Annex IV

FORWARD SCATTER

IV.1 General Discussion

This annex discusses some of the similarities and differences between forward scatter from refractive index turbulence and forward scatter or incoherent reflections from tropospheric layers.

To scatter is to spread at random over a surface or through a space or substance. Scattering which tends to be coherent is more properly called forward scatter, reflection, refraction, focusing, diffraction, or all of these, depending on the circumstances. Modes of scattering as well as mechanisms of propagation bear these names. For example, we may speak of the reflection, refraction, diffraction, focusing, scattering, and absorption of a radio wave by a single spherical hailstone, and all of these modes can be identified in the formal solutions of Maxwell's equations for this problem.

The large volume of beyond-the-horizon radio transmission loss data available in the frequency range 40 to 4000 MHz and corresponding to scattering angles between one and three degrees indicates that the ratio $10^{-A/10}$ corresponding to the transmission loss, A, relative to free space is approximately proportional to the wavelength, λ, or inversely proportional to the radio frequency, f, [Norton, 1960], so that the ratio $10^{-L_b/10}$ corresponding to the forward scatter basic transmission loss is approximately proportional to λ^3 or to f^{-3}. This circumstance is more readily explained in terms of forward scatter from layers [Friis, Crawford, and Hogg, 1957] or in terms of glancing or glinting from brilliant points on randomly disposed "feuillets", [du Castel, Misme, Spizzichino, and Voge, 1962], than in terms of forward scatter from the type of turbulence characterized by the modern Obukhov-Kolmogorov theory [Obukhov, 1941, 1953; Batchelor, 1947, 1953]. There is recent evidence [Norton and Barrows, 1964] that the wavenumber spectrum of refractivity turbulence in a vertical direction has the same form as the more adequately studied spectrum of variations in space in a horizontal direction. Some mechanism other than scatter from refractivity turbulence must be dominant most of the time to explain the observed transmission loss values over a majority of the transhorizon tropospheric paths for which data are available. Scattering from refractivity turbulence and scattering from sharp gradients are mechanisms which coexist at all times in any large scattering volume. Sharp gradients always exist somewhere, and the atmosphere between them is always somewhat turbulent. Power scattered by these mechanisms is occasionally supplemented by diffraction, specular reflection from strong extended layers, and/or ducting.

A tropospheric duct exists, either ground-based or elevated, if a substantial amount of energy is focused toward or defocused away from a receiver as super-refractive gradients of N exceed a critical value called a "ducting gradient." This gradient is about -157 N-units per kilometer at sea level for horizontally launched radio waves. The duct thickness must exceed about $5062 f^{-\frac{2}{3}}$ meters with f in MHz [Kerr, 1964] for a duct to completely trap such radio waves. A few useful references in this connection are cited at the end of section 4, Volume 1.

A more or less horizontally homogeneous "kink" in a refractive index profile may indicate the possibility of ducting for very short wavelengths, the presence of a refracting layer for some of the longer waves, and merely a slight and random perturbation of average atmospheric conditions for other frequencies, antenna locations, or antenna beam patterns and elevation angles. The layer that presents a sharp discontinuity for radio frequencies from 30 to 100 MHz (λ = 10 to 3 meters) may represent a relatively gradual change of refractive index at 300 MHz (λ = 1 meter) and higher frequencies. A tropospheric layer or "feuillet" requires a sufficiently abrupt change in refractive index, usually associated with fine weather conditions, to reflect a substantial amount of radio energy at the grazing angles and frequencies of interest. These may be horizontal changes, in thermals, for instance, as well as changes of refractivity, N, with height.

Almost specular reflection from tropospheric layers is often observed between 30 MHz and 200 MHz. At higher frequencies, where focusing, defocusing, and ducting are common, and where extensive layers are not sufficiently abrupt or sufficiently numerous to provide strong reflections, a number of small and randomly oriented surfaces come into play. A recent summary of the role of the layer structure of the troposphere in explaining tropospheric propagation [Saxton, Lane, Meadows, and Matthews, 1964] includes an extensive list of references. Also useful are general discussions of tropospheric propagation by Bullington [1955], du Castel [1960], Crawford, Hogg, and Kummer [1959], Fengler [1964], Fengler, Jeske, and Stilke [1964], Kirby, Rice, and Maloney [1961], Johnson [1958], Rice and Herbstreit [1964], Shkarofsky [1958], and Vvedenskii and Arenberg [1957].

