1} = 1.504 \times 10^{-11} \text{ km}^2 , \quad a_{e2} = 1.726 \times 10^{-9} \text{ km}^2 . \qquad \text{(II. 119)}$$

The polarization factors are

$$(\hat{\underline{p}}_1 \cdot \hat{\underline{p}}_{r1}) = -0.021 \, i , \quad (\hat{\underline{p}}_2 \cdot \hat{\underline{p}}_{r2}) = 0.062 + 0.236 \, i \qquad \text{(II. 120)}$$

and the phase factors are $\exp(i\tau)$ and $\exp[i(\tau + 3.137)]$, respectively. Substituting these values in (II. 65), the complex voltages are

$$v_1 = -1.175(10^{-6}) i \exp(i\tau) , \quad v_2 = -(1.887 + 7.071 i)(10^{-6}) \exp(i\tau). \qquad \text{(II. 121)}$$

The real voltage at the antenna terminals, as given by (II. 70) is

$$v_\nu = (v_1 + v_2)(v_1 + v_2)^* = 8.33 \times 10^{-6} \text{ volts} = 8.33 \text{ microvolts} \qquad \text{(II. 122)}$$

and the corresponding power $w_a$ available at the terminals of the loss-free receiving antenna is

$$w_a = 0.334 \times 10^{-12} \text{ watts} , \quad W_a = -125 \text{ dbw} = -95 \text{ dbm} \qquad \text{(II. 123)}$$

as given by (II. 71).

II. 3.8 Conclusions

The foregoing exercise demonstrates that:

(1) Even small changes in antenna beam orientation, transmission loss, polarization coupling, and multipath phasing may have a visible effect on the available power at the terminals of a receiving antenna.

(2) If the formulation of the general relationships for a completely polarized wave is programmed for a digital computer, it may be feasible to estimate the complete statistics of a received signal whenever reasonable assumptions can be made about the statistics of the parameters described in this annex.

(3) The measurement of antenna characteristics in a few critical directions will often be sufficient to provide valuable information to be used with the relationships given here. The measurement of Stokes' parameters, for instance, will provide information about $a_{xr}$, $g_r$, $\psi_p$, and both the polarized field intensity $s_r$ and the unpolarized field intensity $s_o$. These parameters [Stokes, 1922 ] are

$$s_r + s_o = \text{total mean field intensity} \tag{II.124}$$

$$Q = s_r \cos(2\beta) \cos(2\psi_p) \tag{II.125}$$

$$U = s_r \cos(2\beta) \sin(2\psi_p) \tag{II.126}$$

$$V = s_r \sin(2\beta) \tag{II.127}$$

where

$$\beta = \tan^{-1} a_{xr} . \tag{II.128}$$

The unpolarized or randomly polarized field intensity $s_o$ is determined from (II.124) and the identity

$$s_r = (Q^2 + U^2 + V^2)^{1/2} . \tag{II.129}$$

Using standard sources and antenna model ranges, the gain $g_r$ may be determined from

$$g_r = s_r/(e_o^2/\eta_o) , \qquad e_o^2 = \eta_o p_t/(4\pi r^2) \tag{II.130}$$

assuming, if $e_o$ is measured, that any power reception efficiency $1/\ell_{er}$ less than unity will affect $s_r$ and $e_o^2$ alike.

Finally, a method for measuring relative phase responses $\tau_r$ is also needed. In individual cases, multipath coupling loss may be insufficient to provide adequate unwanted signal rejection. Variations of $\tau_r$ may lead to phase interference fading of wanted signals, just as variations of $a_p$ are associated with long-term power fading. Because of the complexity of these phenomena, they are usually described in terms of cumulative distributions of signal amplitudes or fade durations. Fortunately, even crude measurements or simple theories may then suffice to provide statistical information about $\tau_r$.

