sources perceivable by the receiver [in the present Case I, cf.Fig.(2.3)II]. Now we use the fact that u_{oA} is nonzero only for $(0 \leq z \leq z_o)$, while $u_{oB} \neq 0$, $(0 \leq z < \infty)$. Since $G_{oA,B} \sim u_{oA,B}$, cf. (2.64c), we see at once that for

$$\underline{\text{Class A:}} \quad \begin{cases} 0 \leq x \leq x_o, \text{ since } u_{oA} = 0,\ z > z_o; \\ \lambda_{max} \geq \lambda \geq 0, \text{ cf. Sec. (2.5.2).} \end{cases} \tag{2.67a}$$

$$\underline{\text{Class B:}} \quad \begin{cases} 0 \leq x < \infty, \text{ since } u_{oB} \neq 0,\ z < \infty\ (u_{oB} \to 0,\ z \to \infty); \\ \lambda_{max} \geq \lambda \geq 0, \text{ cf. Sec. (2.5.2).} \end{cases} \tag{2.67b}$$

Fig. (2.4)II shows the allowed domains of x and λ for these two classes of interference. [For Class C interference, we use (2.42), with the c.f.'s of Class A and Class B determined separately, with the help of (2.67) and the results of Section 2.6, 2.7 below. The details are reserved for a subsequent Report.] In Sections (2.6), (2.7) following we obtain the desired c.f.'s (= $\exp(\hat{I}_\infty)$) for these two basic classes of interference, with the help of (2.67) in (2.65) and the observations presented at the beginning of Sec. 2.5.

2.6 The C.F. for Class A Interference:

With Case I conditions (cf. Fig. (2.3)II) on the source distribution vis-à-vis the receiver range (λ_{max}), and (2.67a) applicable here, we see that $x \to x_o$ for the upper limit on the integrand (for x) in (2.65). Accordingly, since $A_{S,V}$ is now precisely equivalent to the indicated integration over (λ,θ,ϕ) therein, we see that (2.65) becomes at once

$$\hat{I}_\infty(r)_A = -A_{\infty,A}\left\langle \int_0^{z_o} dz \int_0^{x_o} J_1(x)dx \right\rangle_{z_o,\underset{\sim}{\theta}} \quad ; \quad x_o = rG_o/\lambda_{max}^{\gamma} = r\hat{B}_{oA} \ . \tag{2.68}$$

