[We shall reserve the analysis for Class C interference and gauss noise, e.g. based here on

$$\hat{F}_1(ir)_{C+G} = \exp\,[-\sigma_G^2 r^2/2 + A_{\infty,A}\left\langle \int_0^{z_o} [J_o(r\hat{B}_{oA})-1]dz \right\rangle_{z_o,\underset{\sim}{\lambda},\underset{\sim}{\theta}'}$$

$$+A_{\infty,B}\int_0^{\infty}\left\langle [J_o(r\hat{B}_{oB})-1]\right\rangle_{\underset{\sim}{\lambda},\underset{\sim}{\theta}'} dz] \quad , \tag{2.52}$$

to a subsequent Report.]

2.4 Large Impulsive Indexes:

When the impulsive index, A_∞, is large, we expect asymptotically gaussian statistics for the instantaneous amplitude X [Secs.3,p.26; 5,p. 39, Middleton, 1974], and rayleigh statistics here, cf. (2.48), for the instantaneous envelope E. This latter is easily shown by developing $\hat{I}_\infty(r)$, (2.41) or (2.52), as a power series in r about r = 0, in the usual way.* Thus, the c.f. (2.52) for our general Class C case, with gaussian background noise in addition, becomes

$$\hat{F}_1(ir)_{C+G} = \exp\left\{-\frac{r^2}{2}\,\sigma_0^2\right\}\cdot\exp\left\{\sum_{n=2}^{\infty}\frac{(-1)^n r^{2n}}{2^{2n}(n!)^2}\,(A_{\infty,A}b_{2n}^{(A)}+A_{\infty,B}b_{2n}^{(B)})\right\}, \tag{2.53}$$

where

$$\left.\begin{aligned}
\sigma_0^2 &= (\sigma_R^2+\sigma_E^2)+\bar{\nu}_\infty\bar{T}_{sA}\left\langle\int_0^{z_o}\hat{B}_{oA}^2 dz\right\rangle_{z_o,\underset{\sim}{\lambda},\underset{\sim}{\theta}'}+\bar{\nu}_\infty\bar{T}_{sB}\int_0^{\infty}\left\langle\hat{B}_{oB}^2\right\rangle_{\underset{\sim}{\lambda},\underset{\sim}{\theta}'}dz, & (2.53a)\\
b_{2n}^{(A)} &= \left\langle\int_0^{z_o}\hat{B}_{oA}^{2n}dz\right\rangle_{z_o,\underset{\sim}{\lambda},\underset{\sim}{\theta}'} \quad , \quad b_{2n}^{(B)} = \int_0^{\infty}\left\langle\hat{B}_{oB}^{2n}\right\rangle_{\underset{\sim}{\lambda},\underset{\sim}{\theta}'}dz \quad . & (2.53b)\,(2.53c)
\end{aligned}\right\}$$

---

* Provided we consider for the moment <u>finite</u> observation intervals $T(<\infty)$, i.e. finite upper limits on the z-integrals in (2.51), (2.52), so that these integrals are uniformly convergent, <u>proper</u> integrals, permitting a series expansion of their integrands. Then, we ultimately have $\hat{F}_1(ir)_{C+G}$: $\lim_{T\to\infty} F_1(ir|T)_{C+G}$, where $(\lim_{T\to\infty})$ is invoked for each term of the resulting expansions. See the comments in B, Sec. (5.2) below.

Next, expanding the second exponential gives us

$$\hat{F}_1(ir)_{C+G} \doteq e^{-r^2\sigma_0^2/2}\left\{1+\frac{r^4}{2^4(2!)^2}B_4+\left[-\frac{r^6}{2^6(3!)^2}B_6+\frac{r^8}{2^8(4!)^2}18B_4^2\right]+\dots\right\}, \tag{2.54}$$

where now we write

$$B_{2n} \equiv A_{\infty,A}b_{2n}^{(A)} = A_{\infty,B}b_{2n}^{(B)} , \tag{2.54a}$$

cf. (2.53b,c).

From the leading term of (2.54) applied to (2.21), (2.22) we see that the result is indeed the expected rayleigh form, cf. (2.48a,b). Using the Hankel exponential integral relation, for example (A.1-49), [Middleton (1960)], viz.

$$\int_0^\infty J_\nu(az)z^{\mu-1}e^{-b^2z^2}dz = \frac{\Gamma(\frac{\nu+\mu}{2})}{2b^\mu\Gamma(\gamma+1)}\left(\frac{a}{2b}\right)^\nu {}_1F_1\left(\frac{\nu+\mu}{2};\nu+1;-\frac{a^2}{4b^2}\right), \tag{2.55}$$

$$\mathrm{Re}(\mu+\nu) > 0; \quad |\arg b| < \pi/4,$$

we obtain the Edgeworth "correction" terms to $W_1(E)_{C+G}$, $P_1(E > E_0)_{C+G}$, e.g. terms $O(A_\infty^{-1}, A_\infty^{-2}$, etc.) vis-à-vis the leading, rayleigh term. The first-order p.d. W_1 of the envelope and the APD, P_1, with correction terms, are found to be explicitly:

