as expected, for this cumulative rayleigh P.D., cf. (2.48b).

Finally, the above results apply also for the purely Class A or Class B interference, whenever $\bar{\nu}_\infty$ becomes very large (i.e. the impulsive index is large). The variance σ_0^2, (2.53a), is then suitably modified, as is (2.54a) for the correction terms. Since B_{2n} is $O(A_\infty)$, while σ_0^2 is also $O(A_\infty)$, it is clear that the correction coefficients $B_4(\sigma_0^4, [B_6/\sigma_0^6, (B_4^2)\sigma_0^{-8}]$ are $O(A_\infty^{-1}, A_\infty^{-2})$, etc., showing the rate of fall-off of the correction terms with increasing index A_∞.

2.5 Second Reduction of the c.f. $\hat{F}_1$: The Rôle of Source Distribution and Propagation Law:

Our major problem, as stated earlier in Part I, is to obtain analytically tractable results, as well as a pertinent physical foundation for our models of man-made (and natural) electromagnetic interference. Technically, our central problem now is to evaluate the probability densities and cumulative probabilities (2.21), (2.22), when the interference is Class A or Class B, accompanied by gaussian noise, with the respective characteristic functions (2.50), (2.51). [The detailed study of Class C interference, with the more general c.f. (2.52), is reserved to Part IV of this series of Reports.]

The desired evaluation may now be achieved by recalling (as in Section 3 of Part I [Middleton, 1974]) that the general character of the p.d. (and hence of the P.D.) of a random variable at large values is controlled principally by the behavior of the associated characteristic function at and near zero values of its argument. Thus, the behaviour of $\hat{F}_1(ir)$ at, and in the vicinity of, r=0, is determined by the largest r-dependent contribution which establishes the large-amplitude structure of $W_1(E)$, $P_1(E)$, etc., i.e. as $E \to \infty$. In fact, for these general classes of non-gaussian noise this corresponds to the expected slower fall-off of $W_1(E)$, as $E \to \infty$, than the rayleigh p.d. (2.48a), for example here. [See also the discussion in Section (2.7)A following.]

Our preliminary procedure for obtaining the required development of the c.f. $\hat{F}_1$ in the neighborhood of r=0 consists of: (i), expressing J_0-1 as an integral; (ii), using an explicit class of propagation law and source distribution; (iii), reversing the order of integration in (i), (ii) and observing the bounds imposed by the fact that u_{oA} is of finite duration, while $u_{oB}(z) \neq 0$, $0<z<\infty$, cf. Fig. (2.1) above.* As we shall see below, it is this latter condition (on u_{oA} vs. u_{oB}) which critically affects the explicit form of the needed development of F_1.

We begin with the identity $-J_o'(x) = J_1(x)$, from which it follows by integration that

$$J_o(y) - 1 = -\int_0^y J_1(x)dx. \tag{2.58}$$

Then, the exponents of the c.f.'s $\hat{F}_{1A,B}$ are (without the contributions, for the moment, of the background normal noise) but with the help of (2.58),

$$\boxed{-A_{\infty,A,B}\left\langle \int_0^{(z_0,\text{or}\infty)} dz \int_0^{x=r\hat{B}_{oA,B}} J_1(x)dx \right\rangle_{z_0,\underset{\sim}{\lambda},\underset{\sim}{\theta}'} = \hat{I}_\infty(r)_{A,B}.} \tag{2.59}$$

2.5.1 Propagation Law:

We now introduce the somewhat restrictive condition that the source distribution and propagation law are expressible in the factored form: $a(\lambda)[b(\phi)$ or $b(\theta,\phi)]$. The beam patterns are always independent of distance ($c\lambda$), e.g.

$$|\mathcal{Q}_{RT}(\underset{\sim}{\lambda};f_0')| = |\mathcal{Q}_{RT}(\phi;f_o')|_{\text{plane}};\quad |\mathcal{Q}_{RT}(\theta,\phi;f_o')|_{\text{volume}}, \tag{2.60a}$$

e.g.

$$|\mathcal{Q}_{RT}(\underset{\sim}{\lambda};f_o')| = |\mathcal{Q}_{RT}[(\hat{\underset{\sim}{i}}_T,\hat{\underset{\sim}{i}}_R)f_o'/c]|\ ,\ (\hat{\underset{\sim}{i}}_R=-\hat{\underset{\sim}{i}}_T,\text{cf. Fig. (2.1),}$$
Middleton [1974]), (2.60b)

where specifically

---

* Our procedure here is a generalization of that used by Giordano [1970], who, however, considered what in the limit ($T\to\infty$) is ultimately only Class B interference, and only special choices of source distributions and propagation law.