There are at least three distinguishing features in most theories of forward scatter from clouds, precipitation, refractive index turbulence, layers, or feuillets. A calculation is first made of the expected or average forward scattering pattern, reradiation pattern, or diffraction pattern of a scatterer or a group of scatterers, usually located in free space, and usually assuming an incident plane wave and a distant receiver. Second, a decision is made that the relative phases of waves scattered from individual raindrops or subvolumes of refractivity turbulence or feuillets are random, so that we may simply add the power contributions from these elements and ignore the phases. This is an essential feature of a random scatter theory. And third, some way is found to relate the actual terrain, atmosphere, and antenna parameters to the theoretical model so that a comparison may be made between data and theory.

IV.2 Models for Forward Scattering

The mechanisms of scattering from refractivity turbulence, reflection from elevated layers, and ducting are much more sensitive to vertical refractivity gradients than to the horizontal gradients commonly observed. The forward scatter theory used to develop the prediction methods of section 9 assumes that only vertical scales of turbulence or layer thicknesses are important. The radio wave scattered forward by all the scattering subvolumes visible to both antennas or by all the layers of feuillets visible to both antennas is most affected by a particular range of "eddy sizes", ℓ, or by layers of an average thickness $\ell/2$. A stack of eddies of size ℓ must satisfy the Bragg condition that reradiation by adjacent eddies shall add in phase. Reflections from the exterior and interior boundaries of a layer will add in phase if the ray traversing the interior of the layer is an odd number of wavelengths longer than the ray reflected from the exterior boundary. Either the mechanism of forward scatter from refractivity turbulence or the mechanism of reflection from layers or feuillets selects a wavenumber direction $\hat{\kappa}$ that satisfies the specular reflection condition corresponding to Snell's law that angles of incidence and reflection, ψ, are equal. Mathematically, these conditions are represented by the following relations:

$$\ell = \frac{\lambda}{2\sin(\theta/2)} \cong \frac{\lambda}{\theta} \qquad \hat{\kappa} = \frac{\hat{R}_o + \hat{R}}{|\hat{R}_o + \hat{R}|} \tag{IV.1}$$

where $\hat{R}_o$ and $\hat{R}$ are unit vectors from the centers of radiation of the transmitting and receiving antenna, respectively, towards an elementary scattering volume, or towards the point of geometrical reflection from a layer. The angle between $\hat{R}$ and $\hat{R}_o$ is the scattering angle θ illustrated in figure IV-1 and is thus twice the grazing angle ψ for reflection from a layer:

$$\theta = 2\psi = \cos^{-1}(-\hat{R}\cdot\hat{R}_o) \quad \text{radians}. \tag{IV.2}$$

The plane wave Fresnel reflection coefficient q_o for an infinitely extended plane boundary between homogeneous media with refractive indices n_1 and n_2 and for horizontal polarization [Wait, 1962] is

$$q_o = \frac{\sin\psi - \left[2(n_1 - n_2) + (n_1 - n_2)^2 + \sin^2\psi\right]}{\sin\psi + \left[2(n_1 - n_2) + (n_1 - n_2)^2 + \sin^2\psi\right]}. \tag{IV.3}$$

The following approximation, valid for $(n_1 - n_2)^2 < \sin^2\psi << 1$ is also good for vertical polarization:

$$q_o \cong \frac{n_2 - n_1}{2\psi^2} \exp\left[-(n_2 - n_1)^2/(2\psi^2)\right] \cong \frac{n_2 - n_1}{2\psi^2} = \frac{2(n_2 - n_1)}{\theta^2}. \tag{IV.4}$$