# COORDINATE SYSTEMS FOR STUDYING ANTENNA PATTERNS

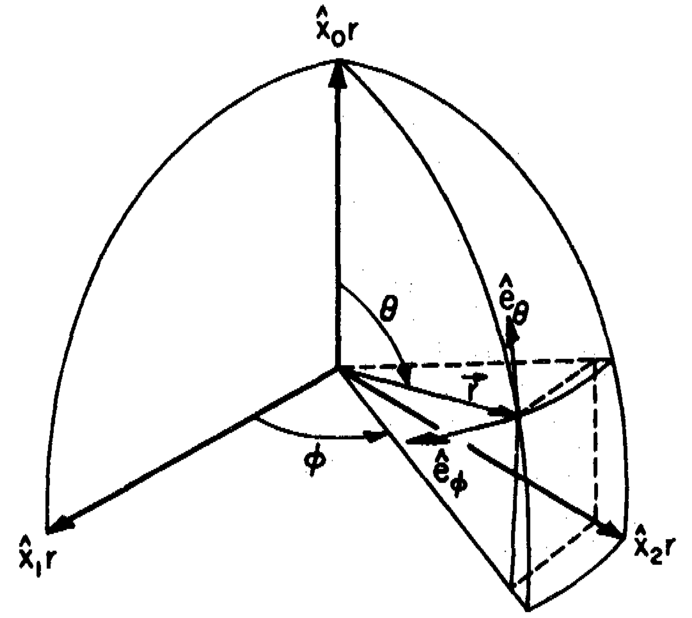


Figure II.1

## II. 4 List of Special Symbols Used in Annex II

$a_{en}$, $a_{e1}$, $a_{e2}$ The effective absorbing area for the $n^{th}$ discrete plane wave incident on an antenna from a single source, (II. 67), and for each of two waves (II. 119).

$a_p$, $a_{pm}$, $a_{pn}$ The fraction of energy absorbed along a ray path, or scattered out of it, (II. 59), and the fraction of energy, $a_p$, for the $m^{th}$ and $n^{th}$ multipath components from a single source, where m and n take on integral values from 1 to N, (II. 72)

$a_{xn}$, $a_{x1}$, $a_{x2}$ Axial ratios of the polarization ellipse of the $n^{th}$, first, and second plane wave from a single source, (II. 68) and (II. 118).

$a_{xrn}$, $a_{xr1}$, $a_{xr2}$ Axial ratios of the polarization ellipse associated with the receiving pattern for the $n^{th}$, first, and second plane wave from a single source, (II. 68) and (II. 115).

$a_1$, $a_2$ Positive or negative amplitudes of real and imaginary components of a complex vector: $\underline{\vec{a}} = \vec{a}_1 + i\vec{a}_2$, $\underline{\vec{a}}^2 = a_1^2 + a_2^2$, (II. 78).

$\vec{a}$, $\hat{a}$ The real vector $a = a\hat{a}$, where $\hat{a}$ is a unit vector.

$\vec{a}_1$, $\vec{a}_2$ Real vectors defining real and imaginary components of a complex vector: $\underline{\vec{a}} = \vec{a}_1 + i\vec{a}_2$, (II. 78).

$\underline{\vec{a}}$ A complex vector: $\underline{\vec{a}} = \vec{a}_1 + i a_2$, (II. 78).

$\underline{\vec{a}}_o$ A complex vector defined in terms of the unit vector system $\hat{x}_0$, $\hat{x}_1$, $\hat{x}_2$, (II. 95).

$e_{cr}$ The positive or negative amplitude of the cross-polarized vector component $\vec{e}_{cr}$ of a receiving antenna response pattern, (II. 50).

$e_i$ The positive or negative amplitude of the real vector $\vec{e}_i$ associated with a complex plane wave $\sqrt{2}\,(\vec{e}_r + i\vec{e}_i) \exp(i\tau)$, where $\vec{e}_r$ and $\vec{e}_i$ are time-invariant and $\exp(i\tau)$ is a time phasor, (II. 41b).