$$\hat{i}_R = \hat{i}_x \cos \phi_R \sin \theta_R + \hat{i}_y \sin \phi_R \cos \theta_R - \hat{i}_z \cos \theta_R \tag{2.60c}$$

in which ϕ_T is an azimuthal angle and θ_T a polar angle, as sketched in Fig.(2.2)II. Thus, for the propagation law, $g(\underset{\sim}{\lambda})$ in (2.41b) we write

$$g(\underset{\sim}{\lambda}) = [g_S(\phi),\ g_V(\theta,\phi)]/(4\pi c\lambda)^{\gamma}\ , \quad 0 < \gamma \tag{2.61}$$

where $g_{S,V}$ are angular factors, usually taken to be unity in the common propagation models. In general, $\gamma > 0$, and, in fact, $\gamma \geq 1/2$: $\gamma = 1/2$ corresponds to the "wave guide" modes often encountered in long-distance propagation in the atmosphere, while $\gamma = 1$ applies for the usual spherical spreading of less distant sources. For practical applications, sources and receiver in a common plane, Fig.(2a)II, is typical of most mobile land transport communication environments, while the "volume" situation of Fig. (2b)II is characteristic of ground/air, or ground/satellite, or air/air environments. Also, for practical purposes, atmospheric noise may often be regarded as essentially coplanar with the surface (and $\gamma \doteq 1/2$), unless the principal discrete sources are comparatively near to the receiver, i.e. $\theta_R(=\theta_T)$ is large, e.g. $[>O(5\text{-}10°)]$.

2.5.2 Source Distributions:

For the moment we continue to assume that the source distributions are factorable into the form $\sigma \equiv a(\lambda)b(\theta,\phi)$, cf. remarks at the beginning of Sec. (2.5.1) above. Then, the density $w_1(\lambda)$ required in the averages $\langle\ \rangle_\lambda$ in (2.59) is now from (2.26), (2.28)

$$w_1(\underset{\sim}{\lambda}) = \begin{bmatrix} \sigma_S(\lambda)c^2\lambda\sigma_S(\phi) \\ \\ \sigma_V(\lambda)c^3\lambda^2\sin\theta\ \sigma_V(\theta,\phi) \end{bmatrix} A_{S,V}^{-1}\ , \tag{2.62}$$

for the surface and volume régimes, where the normalizing factors $A_{S,V}$ are given by