Next we use (2.58) to reëxpress (2.68) as

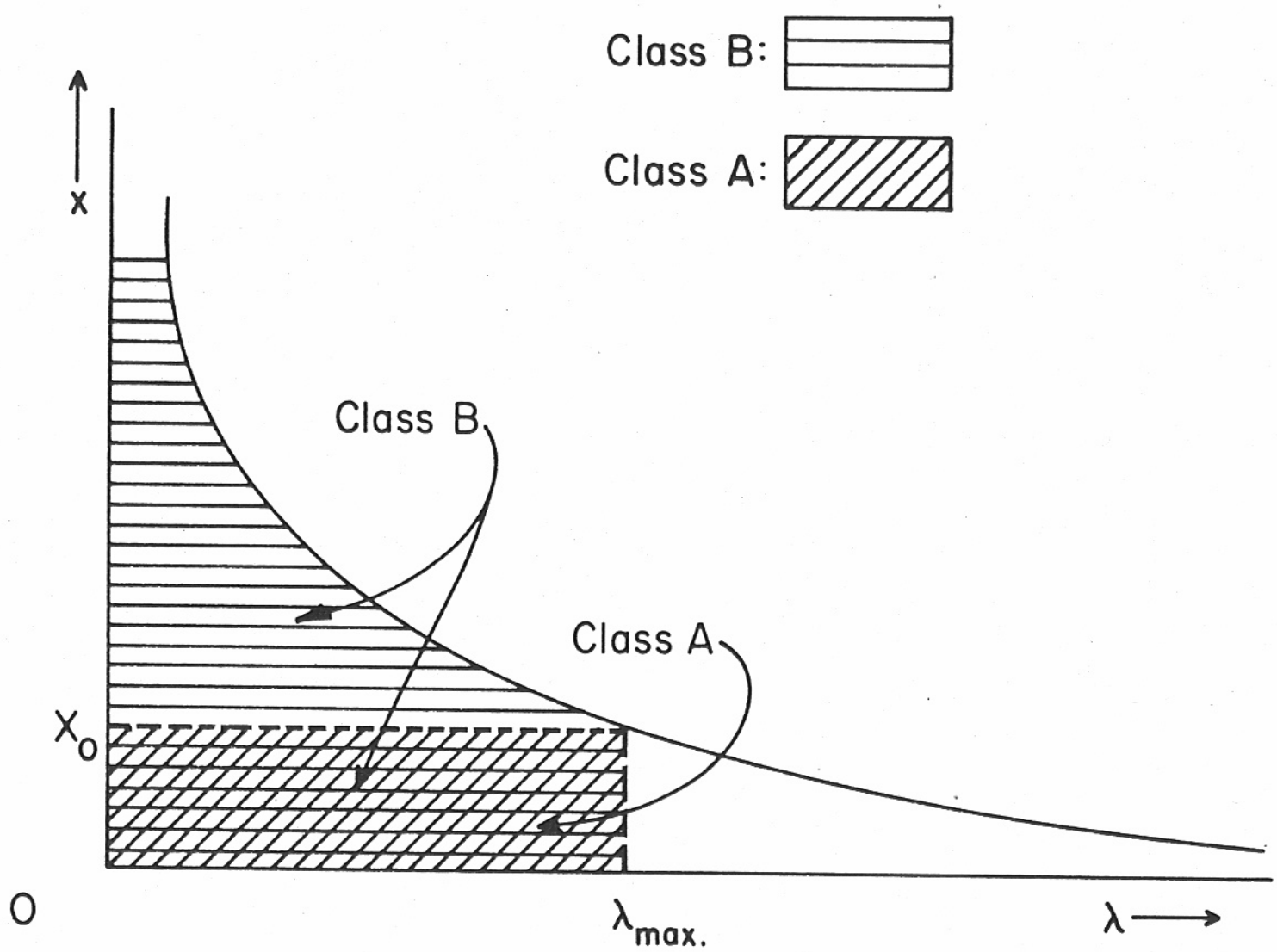

Figure 2.4 (II). Schema of the (x,λ)-domains of intergration for $\hat{I}_\infty$, (2.65), for Class A,B interference.

$$\hat{I}_{\infty}(r)_A = -A_{\infty,A}\left[1-\left\langle\int_0^{z_0} J_o(x_o)dz\right\rangle_{\underset{\sim}{\theta}}\right], \tag{2.69}$$

since $\bar{z}_o = \langle T_s\rangle/\bar{T}_s = 1$), and then employ the steepest descent method of Report I [Middleton, 1974, Sec. 3, Eq. (3.10) et seq; as mentioned earlier [cf. Section 2.5)] it is the largest contribution to the exponent in the neighborhood of r=0, which controls the behavior for E >> 0)] to obtain the desired development in r in the neighborhood of r = 0. For this we use the identity

$$\left\langle J_o(x_o)\right\rangle_{\underset{\sim}{\theta}'} = e^{-r^2\langle\hat{B}_{oA}^2\rangle/4}\left\langle J_o(r\hat{B}_{oA})e^{r^2\langle\hat{B}_{oA}^2\rangle/4}\right\rangle_{\underset{\sim}{\theta}'=A_o,\text{etc.}} ; \ \underset{\sim}{\theta} = (\underset{\sim}{\theta}',z_o) \tag{2.70}$$

to write

$$\left\langle\int_0^{z_o} J_o(x_o)dz\right\rangle_{\underset{\sim}{\theta}} = e^{-r^2\langle\hat{B}_{oA}^2\rangle/4}\left\langle\int_0^{z_o}\left\langle J_o(r\hat{B}_{oA})\right\rangle_{\underset{\sim}{\theta}'} e^{r^2\langle\hat{B}_{oA}^2\rangle/4}dz\right\rangle_{z_o} \tag{2.71}$$