A differential amplitude reflection coefficient dq for a tropospheric layer is next defined as proportional to the difference between two gradients of refractive index, m and m_o, where m is the average refractive index gradient dn/dz across the layer, and m_o is the average refractive index gradient for the region in which the layer is embedded. Let the layer extend in depth from $z = 0$ to $z = z_o$ in the wavenumber direction $\hat{\kappa}$ defined by (IV. 1), and write the differential reflection coefficient as

$$dq = dz\,(m - m_o)/(2\psi^2). \tag{IV. 5}$$

A phasor $\exp[-iz(4\pi\psi/\lambda)]$ is associated with dq, and the power reflection coefficient q^2 for a tropospheric layer of thickness z_o is approximated as

$$q^2 = \left| \int_{z=o}^{z=z_o} dq \exp[-i\,z(4\pi\psi/\lambda)] \right|^2 = (4\pi)^2 \lambda^2 \psi^{-6} M \tag{IV. 6a}$$

$$M = (m - m_o)^2 \;\; [1 - \cos(4\pi\psi z_o/\lambda)]/(4\pi)^4. \tag{IV. 6b}$$

If M is assumed continuous at $z = 0$ and $z = z_o$, somewhat smaller values of q^2 and m will result [Wait, 1962].

Friis, Crawford, and Hogg [1957] point out that the power received by reflection from a finite layer can be approximated as the diffracted power through an absorbing screen with the dimensions of the layer projection normal to the direction of propagation. They then consider layers of large, small, and medium size compared to

$$2x = 2(\lambda R_o R/d)^{1/2}, \quad d \cong R_o + R \tag{IV. 7}$$

which is the width of a first Fresnel zone. Let b represent the dimensions of a layer or feuillet in any direction perpendicular to $\hat{\kappa}$. Since $\hat{\kappa}$ is usually nearly vertical, b is usually a horizontal dimension. Adopting a notation which conforms to that used elsewhere in this report, the available power w_a at a receiver at a distance d from a transmitting antenna radiating w_t watts is

$$w_a = \frac{4 w_t g_t g_r \lambda^2 q^2}{(4\pi d)^2} \;[C^2(u) + S^2(u)]\;[C^2(v) + S^2(v)] \tag{IV. 8}$$

in terms of Fresnel integrals given by (III. 33), where

$$u = b\sqrt{2}/x, \qquad v = b\,\psi\sqrt{2}/x \tag{IV. 9}$$

and g_t and g_r are antenna directive gains. For large u and v,

$$C^2(u) = S^2(u) = C^2(v) = S^2(v) = 1/4 \, ,$$

and for small u and v, $C^2(u) = u^2$, $C^2(v) = v^2$, and $S^2(u) = S^2(v) = 0$.

For large layers, where $b >> 2x$:

$$w_a = w_t g_t g_r \lambda^4 \psi^{-6} d^{-2} M \cdot \qquad \text{(IV.10a)}$$

For intermediate layers, where $b \cong 2x$:

$$w_a = w_t g_t g_r \lambda^3 \psi^{-4} (R R_o d)^{-1} b^2 M \cdot \qquad \text{(IV.10b)}$$

For small layers, where $b << 2x$:

$$w_a = w_t g_t g_r \lambda^2 \psi^{-4} (R R_o)^{-2} b^4 M \cdot \qquad \text{(IV.10c)}$$

Forward scatter from layers depends on the statistics of sharp refractive index gradients in the directions $\hat{\kappa}$ defined by (IV.1). The determination of these statistics from radio and meteorological measurements is only gradually becoming practical. A study of likely statistical averages of the meteorological parameters M, b^2M, and b^4M indicates that these expected values should depend only slightly on the wavelength λ and the grazing angle ψ, as was assumed by Friis, et al.[1957]. The expected value of

$$[1 - \cos(4\pi \psi z_o/\lambda)]$$

can vary only between 0 and 2 and is not likely to be either 0 or 2 for any reasonable assumptions about the statistics of z_o.

Available long-term median radio transmission loss data usually show the frequency law given by (IV.10b) for medium-size layers. Long-term cumulative distributions of short-term available power ratios on spaced frequencies rarely show a wavelength law outside the range from λ^2 to λ^4 [Crawford, Hogg, and Kummer, 1959; Norton 1960]. An unreported analysis of 8978 hours of matched simultaneous recordings at 159.5, 599, and 2120 MHz over a 310-km path in Japan shows that this wavelength exponent for transmission loss w_a/w_t is within the range from 2 to 4 ninety-eight percent of the time. This corresponds to a wavelength exponent range from 0 to 2 or a frequency exponent range from 0 to -2 for attenuation relative to free space values, and to corresponding ranges λ^2 to λ^4 or f^{-2} to f^{-4} for values of basic transmission loss, L_b.