$$\boxed{\begin{aligned} W_1(E)_{C+G} \simeq \frac{Ee^{-E^2 2\sigma_0^2}}{\sigma_0^2}\Bigg\{1+&\frac{(B_4/\sigma_0^4)}{2^2 2!}\,{}_1F_1(-2;1;E_0^2/2\sigma_0^2) \\ &+\left[-\frac{(B_6/\sigma_0^6)}{2^3 3!}\,{}_1F_1(-3;1;E_0^2/2\sigma_0^2)\right. \\ &+\left.\left[\frac{18B_4^2}{2^4 4!\sigma_0^8}\right]{}_1F_1(-4;1;E_0^2/2\sigma_0^2)\right] + \dots\Bigg\}, \end{aligned}} \tag{2.56}$$

where

$$ {}_1F_1(-2;1;x^2) = 1-2x^2+x^4/4;\ {}_1F_1(-3;1;x^2) = 1-3x^2 + \frac{3}{2}x^4 - \frac{x^6}{6}; $$

$$ {}_1F_1(-4;1;x^2) = 1-4x^2 + 6x^4 - \frac{2}{3}x^6 + \frac{x^8}{24}\ , \text{ etc.,} \tag{2.56a} $$

with $x^2 = E^2/2\sigma_0^2$ here.

Similarly, we get for $P_1(E > E_o)_{C+G}$, (2.22b), here

$$ \boxed{\begin{aligned} P_1(E>E_o)_{C+G} \cong 1 - \left(\frac{E_o^2}{2\sigma_o^2}\right) e^{-E_o^2/2\sigma_o^2} \Bigg\{ & {}_1F_1(1;2;E_o^2/2\sigma_o^2) \\ & + \frac{(B_4/\sigma_o^4)}{2^2 3!}\, {}_1F_1(-1;2;E_o^2/2\sigma_o^2) \\ & + \Bigg[- \frac{(B_6/\sigma_o^6)}{2^3 3!}\, {}_1F_1(-2;2;E_o^2/2\sigma_o^2) \\ & + \frac{(18B_4^2)\sigma_o^{-8}}{2^4 4!}\ {}_1F_1(-3;2;E_o^2/2\sigma_o^2)\Bigg] + \ldots \Bigg\}, \end{aligned}} \tag{2.57} $$

with

$$ {}_1F_1(1;2;x^2) = (e^{x^2}-1)/x^2\ ;\quad \text{[Eq. (A.1.19b), Middleton (1960)]} $$

$$ \begin{aligned} {}_1F_1(-1;2;x^2) &= 1 - x^2/2\ ; \\ {}_1F_1(-2;2;x^2) &= 1 - x^2 + x^4/6\ ; \\ {}_1F_1(-3;2;x^2) &= 1 - 3x^2/2 + x^4/2 - x^6/24, \text{ etc.} \end{aligned} \tag{2.57a} $$

The first two terms of (2.57) reduce to

$$ P_1(E>E_o)_{C+G} \simeq e^{-E_o^2/2\sigma_o^2}\ , \tag{2.57b} $$

as expected, for this cumulative rayleigh P.D., cf. (2.48b).

Finally, the above results apply also for the purely Class A or Class B interference, whenever $\bar{\nu}_\infty$ becomes very large (i.e. the impulsive index is large). The variance σ_0^2, (2.53a), is then suitably modified, as is (2.54a) for the correction terms. Since B_{2n} is $O(A_\infty)$, while σ_0^2 is also $O(A_\infty)$, it is clear that the correction coefficients $B_4(\sigma_0^4$, $[B_6/\sigma_0^6$, $(B_4^2)\sigma_0^{-8}]$ are $O(A_\infty^{-1}, A_\infty^{-2})$, etc., showing the rate of fall-off of the correction terms with increasing index A_∞.

<u>2.5 Second Reduction of the c.f. $\hat{F}_1$: The Rôle of Source Distribution and Propagation Law:</u>

Our major problem, as stated earlier in Part I, is to obtain <u>analytically</u> tractable results, as well as a pertinent physical foundation for our models of man-made (and natural) electromagnetic interference. Technically, our central problem now is to evaluate the probability densities and cumulative probabilities (2.21), (2.22), when the interference is Class A or Class B, accompanied by gaussian noise, with the respective characteristic functions (2.50), (2.51). [The detailed study of Class C interference, with the more general c.f. (2.52), is reserved to Part IV of this series of Reports.]

The desired evaluation may now be achieved by recalling (as in Section 3 of Part I [Middleton, 1974]) that the general character of the p.d. (and hence of the P.D.) of a random variable at large values is controlled principally by the behavior of the associated characteristic function at and near zero values of its argument. Thus, the behaviour of $\hat{F}_1(ir)$ at, <u>and in the vicinity of</u>, r=0, is determined by the <u>largest r-dependent contribution</u> which establishes the large-amplitude structure of $W_1(E)$, $P_1(E)$, etc., i.e. as $E \to \infty$. In fact, for these general classes of non-gaussian noise this corresponds to the expected <u>slower</u> fall-off of $W_1(E)$, as $E \to \infty$, than the rayleigh p.d. (2.48a), for example here. [See also the discussion in Section (2.7)A following.]

Our preliminary procedure for obtaining the required development of the c.f. $\hat{F}_1$ in the neighborhood of r=0 consists of: (i), expressing J_0-1 as an integral; (ii), using an explicit class of propagation law and source distribution; (iii), reversing the order of integration in (i), (ii)