in (2.69). From (3.11), [Middleton, 1974] we have

$$\left\langle J_o(rA)\right\rangle e^{r^2\langle A^2\rangle/4} = 1 + \sum_{\ell=2}^{\infty}\frac{\langle A^2\rangle^{\ell} r^{2\ell}}{2^{2\ell}\ \ell!}\left\langle {}_1F_1(-\ell;1;A^2/\langle A^2\rangle)\right\rangle \tag{2.72}$$

where ${}_1F_1(-\ell;1;y^2)$ is given explicitly by (2.56a). The result is that (2.71) becomes

$$\left\langle\int_0^{z_o} J_o(x_o)dz\right\rangle_{\underset{\sim}{\theta}} = e^{-r^2\langle B_{oA}^2\rangle/4}\left\{1 + \sum_{\ell=2}^{\infty}\frac{\langle\hat{B}_{oA}^2\rangle_{\theta}^{\ell} r^{2\ell}}{2^{2\ell}\ \ell!}F_{\ell}\right\}, \tag{2.73}$$

in which

$$F_{\ell} \equiv \left\langle\int_0^{z_0} {}_1F_1(-\ell;1;\hat{B}_{oA}^2/\langle\hat{B}_{oA}^2\rangle_{\theta})dz\right\rangle_{z_0} \ ; \ \langle\hat{B}_{oA}^2\rangle_{\theta=\theta',z_o} = \langle G_o/\lambda_{max}^r\rangle_{\underset{\sim}{\theta}}. \tag{2.73a}$$

Equation (2.73) in (2.69) now gives us the desired expansion of $\hat{I}_{\infty,A}$ about r=0, which governs the p.d. (and P.D.) for large values of the

envelope E. We have explicitly

$$\hat{I}_\infty(r)_A = -A_{\infty,A} + A_{\infty,A} e^{-r^2\langle\hat{B}^2_{oA}\rangle/4}\left\{1 + \sum_{\ell=2}^{\infty} \frac{\langle\hat{B}^2_{oA}\rangle^\ell}{2^{2\ell}\,\ell!} r^{2\ell} F_\ell\right\} \tag{2.74a}$$

$$\doteq -A_{\infty,A} + A_{\infty,A} e^{-r^2\langle\hat{B}^2_{oA}\rangle/4}\left\{1+O(r^4)\right\}, \tag{2.74b}$$

this last if we neglect the correction terms [$O(r^4$ in $\hat{I}_\infty)$].

When we consider correction terms [in a later Report] it is convenient to define, as we did in Report I [Middleton, 1974] a set of coefficients, now extended to include the average over z_o:

$$\hat{C}_{2\ell} \equiv \ell!(-1)^\ell \left\langle \int_0^{z_o} {}_1F_1(-\ell;1;\hat{B}^2_{oA}/\langle\hat{B}^2_{oA}\rangle)dz \right\rangle_{z_o} = (-1)^\ell \ell!\ F_\ell, \tag{2.75}$$

cf. (2.73a), which contain the 2ℓ, $2\ell-2,\ldots,$ moments of the envelope $\hat{B}_{oA}$ at the output of the ARI stages of the receiver. Specifically, we have here

$$\hat{C}_4 = [\langle\hat{B}^4_{oA}\rangle - 2\langle\hat{B}^2_{oA}\rangle^2]/\langle\hat{B}^2_{oA}\rangle^2 \quad ; \ (\hat{C}_2 = 0) \tag{2.75a}$$

$$\hat{C}_6 = [\langle\hat{B}^6_{oA}\rangle - 9\langle\hat{B}^4_{oA}\rangle\langle\hat{B}^2_{oA}\rangle + 12\langle\hat{B}^2_{oA}\rangle^3]/\langle\hat{B}^2_{oA}\rangle^3 \ ; \tag{2.75b}$$

$$\hat{C}_8 = [\langle\hat{B}^8_{oA}\rangle - 16\langle\hat{B}^6_{oA}\rangle\langle\hat{B}^2_{oA}\rangle + 72\langle\hat{B}^4_{oA}\rangle\langle\hat{B}^2_{oA}\rangle^2 - 72\langle\hat{B}^2_{oA}\rangle^4]/\langle\hat{B}^2_{oA}\rangle^4; \text{ etc.}, \tag{2.75c}$$

with

$$\langle\hat{B}^{2n}_{oA}\rangle \equiv \left\langle \int_0^{z_o} \langle\hat{B}^{2n}_{oA}\rangle_\theta, dz \right\rangle_{z_o} = \left\langle \int_0^{z_o} \langle G^{2n}_{oA}\rangle_{\underset{\sim}{\theta}}, \lambda^{2\gamma n}_{max} dz \right\rangle_{z_o} . \tag{2.75d}$$