Figures IV.1(a) and IV.1(b) illustrate forward scattering from a single small layer and from refractivity turbulence in a single small scattering subvolume of the volume V of space visible to two antennas. Figures IV.1(c) and IV.1(d) illustrate models for the addition of power contributions from large parallel layers, and from scattering or reflection subvolumes, respectively. Contributions from diffraction or ducting are ignored, as well as returns from well-developed layers for which a geometrical reflection point is not visible to both antennas. Combinations of these mechanisms, though sometimes important, are also not considered here.

For each of the cases shown in figure IV.1, coherently scattered or reflected power w_{ai} from the neighborhood of a point $\vec{R}_{oi}$ is conveniently associated with a scattering subvolume $d^3R_o = dv = v_i(\vec{R}_{oi})$, so that the total available forward scattered power at a receiver is

$$w_a = \sum_{i=1}^{N_v} w_{ai} = \sum_{i=1}^{N_v} v_i w_{vi} \cong \int_V d^3R_o \, w_v(\vec{R}_o, \vec{R}) \text{ watts} \tag{IV.11}$$

where

$$w_{vi} = w_{ai}/v_i = w_v(\vec{R}_o, \vec{R}) \tag{IV.12}$$

is the available power per unit scattering volume for the i^{th} scattering subvolume, feuillet, or layer, and it is assumed that only N_v such contributions to w_a are important.

Each of the power contributions w_{ai} is governed by the bistatic radar equation. Omitting the subscript i, this equation may be written as

$$w_a = \left(\frac{w_t g_t}{4\pi R_o^2}\right) \left(\frac{a_s c_p}{4\pi R^2}\right) \left(\frac{\lambda^2 g_r}{4\pi}\right) \text{ watts ,} \tag{IV.13}$$

where $a_s c_p$ is the effective scattering cross-section of a single scatterer or group of scatterers, including the polarization efficiency c_p of the power transfer from transmitter to receiver. The first set of parentheses in (IV.13) represents the field strength in watts per square kilometer at the point $\vec{R}_o$, the second factor enclosed in parentheses shows what fraction of this field strength is available at the receiver, and $\lambda^2 g_r/(4\pi)$ is the absorbing area of the receiving antenna.

The key to an understanding of scattering from spacecraft, aircraft, rain, hail, snow, refractivity turbulence, or inhomogeneities such as layers or feuillets is the scattering cross-section $a_s c_p$ defined by (IV.13) or the corresponding scattering cross-section per unit volume a_v, defined from (IV.12) and (IV.13) as

$$a_v = (4\pi)^3 (R_o R)^2 w_v/(w_t g_t g_r \lambda^2) \quad \text{per km} \cdot \tag{IV. 14}$$

This quantity is usually estimated by isolating a small volume of scatterers in free space at large vector distances $\vec{R}_o$ and $\vec{R}$, respectively, from the transmitter and receiver. If both antennas are at the same place, (IV. 13) becomes the monostatic radar equation, corresponding to backscatter instead of to forward scatter.

The scattering cross-sections per unit volume for large, medium, and small layers, assuming a density of N_ℓ layers per unit volume, may be obtained by substituting (IV. 10a) to (IV. 10c) in (IV. 14):

For large layers, where $b >> 2x$:

$$a_{v1} = x^4 \psi^{-6} M N_\ell = \lambda^2 \psi^{-6} (R_o R/d)^2 M N_\ell \cdot \tag{IV. 15}$$

For intermediate layers, where $b \cong 2x$:

$$a_{v2} = x^2 \psi^{-4} b^2 M N_\ell = \lambda \psi^{-4} (R_o R/d) b^2 M N_\ell \cdot \tag{IV. 16}$$