$e_{pr}$ The positive or negative amplitude of the principal polarization component $\vec{e}_p$ of a receiving antenna response pattern, (II. 50).

$e_r$ The positive or negative amplitude of the real vector component $\vec{e}_r$ associated with a complex plane wave $\sqrt{2}\,(\vec{e}_r + i\vec{e}_i) \exp(i\tau)$, where $\vec{e}_r$ and $\vec{e}_i$ are time invariant and $\exp(i\tau)$ is a time phasor, (II. 41a).

$e_o$ Equivalent free space field strength, (II. 38).

$e_1$, $e_2$ The positive or negative real amplitudes of real and imaginary components of the complex polarization vector $\underline{\vec{e}}$, (II. 43).

$e_\theta$, $e_\phi$ The positive amplitudes of real vectors $\vec{e}_\theta$ and $\vec{e}_\phi$ associated with the $\theta$ and $\phi$ components of a complex plane wave, (II. 4) figure II. 1.

$\vec{e}_c$, $\vec{e}_p$ Real vectors associated with cross and principal polarization components of a uniform elliptically polarized plane wave, annex II, section II. 3. 2.

$\hat{e}_c$, $\hat{e}_p$ Directions of cross and principal polarization, chosen so that their vector product $\hat{e}_p \times \hat{e}_c$ is a unit vector in the direction of propagation, (II. 47).

| | |
|---|---|
| $\vec{e}_{cr}$, $\vec{e}_{pr}$ | Cross and principal polarization field components of a receiving antenna response pattern, (II.49). |
| $\hat{e}_{cr}$, $\hat{e}_{pr}$ | Directions of cross and principal polarization components of a receiving antenna response pattern, (II.51), (II.53). |
| $\vec{e}_i$ | The real vector associated with the imaginary component of the time-invariant part of a complex plane wave $\sqrt{2}\ (\vec{e}_r + i\vec{e}_i)\ \exp(i\tau)$, (II.41b). |
| $\vec{e}_r$ | The real vector associated with the real component of the time-invariant part of a complex plane wave $\sqrt{2}\ (\vec{e}_r + i\vec{e}_i)\ \exp(i\tau)$, (II.41a). |
| $\vec{e}_1$, $\vec{e}_2$ | Real vector components of a complex polarization vector $\underline{\vec{e}}$ which has been resolved into components which are orthogonal in both space and time, (II.43). |
| $\vec{e}_\theta$, $\vec{e}_\phi$ | Real vectors associated with the $\theta$ and $\phi$ components of a complex plane wave $\sqrt{2}\ [\vec{e}_\theta \exp(i\tau_\theta) + \vec{e}_\phi \exp(i\tau_\phi)] \exp(i\tau)$, where only the phasor $\exp(i\tau)$ depends on time, (II.40) figure II.1. |
| $\hat{e}_\theta$ | A unit vector $\hat{e}_\phi \times \hat{r}$ perpendicular to $\hat{e}_\phi$ and $\hat{r}$, (II.36b) figure II.1. |
| $\hat{e}_\phi$ | A unit vector $(\hat{r} \times \hat{x}_o)/\sin\theta$ perpendicular to $\hat{r}$ and $\hat{x}_o$, (II.36a) figure II.1. |
| $\underline{\vec{e}}$, $\underline{\vec{e}}_r$ | A bar is used under the symbol to indicate a complex vector: $\underline{\vec{e}} = \vec{e}_p + i\vec{e}_c$, $\underline{\vec{e}}_r = \vec{e}_{pr} + i\vec{e}_{cr}$, (II.46). |
| $\vec{e}^{\,*}$ | The complex conjugate of $\underline{\vec{e}}$: $\underline{\vec{e}}^{\,*} = \vec{e}_p - i\vec{e}_c$. |
| $\lvert\underline{\vec{e}}\rvert$, $\lvert\underline{\vec{e}}_r\rvert$ | The magnitudes of the complex vectors $\underline{\vec{e}}$ and $\underline{\vec{e}}_r$, (II.55). |
| $\lvert e_c\rvert$, $\lvert e_{cr}\rvert$, $\lvert e_p\rvert$, $\lvert e_{pr}\rvert$ | The amplitudes of the cross and principal polarization components $\vec{e}_c$, $\vec{e}_{cr}$, $\vec{e}_p$, and $\vec{e}_{pr}$, section II.3.3. |
| $E$ | Field strength in dbu, (II.20). |
| $E_o$ | The equivalent free space field strength in dbu, (II.26). |
| $E_I$ | The equivalent inverse distance field, (II.28). |
| $E_{1kw}$ | Field strength in dBu per kilowatt effective radiated power, (II.23) -(II.25). |
| $g$ | Maximum free space directive gain, or directivity, Section II.3.4. |
| $g_c$ | The cross-polarization component of the directive gain, (II.57). |
| $g_{cr}$ | The cross-polarization component of the directive gain of a receiver, (II.51). |
| $g_p$ | Principal polarization directive gain, (II.57). |
| $g_{pr}$, $g_{pt}$ | Principal polarization directive gains for the receiving and transmitting antennas, respectively, (II.59). |
| $g_{rn}$, $g_{tn}$ | The directive gains $g_r$ and $g_t$ for the $n^{th}$ of a series of plane waves, (II.66) and (II.67). |
| $g_\theta$, $g_\phi$ | Directive gains associated with the field components $\vec{e}_\theta$, $\vec{e}_\phi$, (II.37). |
| $g(\hat{r})$ | Directive gain in the direction $\hat{r}$, (II.89). |
| $g_c(\hat{r})$, $g_p(\hat{r})$ | Cross polarization and principal polarization directive gains in the direction $\hat{r}$, (II.94). |
| $g_r(\hat{r}_1)$, $g_r(\hat{r}_2)$ | Directive gains associated with direct and ground-reflected rays, respectively, (II.117). |