Thus, (2.74) becomes here, equivalently,

$$\hat{I}_\infty(r)_A = -A_{\infty,A} + A_{\infty,A} e^{-r^2\langle\hat{B}^2_{oA}\rangle/4}\left\{1 + \sum_{\ell=2}^{\infty} \frac{\langle\hat{B}^2_{oA}\rangle^\ell(-1)^\ell}{2^{2\ell}(\ell!)^2} \hat{C}_{2\ell} r^{2\ell}\right\}. \tag{2.76}$$

Combining (2.74), or (2.76), with the practical situation involving an additive background gaussian component, cf. Sec. (2.3.1), we get finally for the desired form of the c.f. for Class A interference

$$\hat{I}_\infty(r)_{A+G} = -\sigma_G^2 r^2/2 + \hat{I}_\infty(r)_A,$$

so that

$$\boxed{\hat{F}_1(ir)_{A+G} = \exp\left\{-\sigma_G^2 r^2/2 - A_{\infty,A} + A_{\infty,A} e^{-r^2\langle \hat{B}_{oA}^2\rangle/4}\,[1+O(r^4)]\right\},} \qquad (2.77)$$

where $[1+O(r^4)]$ in (2.77) is given explicitly by the expression { } in (2.76). The final step in the reduction of these c.f.'s to the desired technically manageable form, particularly for the smaller values $O(\leq 10^0)$ of the impulsive index $A_{\infty,A}$, (2.39), which are typical of this class of interference, is now the following direct expansion of the terms containing $A_{\infty,A}$ in (2.76) or (2.77):

$$\boxed{\begin{aligned}\hat{F}_1(ir)_{A+G} = e^{-A_{\infty,A}} \sum_{m=0}^{\infty} \frac{A_{\infty,A}^m}{m!}\, e^{-[m\langle \hat{B}_{oA}^2\rangle/2+\sigma_G^2]r^2/2} \\ \cdot \left\{1 + \frac{A_{\infty,A}\langle \hat{B}_{oA}^2\rangle r^4 \hat{C}_4}{4^3}\, e^{-r^2\langle \hat{B}_{oA}^2\rangle/4} \right. \\ \left. + \frac{A_{\infty,A}\langle \hat{B}_{oA}^2\rangle^3 r^6 \hat{C}_6}{4^3(3!)^2}\, e^{-r^2\langle \hat{B}_{oA}^2\rangle/4} + \ldots\right\}\end{aligned}} \qquad (2.78)$$

[Equation (2.78) is formally the same as our earlier result (3.16) [Middleton, 1974], which is not surprising, since the c.f.'s have the same

form, cf. (2.50). The parameters differ somewhat in detail, of course.]

However, in approximating the exact (2.74a) c.f. by the approximate forms (2.74b), (2.77), (2.78), we obtain ultimately (cf. Section 3 following) approximate P.D.'s, and pdf's (Section 4, ff), which may not be properly normalized, in the sense of yielding a mean square value, $\langle E^2\rangle_A$, of the envelope, different from the exact relation $\langle E^2\rangle_A = 2\Omega_{2A}(1+\Gamma'_A)$, cf. Eq. (5.12b), which is derived from the exact c.f. (5.10), or (5.10a).

Accordingly, we choose the c.f., here and subsequently the leading term of (2.78), and from it determine the associated P.D. and pdf. Then, from the pdf we determine

$$\langle \mathcal{E}^2\rangle_A = \int_0^\infty w_1(\mathcal{E})_A \mathcal{E}^2 d\mathcal{E}\ , \quad \mathcal{E} \equiv \frac{E}{\sqrt{2\Omega_{2A}(1+\Gamma'_A)}}\ . \tag{2.78a}$$