For small layers, where $b << 2x$:

$$a_{v3} = \psi^{-4} b^2 M N_\ell = \lambda^0 \psi^{-4} b^2 M N_\ell \cdot \tag{IV. 17}$$

The modern Obukhov-Kolmogorov theory of homogeneous turbulence in a horizontal direction, when extended to apply to the wavenumber spectrum of instantaneous variations of refractive index in a vertical direction, predicts a $\lambda^{-\frac{1}{3}}$ or $f^{\frac{1}{3}}$ law for the variation with wavelength λ or carrier frequency f of either a_v or attenuation relative to free space, or a $\lambda^{5/3}$ or $f^{-5/3}$ law for variations of the transmission loss w_a/w_t. Theoretical studies of multiple scattering by Beckmann [1961a], Bugnolo [1958], Vysokovskii [1957, 1958], and others suggest that single scattering adequately explains observed phenomena. Descriptions of atmospheric turbulence are given by Batchelor [1947, 1953], de Jager [1952], Heisenberg [1948], Kolmogoroff [1941], Merkulov [1957], Norton [1960], Obukhov [1941, 1953], Rice and Herbstreit [1964], Sutton [1955], Taylor [1922], and Wheelon [1957, 1959].

The observed wavelength exponent for the Japanese transmission loss data previously noted was below 5/3 less than two tenths of one percent of the time, and an examination of other data also leads to the conclusion that forward scatter from Obukhov-Kolmogorov turbulence can rarely explain what is observed with frequencies from 40 to 4000 MHz and scattering angles from one to three degrees.

Early recognition of this fact by Norton, Rice, and Vogler [1955] led to the proposal of a mathematical form for the vertical wavenumber spectrum which would achieve agreement between radio data and the theory of forward scatter from refractivity turbulence [Norton, 1960]. Radio data were used to determine the following empirical form for a_v, upon which the predictions of section 9 are based:

$$a_v = \lambda \psi^{-5} M \qquad \text{(IV.18)}$$

$$M = 3 \langle(\Delta n)^2\rangle / (32 \ell_o^2) \qquad \text{(IV.19)}$$

where

$$\Delta n = n - \langle \Delta n \rangle \qquad \text{(IV.20)}$$

is the deviation of refractive index from its expected value $\langle \Delta n \rangle$, and ℓ_o is a "scale of turbulence" [Rice and Herbstreit, 1964].

Values of the variance $\langle(\Delta n)^2\rangle$ of refractivity fluctuations and scales of turbulence ℓ_o obtained from meteorological data lead to good agreement between (IV.18) and radio data when an exponential dependence of M on height is assumed, substituting the corresponding value of w_v in (IV.11). It is not yet clear how the estimates of m, m_o, z_o, b, and N_ℓ required by the theory of forward scatter from layers of a given type can be obtained from direct meteorological measurements, nor how these parameters will vary throughout the large volume of space visible to both antennas over a long scatter path. It does seem clear that this needs to be done.

Data from elevated narrow-beam antennas that avoid some of the complex phenomena due to reflection and diffraction by terrain, and which select small scattering volumes, suggest that for scattering angles exceeding ten degrees, reflections from large layers can hardly be dominant over reflection from intermediate and small layers or from refractivity turbulence. Preliminary results indicate that field strengths decrease more slowly at a fixed distance and with scattering angles θ increasing up to fifteen degrees than would be possible with the θ^{-6} dependence of a_v given by (IV.15) added to a probable exponential decay with height of the expected value of the meteorological parameter $M N_\ell$ for large layers.

The wavelength and angle dependence of forward scatter characterized by the Obukhov-Kolmogorov turbulence theory is nearly the same as that for small layers, given by (IV.17). For scattering from refractivity turbulence:

$$a_{vo} = \lambda^{-1/3} \psi^{-11/3} M_o \qquad \text{(IV.21)}$$

$$M_o = \frac{\Gamma(11/6) \langle (\Delta n)^2 \rangle}{4(2\pi)^{13/3} \Gamma(1/3) \ell_o^{2/3}} \qquad \text{(IV.22)}$$

Although most of the propagation paths which have been studied rarely show this frequency dependence, some occasionally do agree with (IV.21). In general, the radiowave scattering cross-section per unit volume a_v is a weighted average of scattering from all kinds of layers or feuillets and the turbulence between them.