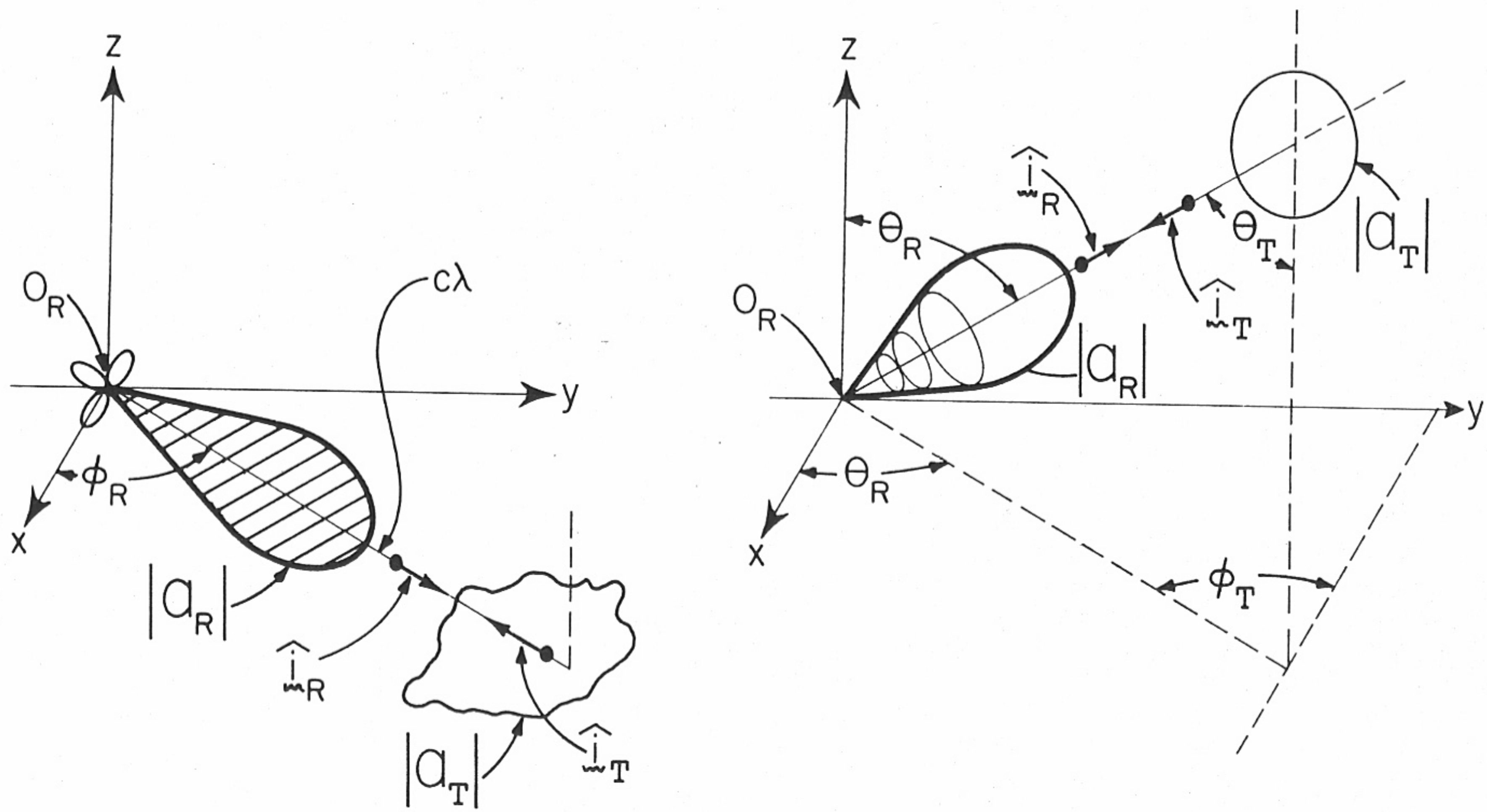

Figure 2.2 (II). Geometries of source and receiver beam patterns: (a), in a plane; (b), in a volume.

$$\begin{cases} A_S = \int_{\Lambda_{S\text{-eff}}} \sigma_S(\lambda)\sigma_S(\phi)c^2\lambda d\lambda d\phi = \int_{[\phi]\text{-eff}} d\phi\sigma_S(\phi)\int_{[\lambda]\text{-eff}} c^2\lambda\sigma_S(\lambda)d\lambda & (2.62a) \\ A_V = \int_{\Lambda_{V\text{-eff}}} \sigma_V(\lambda)\sigma_S(\theta,\phi)c^3\lambda^2\sin\theta\, d\lambda d\theta d\phi & \end{cases}$$

$$= \int_{(\theta,\phi)\text{-eff}} \sigma_V(\theta,\phi)\sin\theta\, d\theta d\phi \int_{[\lambda]\text{-eff}} c^3\lambda^2\sigma_V(\lambda)d\lambda. \tag{2.62b}$$

The Λ_{eff} are the effective domains of the possible interfering sources, namely, those capable of registering at our receiver [O_R, cf. Fig. (2.2)II], i.e. observeable in the receiver background noise. The receiver accordingly has a limiting range $c\lambda_{max}$, which depends on $e_{0\gamma}$, cf. (2.34), e.g. $\lambda_{max} = \lambda_{max}(e_{0_\gamma})$. Several cases are distinguished, as shown in Fig.(2.3)II, [as far as dependence on λ is concerned]: From Fig.(2.3)II it is clear that the source domain to be used for $\Lambda_{(SorV)}$ is $\Lambda_{(SorV)\text{-eff}}$: the domain of sources perceivable by the receiver. This is determined by either $\lambda_{eff} = \lambda_{max}$ or λ_Λ [Cases I, II)], whichever is the lesser, or by a pair of λ's, e.g. $\lambda_0 \leq \lambda \leq \lambda_{max}$, Case III, for such distributions. [Case IV, not shown in Fig.(2.3)II, is a combination of regions ($\pi\lambda_{max}^2$ and Λ) which partially overlap. Here we must consider the overlapping and pertinent nonoverlapping regions separately, which will involve the angles ϕ, or (θ,ϕ) explicitly.] In our present applications, however, we shall assume Case I, e.g. $\lambda_{eff} = \lambda_{max} < \lambda_\Lambda$, which is by far the more prevelant situation in practice: the potential source domain always exceeds that of the receiver's acceptance region.

Finally, we shall, where necessary, postulate the following range dependence of source density:

$$\sigma_{S,V}(\lambda) = 1/\lambda^\mu \quad , \qquad 0 < \mu \quad , \tag{2.63}$$

for both the volume and surface situations, in accordance with our remarks at the beginning of Sec. 2.5.1.