If $w_1(\mathcal{E})_A$ is properly normalized, $\langle \mathcal{E}^2\rangle_A$ should be unity. If not, and $\langle \mathcal{E}^2\rangle_A$ is bounded, such that $\langle \mathcal{E}^2\rangle_A \equiv N_A^2\ (<\infty, \neq 1)$, then the properly normalized pdf ($\therefore$ PD, also) is given by

$$w_1(\mathcal{E})_{A=\text{norm}} = w_1(\mathcal{E})_A N_A^{-2} \tag{2.78b}$$

so that

$$\langle \mathcal{E}^2\rangle_{(A)} = \int_0^\infty w_1(\mathcal{E})_A N_A^{-2}\mathcal{E}^2 d\mathcal{E} = \int_0^\infty w_1(\mathcal{E})_{A\text{-norm}}\mathcal{E}^2 d\mathcal{E} = 1\ , \tag{2.78c}$$

as required. From (5.3) following we have (cf. Sec. 5.1):

$$\langle \mathcal{E}^2\rangle_A = e^{-A_{\infty,A}} \sum_{m=0}^{\infty} \left(\frac{m/A_{\infty,A}+\Gamma'_A}{1+\Gamma'_A}\right) \frac{A^m_{\infty,A}}{m!} = 1 \tag{2.78d}$$

exactly, showing that $w_1(\mathcal{E})_A$ here is indeed suitably normalized (and $\therefore N_A^2 = 1$), as is the PD, $P_1(\mathcal{E} > \mathcal{E}_0)_A$, consequently. Accordingly, the Class A cases discussed subsequently need no scale or other adjustments (unlike the Class B model, where the approximating forms do require scale and level adjustments, cf. remarks following Eq. (2.94), and the discussion in

Section 3.2-A.

An interesting feature of our results here, for Class A interference, is that the c.f., and hence the p.d., and P.D., <u>are not explicitly dependent on the interference source distribution</u> [for the usual Case I, II, and not so common Case III source-receiver conditions, cf. Fig. (2.3)II]. Furthermore, these statistics are insensitive to the propagation law ($\sim \lambda_{max}^{-\gamma}$) involved, which merely affects scale, through the average $\langle \hat{B}_{oA}^2 \rangle = \langle G_o/\lambda^\gamma \rangle$. The source distribution does appear, but in averaged form and only in the impulsive index $A_{\infty,A}$, [cf. (2.35), (2.38), (2.39), in conjunction with (2.27)]. Physically, this is understandable, since it is only the average number of emissions per second times the mean duration of these finite emissions (in the receiver, cf. Fig. (2.1)II) to which the receiver can respond. It has no way to distinguish where and in what concentration, or by what propagation law, the sources may be acting [for a given position of the receiver beam $\mathcal{A}_R$, or for any position if $\mathcal{A}_R$ is omnidirectional]. The only thing that counts here in determining the (first-order) statistics of the received input is total input level and process "density", $A_{\infty,A}$. As we shall see in Section (2.7), and in later Sections, this insensitivity to source distribution and propagation law is definitely <u>not</u> characteristic of Class B interference, and, consequently, Class C noise, to the extent that its Class B component is significant.

2.7 The C.F. for Class B Interference:

Here we use (2.67b) for the exponent (2.65). The result is a term like (2.68), plus an additional term for $x_o < x < \infty$, with $\lambda_{max} \geq \lambda \geq 0$, viz:

$$\hat{I}_\infty(r)_B = -A_{\infty,B}\Bigg\{\Bigg\langle \int_0^\infty dz \int_0^{x_o = rG_o/\lambda_{max}^\gamma} J_1(x)dx \Bigg\rangle_{\underset{\sim}{\theta}'}$$

$$+\Bigg\langle \int_0^\infty dz \int_{\Delta(\theta,\phi)} A_{S,V}^{-1}\sigma_{S,V}(\theta,\phi)d(\theta,\phi) \int_{x_o=rG_o/\lambda_{max}^\gamma}^\infty J_1(x)dx$$

$$\cdot \int_{\lambda=(rG_o)^{1/\gamma}/x^{1/\gamma}}^0 \begin{bmatrix} c^2/\lambda^{\mu-1} \\ c^3/\lambda^{\mu-2} \end{bmatrix} d\lambda \Bigg\rangle_{\underset{\sim}{\theta}'} \Bigg\} , \qquad (2.79)$$