4. PROBABILITY DENSITIES: $w_1(\mathcal{E})_{A,B}$

It is now a simple matter to determine the probability densities (pdf's) (pdf's) associated with the exceedance probabilities (PD's) derived in Section 3 preceeding. Because the PD's are continuous, at least through the second derivative ($0 \leq \mathcal{E} < \infty$), and because

$$w_1(\mathcal{E}) = -\left.\frac{dP_1}{d\mathcal{E}_0}\right|_{\mathcal{E}_0 \to \mathcal{E}} = \mathcal{E}\int_0^\infty \lambda J_0(\lambda\mathcal{E})\hat{F}_1(ia\lambda)d\lambda, \quad 0 \leq \mathcal{E} < \infty, \tag{4.1}$$

cf. (2.21)(3.1)(3.3)

we may apply this to (3.7), (3.11) and (3.17), etc., to obtain directly the desired pdf's. We have first:

4.1 Class A Interference:

From (3.7b) and (4.1) we find that

$$\boxed{w_1(\mathcal{E})_A \simeq e^{-A_A}\sum_{m=0}^{\infty}\frac{A_A^m}{m!}\frac{\mathcal{E}e^{-\mathcal{E}^2/2\hat{\sigma}_{mA}^2}}{\hat{\sigma}_{mA}^2}, \quad 0 \leq \mathcal{E}.} \tag{4.2}$$

Thus, as expected from our earlier result (3.7b), $w_1(\mathcal{E})_A$ (in its principal contribution*) is the weighted sum of rayleigh pdf's, whose variances $\hat{\sigma}_{mA}^2$ cf. (3.5), increase with order (m). Figs.(4.1)II and (4.2)II show $w_1(\mathcal{E})_A$ for various combinations of the controlling parameters A_A, Γ_A'. With A_A small the pdf's are seen to be highly nongaussian (e.g. nonrayleigh in $\mathcal{E}$), unless Γ_A' is very large, in which case the gaussian (e.g. rayleigh) component (here) dominates. As the Impulsive Index A_A gets larger, the pdf approaches the purely rayleigh form, cf. (2.57b). Also, for $\Gamma_A' > 0$, the pdf near $\mathcal{E} = 0$ has finite width, shouldering off into a broad, rather low level (in w_1) form as $\mathcal{E} \to \infty$, which represents the strongly non-rayleigh structure of this class of noise. The larger Γ_A', the wider and less "peaked" is the "spike" at $\mathcal{E} \gtrapprox 0$, and the more "shoulder" there is to the rest of the pdf.

When $\Gamma_A' = 0$, e.g. $\sigma_G^2 = 0$, i.e., when there is no independent,

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* See the comments following Eq. (3.7b).