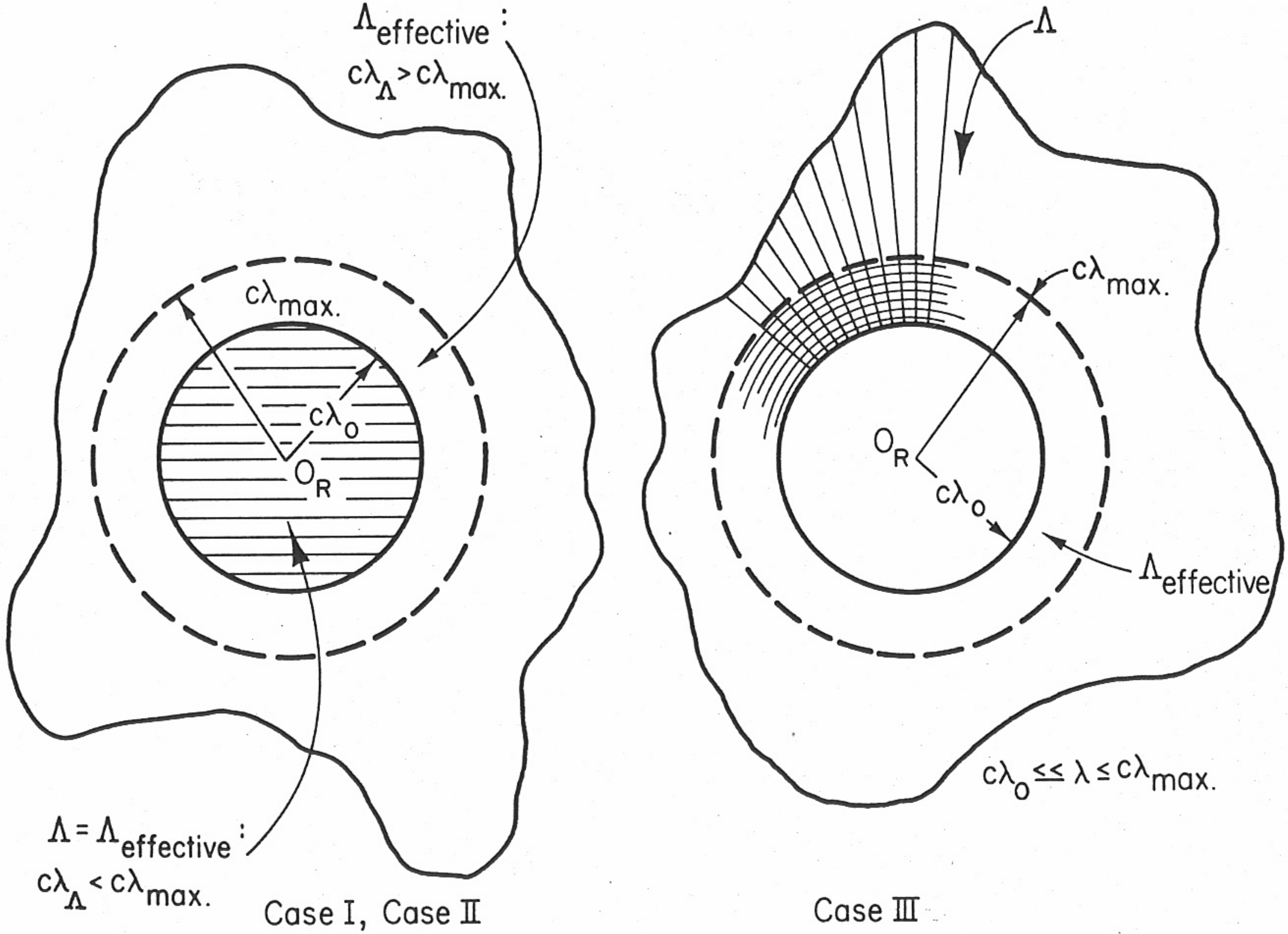

Figure 2.3 (II). Several typical cases (λ only) of source and receiver domains.

