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STATISTICAL-PHYSICAL MODELS OF MAN-MADE AND NATURAL RADIO NOISE

PART II: FIRST ORDER PROBABILITY MODELS OF THE ENVELOPE AND PHASE

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PREFACE

This Report is the second (i.e. Part II) in a series of ongoing studies [Middleton, 1974] of the general electromagnetic (EM) interference environment arising from man-made and natural EM noise sources, and is also part of the continuing analytical and experimental effort whose general aims are [Spaulding and Middleton, 1975]:

(1). to provide quantitative, statistical descriptions of man-made and natural electromagnetic interference (as in this series);

(2). to indicate and to guide experiment, not only to obtain pertinent data for urban and other EM environments, but also to generate standard procedures and techniques for assessing such environments;

(3). to determine and predict system performance in these general electromagnetic milieux, and to obtain and evaluate optimal system structures therein, for

(a). the general purposes of spectrum management;

(b). the establishment of appropriate data bases thereto; and

(c). the analysis and evaluation of large-scale telecommunication systems.

With the aid of (1) and (2) one can predict the interference characteristics of selected regions of the electromagnetic spectrum, and with the results of (3), rational criteria of performance can be developed to predict the successful or unsuccessful operation of telecommunication links and systems in various classes of interference. Thus, the combination of the results of (1)-(3) provide specific, quantitative procedures for spectral management, and a reliable technical base for the choice and implementation of policy decisions thereto.

The man-made EM environment, and most natural EM noise sources as well, are basically "impulsive", in the sense that the emitted waveforms have a highly structured character, with significant probabilities of large

interference levels. This is noticeably different from the usual normal (gaussian) noise processes inherent in transmitting and receiving elements. This highly structured character of the interference can drastically degrade performance of conventional systems, which are optimized, i.e. designed to operate most effectively, against the customarily assumed normal background noise processes. The present Report is devoted to the problems of (1), (2) above, namely, to provide adequate statistical physical models, verified by experiment, of these general "impulsive", highly non-gaussian interference processes, which constitute a principal corpus of the interference environment, and which are required in the successful pursuit of (3), as well. The principal new results here are:

(i). Canonical analytical models, experimentally corroberated, for the first-order statistics of the envelope and phase of Class A and Class B noise*;

(ii). Procedures for estimating the (canonical) model parameters, calculation of moments, APD's (= exceedance probabilities, $P_1(\mathcal{E}>\mathcal{E}_0)$, etc.) and probability density functions (pdf's), and a variety of other pertinent statistics; [see the Table of Contents].

Finally, we emphasize, again, that it is the quantitative interplay between the experimentally established, analytical model-building for the electromagnetic environment, and the evaluation of system performance therein, which provides essential tools for prediction and performance, for the development of adequate, appropriate data bases, procedures for effective standardizations, and spectrum assessment, required for the effective management of the spectral-use environment.

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* Class A and Class B noise are distinguished, qualitatively, by having input bandwidths which are respectively narrower and broader than that of the (linear) front-end stages of the typical (narrow-band) receiver in use. More precise definitions are developed in the text following.

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