| | |
|---|---|
| $G_{pt}(\hat{r}_1)$ | Principal polarization directive gain of the transmitter in the direction $\hat{r}_1$, which is the initial direction of the most important propagation path to the receiver, (II.18) and (II.23) to (II.25). |
| $i$ | $i = \sqrt{-1}$. |
| $I_m$ | Current in r.m.s. amperes where m = 0, 1, 2, (II.76). |
| $I_0$, $I_1$, $I_2$ | Current in r.m.s. amperes corresponding to three elementary dipoles in three mutually perpendicular directions, (II.76). |
| $k$ | Propagation constant, $k = 2\pi/\lambda$, (II.34). |
| $\ell$ | Used as a subscript to indicate a load, for example, $z_{\ell\nu}$, represents the impedance of a load at a radio frequency $\nu$, (II.1). |
| $\ell_{er\nu}$; $L_{er\nu}$ | The effective loss factor for a receiving antenna at a frequency $\nu$ hertz (II.6), $L_{er\nu} = 10 \log \ell_{er\nu}$ db, (II.8). |
| $\ell_{et\nu}$, $L_{et\nu}$ | The effective loss factor for a transmitting antenna at a radio frequency $\nu$ hertz, (II.7), $L_{et\nu} = 10 \log \ell_{et\nu}$ db, (II.9). |
| $\ell_{m\nu}$ | A mismatch loss factor defined by (II.4). |
| $L_{fr}$, $L_{ft}$ | The decibel ratio of the resistance component of antenna input impedance to the free space antenna radiation resistance for the receiving and transmitting antennas, respectively, (II.11). |
| $L_{rr}$, $L_{rt}$ | The ratio of the actual radiation resistance of the receiving or transmitting antenna to its radiation resistance in free space, (II.12), (II.13). |
| $L_p$ | Propagation loss, (II.14). |
| $L_{pb}$ | Basic propagation loss, (II.15). Basic propagation loss in free space is the same as basic transmission loss in free space. |
| $\hat{\underline{p}}$, $\hat{\underline{p}}_n$ | Unit complex polarization vector for the incident wave, (II.54), and (II.68). |
| $\hat{\underline{p}}_r$, $\hat{\underline{p}}_{rn}$ | Unit complex polarization vector associated with a receiving pattern, (II.51) and with the receiving pattern of the $n^{th}$ incident wave, (II.68). |
| $\hat{\underline{p}}_{r1}$, $\hat{\underline{p}}_{r2}$ | The complex receiving antenna polarization vectors $\hat{\underline{p}}_r$ for each of two ray paths between transmitter and receiver, (II.116). |
| $r$ | Resistance of an antenna, (II.1). |
| $r$ | Magnitude of the vector $\vec{r} = r\,\hat{r}$ in the direction $\hat{r}(\theta, \phi)$, and a coordinate of the polar coordinate system r, $\theta$, $\phi$, section II.3.1. |
| $r_{fr}$, $r_{ft}$ | Antenna radiation resistance in free space for the receiving and transmitting antennas, respectively, (II.11), (II.12) and (II.13). |
| $r_{\ell\nu}$ | Resistance of a load, (II.1). |
| $r_r$, $r_t$ | Antenna radiation resistance of the receiving and transmitting antennas, respectively, (II.10). |
| $r'_r$, $r'_t$ | Resistance component of antenna input impedance for the receiving and transmitting antennas, respectively, (II.10). |
| $r_\nu$ | Resistance of an equivalent loss-free antenna, (II.1c). |
| $r'_\nu$ | Resistance of an actual antenna in its actual environment, (II 1b). |
| $\vec{r}$ | The vector distance between two points, $\vec{r} = r\,\hat{r}$, (II.80). |