Summarizing the argument:

$$a_v = a_{vo} + a_{v1} + a_{v2} + a_{v3} \cong \lambda \psi^{-5} M \qquad \text{(IV.23)}$$

for $10^{-4} < \lambda < 10^{-2}$ km, $0.01 < \psi < 0.03$ radians, where M has been determined from radio data, subject to the assumption that M decreases exponentially with height above the earth's surface. Equation (IV.23) is intended to indicate the present state of the twin arts of formulating theories of tropospheric forward scatter and comparing these theories with available long-term median transmission loss data. A great deal of available data is not forward scatter data, and it is for this reason that estimates of long-term variability as given in section 10 and annex III are almost entirely empirical.

Also, for this reason, estimates of L_{gp} as given in section 9 are restricted to long-term median forward scatter transmission loss. Available measurements of differences in path antenna gain agree within the limits of experimental error with the values predicted by the method of section 9 whenever the dominant propagation mechanism is forward scatter.

GEOMETRY FOR FORWARD SCATTER

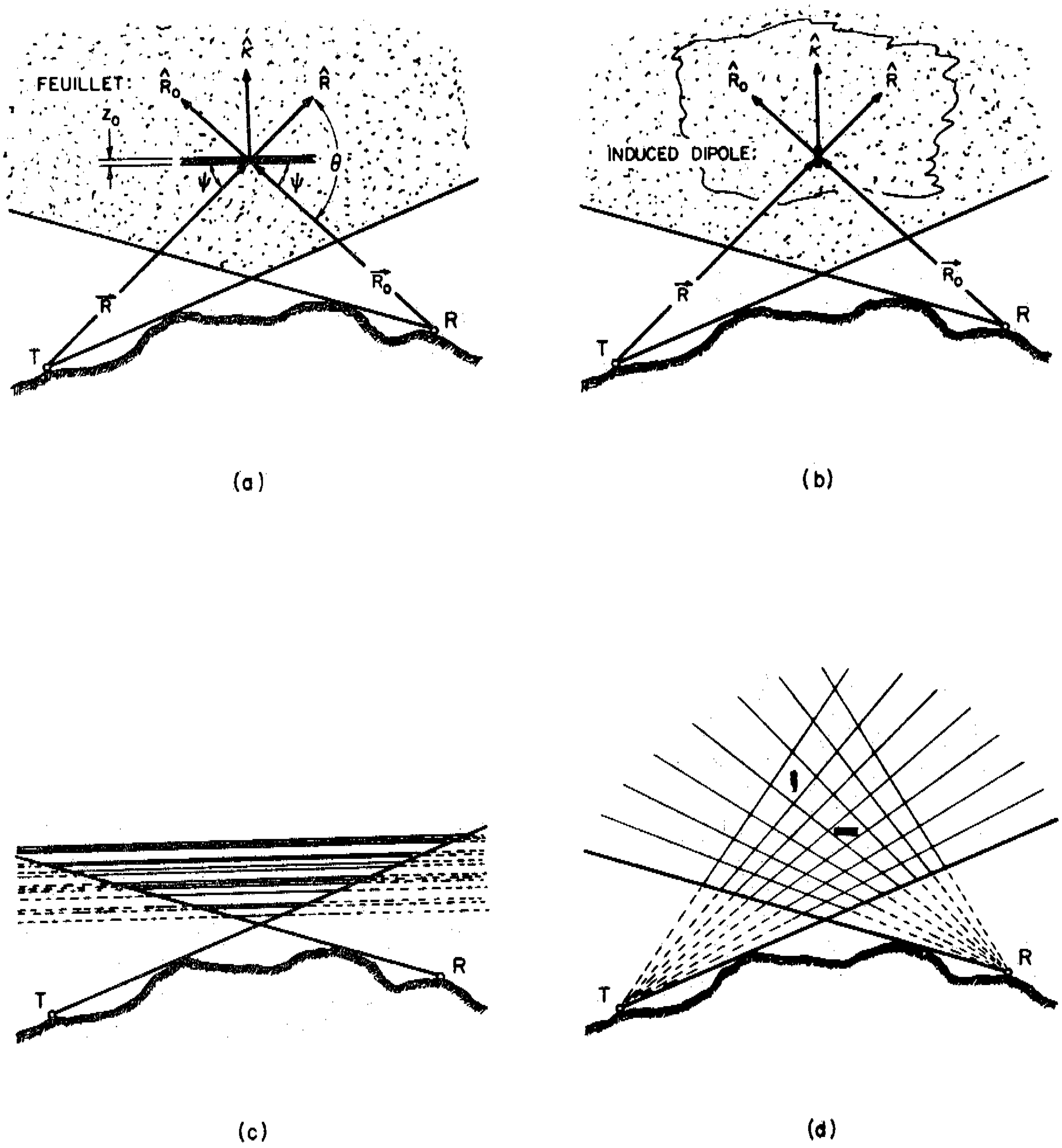

Figure IV.1

IV.3 List of Special Symbols Used in Annex IV

a_s	Radiowave scattering cross-section of a single scatterer or group of scatterers, (IV.13).
a_v	Radiowave scattering cross-section per unit volume, (IV.14).
a_{vo}	Radiowave scattering cross-section from refractivity turbulence, (IV.21).
a_{v1}, a_{v2}, a_{v3}	Radiowave scattering cross-sections per unit volume for large, medium, and small layers, (IV.15) to (IV.17).
b	The dimensions of an atmospheric layer or feuillet in any direction perpendicular to $\hat{\kappa}$, (IV.9).
c_p	Polarization efficiency of the power transfer from transmitter to receiver, (IV.13).
C(u), C(v)	Fresnel cosine integrals, (IV.8).
ℓ	A range of eddy sizes or layers; the radio wave scattered forward is most affected by a particular range of "eddy sizes," ℓ, or by layers of an average thickness $\ell/2$, that are visible to both antennas, (IV.1).