2.5.3: Rôle of Input Signal Duration, T_S

We begin by developing in fuller detail the structure of the basic received envelope $\hat{B}_{oA,B}$, cf. (2.41b). Using (2.60), (2.61) in (2.41b) allows us to write

$$\hat{B}_{oA,B} = e_{o\gamma}^{(A,B)} A_o u_{oA,B}(z) | \mathcal{A}_{RT}(\phi;\text{or } \theta,\phi | f_o') | g_{S,V}(\phi;\text{or } \theta,\phi)(4\pi c)^{-\gamma}/\lambda^{\gamma} \tag{2.64a}$$

$$= G_{oA,B}(z,A_o,e_{o\gamma},|\mathcal{A}_{RT}|;\ \phi,\text{or } \theta,\phi)/\lambda^{\gamma}, \tag{2.64b}$$

with

$$G_{oA,B} \equiv e_{o\gamma}^{(A,B)} A_o u_{oA,B} | \mathcal{A}_{RT} | g_{S,V} (4\pi c)^{-\gamma} \tag{2.64c}$$

containing the (possibly) random parameters A_o, $e_{o\gamma}$, $\mathcal{A}_{RT}$, for both surface and volume régimes and Class A and Class B interference. Next, we use (2.62), (2.63) to write (2.59) explicitly as

$$\hat{I}_{\infty}(r)_{A,B} = -A_{\infty}\Big|_{A,B} \left\langle \int_0^{z_o,\infty} dz \int_{\Delta(\theta,\phi)} A_{S,V}^{-1} \sigma_{S,V} d(\theta,\phi) \int_{[\lambda]} d\lambda \begin{bmatrix} c^2/\lambda^{\mu-1} \\ \\ c^3/\lambda^{\mu-2} \end{bmatrix} \right.$$

$$\left. \cdot \int_0^{x=rG_o/\lambda^{\gamma}} J_1(x)dx \right\rangle_{\underset{\sim}{\theta}} , \tag{2.65}$$

with $\underset{\sim}{\theta} = z_o, A_o$, etc., where the upper term applies for surface sources and the lower for those distributed in the volume.

Next, we implement the key step, (iii), in order to interchange the order of integrations over λ and x in (2.65). This permits us to develop $\hat{I}_{\infty}$ explicitly as a function in r, to which we can then apply the approach indicated at the beginning of Section 2.5, to obtain the controlling term(s) at and near $r = 0$ for the characteristic function. Since from (2.59)

$$x = rG_o/\lambda^{\gamma};\ \therefore \lambda = (r,G_o)^{1/\gamma} x^{-1/\gamma}\ ;\ \text{and } \therefore x_o = rG_o/\lambda_{max}^{\gamma} \tag{2.66}$$

is the value of x corresponding to λ_{max}, which establishes the domain of sources perceivable by the receiver [in the present Case I, cf.Fig.(2.3)II]. Now we use the fact that u_{oA} is nonzero only for $(0 \leq z \leq z_o)$, while $u_{oB} \neq 0$, $(0 \leq z < \infty)$. Since $G_{oA,B} \sim u_{oA,B}$, cf. (2.64c), we see at once that for

$$\text{\underline{Class A:}} \quad \begin{cases} 0 \leq x \leq x_o, \text{ since } u_{oA} = 0,\ z > z_o; \\ \\ \lambda_{max} \geq \lambda \geq 0, \text{ cf. Sec. (2.5.2).} \end{cases} \tag{2.67a}$$

$$\text{\underline{Class B:}} \quad \begin{cases} 0 \leq x < \infty, \text{ since } u_{oB} \neq 0,\ z < \infty\ (u_{oB} \to 0,\ z \to \infty); \\ \\ \lambda_{max} \geq \lambda \geq 0, \text{ cf. Sec. (2.5.2).} \end{cases} \tag{2.67b}$$

Fig. (2.4)II shows the allowed domains of x and λ for these two classes of interference. [For Class C interference, we use (2.42), with the c.f.'s of Class A and Class B determined separately, with the help of (2.67) and the results of Section 2.6, 2.7 below. The details are reserved for a subsequent Report.] In Sections (2.6), (2.7) following we obtain the desired c.f.'s (= $\exp(\hat{I}_\infty)$) for these two basic classes of interference, with the help of (2.67) in (2.65) and the observations presented at the beginning of Sec. 2.5.

2.6 The C.F. for Class A Interference:

With Case I conditions (cf. Fig. (2.3)II) on the source distribution vis-à-vis the receiver range (λ_{max}), and (2.67a) applicable here, we see that $x \to x_o$ for the upper limit on the integrand (for x) in (2.65). Accordingly, since $A_{S,V}$ is now precisely equivalent to the indicated integration over (λ,θ,ϕ) therein, we see that (2.65) becomes at once

$$\hat{I}_\infty(r)_A = -A_{\infty,A}\left\langle \int_0^{z_o} dz \int_0^{x_o} J_1(x)dx \right\rangle_{z_o,\underset{\sim}{\theta}} \quad ; \quad x_o = rG_o/\lambda_{max}^{\gamma} = r\hat{B}_{oA} \ . \tag{2.68}$$

Next we use (2.58) to reëxpress (2.68) as