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| $\hat{r}$ | A unit vector, (II. 35) |
| $\hat{r}$, $\hat{e}_\theta$, $\hat{e}_\phi$ | A cartesian unit vector coordinate system, (II. 35) and (II. 36). |
| $s$ | Total mean power flux density, (II. 58). |
| $s_c$, $s_p$ | Mean power flux densities associated with cross-polarization, and principal polarization components, (II. 56). |
| $s_e$ | The fraction of the total flux density that contributes to the available power, (II. 75). |
| $s_\ell$, $s_r$ | Mean power flux densities associated with left-handed, and right-handed polarization, respectively, (II. 61). |
| $s_o$ | Free space field intensity in watts per square kilometer, (II. 16). |
| $< s_e >$ | The statistical "expected value of $s_e$, (II. 76). |
| $s_c(\vec{r})$, $s_p(\vec{r})$ | Mean power flux densities associated with the cross and principal polarization components of $\underline{\vec{e}}$ in the direction $\vec{r}$, (II. 17). |
| $v_n$ | Complex open-circuit r.m.s. signal voltage for coherently phased multipath components, (II. 65). |
| $v_\nu$ | The open-circuit r.m.s. voltage for an equivalent loss-free antenna at a frequency $\nu$, (II. 5). |
| $v'_\nu$ | The actual open-circuit r.m.s. voltage at the antenna terminals at a frequency $\nu$, (II. 2). |
| $w_{ab}$ | The available power corresponding to propagation between hypothetical isotropic antennas, (II. 72). |
| $w_{a\nu}$ | Available power at the terminals of an equivalent loss-free receiving antenna at a radio frequency $\nu$, (II. 5). |
| $w'_{a\nu}$ | Available power at the terminals of the actual receiving antenna at a radio frequency $\nu$, (II. 3). |
| $w_{\ell\nu}$ | Power delivered to the receiving antenna load, at a radio frequency $\nu$, (II. 2) |
| $w_{t\nu}$ | Total power radiated at a frequency $\nu$, (II. 7). |
| $w'_{t\nu}$ | Total power delivered to the transmitting antenna at a frequency $\nu$, (II. 7). |
| $x_{\ell\nu}$, $x'_\nu$, $x_\nu$ | Reactance of a load, an actual lossy antenna, and an equivalent loss-free antenna, respectively, (II. 1). |
| $\hat{x}_m$ | One of three mutually perpendicular directions, m=0, 1, 2, section II. 3. 7. |
| $\hat{x}_o$, $\hat{x}_1$, $\hat{x}_2$ | Axes of a cartesian unit vector coordinate system, (II. 35) figure II. 1. |
| $z_{\ell\nu}$ | Impedance of a load, (II. 1). |
| $z_\nu$ | Impedance of an equivalent loss-free antenna (II. 1). |
| $z'_\nu$ | Impedance of an actual lossy antenna, (II. 1). |
| $z'^*_\nu$ | The conjugate of $z'_\nu$, following (II. 2). |