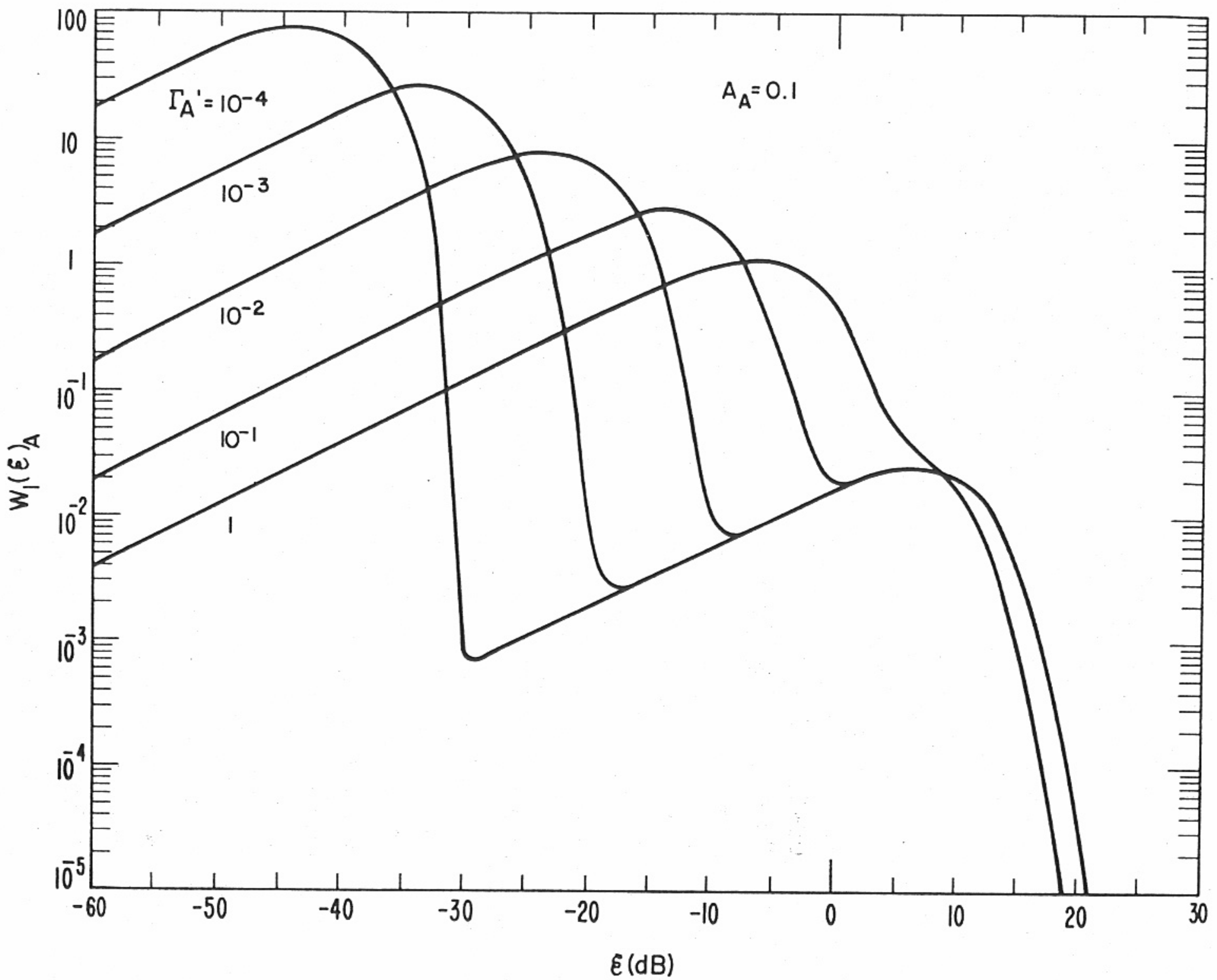

Figure 4.1 (II). The pdf, $w_1(\mathcal{E})_A$, of the envelope for Class A interference, calculated from eq. (4.2) for various Γ_A', given A_A [cf. fig. 3.1 (II) for $P_1(\mathcal{E} > \mathcal{E}_0)_A$].