ℓ_o	Scale of turbulence, (IV.19).
m	Average refractive index gradient, dn/dz, across a layer, (IV.5).
m_o	Average refractive index gradient for the region in which a layer is imbedded, (IV.5).
M	A term defined by (IV.6) used in the power reflection coefficient q^2.
M_o	A term defined by (IV.22) used in defining a_{vo}, the scattering cross-section from refractivity turbulence.
n_1, n_2	Refractive indices of adjacent layers of homogeneous media, (IV.3).
N_ℓ	The number of layers per unit volume of a scattering cross-section, (IV.15) to (IV.17).
N_v	The number of scattering subvolumes that make an appreciable contribution to the total available power, (IV.11).
q	The power reflection coefficient, q^2, for a tropospheric layer is approximated by (IV.6).
q_o	The plane wave Fresnel reflection coefficient for an infinitely extended plane boundary, (IV.3).
$\vec{R}, \vec{R}_o$	Vector distances from transmitter and receiver, respectively, to a point $\vec{R}_o$.
$\hat{R}, \hat{R}_o$	Unit vectors from the centers of radiation of the receiving and transmitting antennas, respectively, (IV.1).
$\vec{R}_{oi}$	A point from which power is coherently scattered or reflected, (IV.11).
S(u), S(v)	Fresnel sine integrals, (IV.8).
u	A parameter defined by (IV.9).
v	A parameter defined by (IV.9).
v_i	The i^{th} scattering subvolume, (IV.11).

w_v	Available power per unit scattering volume, (IV.11).
w_{vi}	Available power per unit scattering volume for the i^{th} scattering subvolume, (IV.12)
x	Half the width of a first Fresnel zone, (IV.7).
z	Thickness of a tropospheric layer, (IV.6).
z_o	The thickness of a tropospheric layer, (IV.6).
Δn	The deviation of refractive index from its expected value, (IV.20).
$<\Delta n>$	The expected value of refractive index, (IV.20).
$<(\Delta n)^2>$	The variance of fluctuations in refractive index, (IV.19).
$\hat{\kappa}$	A wave number direction defined by (IV.1).
ψ	The grazing angle for reflection from a layer, (IV.2).