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| $\epsilon$ | A small increment used in (II. 97) and (II. 98). |
| $\eta_o$ | Characteristic impedance of free space, $\eta_o = 4\pi c . 10^{-7}$, (II. 38). |
| $\theta$ | A polar coordinate, (II. 35). |
| $\nu$ | **Radio frequency in hertz (cycles per second), section II. 1.** |
| $\nu_\ell$, $\nu_m$ | Limits of integration (II. 8), (II. 9). |
| $\tau$ | The time-varying phase $\tau = k\,(ct - r)$, where c is the free space velocity of radio-waves, t is the time at the radio source, and r is the length of the radio ray, (II. 34). |
| $\tau_a$ | Time element defined by (II. 103). |
| $\tau_{a1}$, $\tau_{a2}$ | The time element $\tau_a$ corresponding to direct and ground-reflected waves at the receiving antenna, (II. 112). |
| $\tau_i$ | A time-independent phase which is a function of $\vec{r}$, (II. 42), (II. 64). |
| $\tau_{in}$ | The time-independent phase for the $n^{th}$ component of an incident wave, section II. 3. 6. |
| $\tau_{i1}$, $\tau_{i2}$ | The time-independent phase for two components of an incident wave, (II. 118). |
| $\tau_m$, $\tau_o$, $\tau_1$, $\tau_2$ | Initial phase of the current supported by one of m elementary dipoles, where m = 0, 1, 2, (II. 79). |
| $\tau_p$ | A function of the ray path, including allowances for path length differences and diffraction or reflection phase shifts, (II. 64). |
| $\tau_{pn}$, $\tau_{p1}$, $\tau_{p2}$ | The phase function $\tau_p$ for the $n^{th}$, first, and second plane wave incident on an antenna from a single source, (II. 65) and (II. 118). |
| $\tau_r$ | Antenna phase response for the receiving antenna, (II. 49). |
| $\tau_{rn}$, $\tau_{r1}$, $\tau_{r2}$ | The antenna phase response, $\tau_r$, for the $n^{th}$, first, and second plane wave incident on the receiving antenna, (II. 65) and section II. 3. 7. |
| $\tau_t$ | Antenna phase response for a transmitting antenna, (II. 64). |
| $\tau_{tn}$, $\tau_{t1}$, $\tau_{t2}$ | The antenna phase response $\tau_t$ for the $n^{th}$, first, and second plane wave, (II. 65) and (II. 118). |
| $\tau_\theta$, $\tau_\phi$ | Phases associated with the electrical field components $\vec{e}_\theta$, $\vec{e}_\phi$, (II. 40). |
| $\phi$ | One of the polar coordinates, r, $\theta$, $\phi$, (II. 89) and figure II. |
| $\psi_{p1}$, $\psi_{p2}$ | The acute angle, $\psi_p$, for each of two waves, (II. 118). |