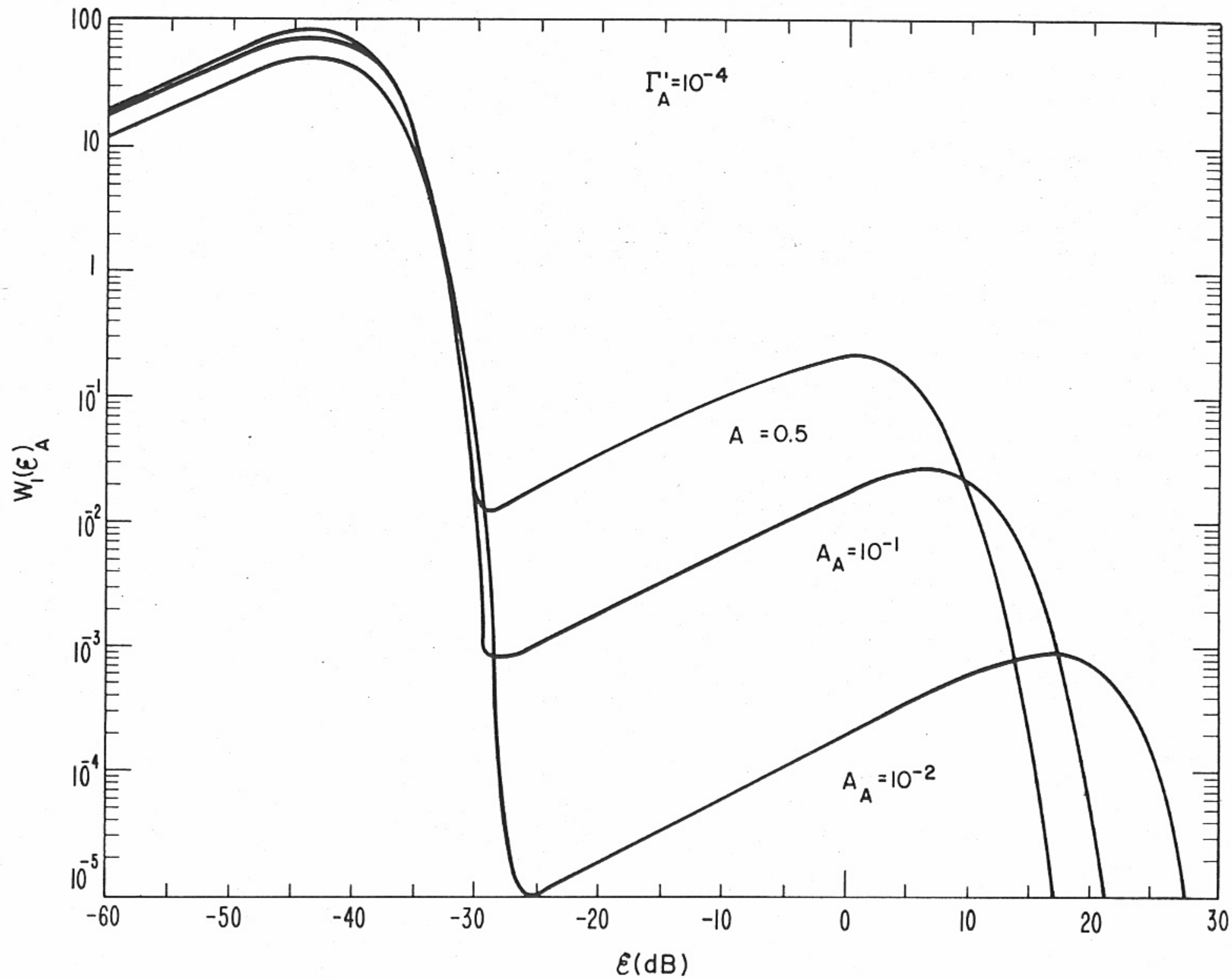

Figure 4.2 (II). The pdf, $w_1(\mathcal{E})_A$, of the envelope for Class A interference, calculated from eq. (4.2) for various A_A, given Γ'_A [cf. fig. 3.2 (II) for $P_1(\mathcal{E} > \mathcal{E}_0)_A$].

additive gaussian component to the interference, Eq. (4.2) reduces to

$$w_1(\mathcal{E})_A \simeq e^{-A_A}\left\{\delta(\mathcal{E}-0) + \sum_{m=1}^{\infty} \frac{A_A^{m+1}}{m!\, m}\, 2\mathcal{E} e^{-\mathcal{E}^2 A_A/m}\right\}, \quad 0 \leq \mathcal{E} < \infty), \tag{4.2a}$$

and the "spike" at $\mathcal{E} = 0$ is truly a delta-function. The variance is now m/A_A. In this case we have an example of "holes in time": there is a nonzero probability that $\mathcal{E} = 0$, an idealized limiting case, since there is always in practice some system noise, which means an additive gaussian term, so that (4.2) applies, with $w_1(0)_A = 0$, of course.

4.2 Class B Interference:

As expected for our general canonical approximation [(3.11b) plus (3.17)], cf. (3.20), we have also two relations for $w_1(\mathcal{E})_B$: $w_1(\mathcal{E})_{B-I}$ applies for small and intermediate values of $\mathcal{E}$, while $w_1(\mathcal{E})_{B-II}$ is appropriate for $\mathcal{E} \geq \mathcal{E}_B$. Again, $\mathcal{E}_B$ is a point of inflexion, or the "bend-over" point, where the Class A form, (3.17) applies, with, of course, the appropriate Class B parameters (A_B, Γ_B'), as determined analytically by the procedures described in (3.18), (3.19). Accordingly, we use (3.11b) in (4.1), to obtain specifically:

$$\boxed{\begin{aligned} &\hat{w}_1(\hat{\mathcal{E}})_{B-I} \equiv \\ &w_1(\mathcal{E})_{B-I} \simeq 2\hat{\mathcal{E}} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \hat{A}_\alpha^n \;\Gamma\left(1 + \frac{n\alpha}{2}\right) {}_1F_1(1+n\alpha/2;1;-\hat{\mathcal{E}}^2), \\ &\hat{\mathcal{E}} \equiv (\mathcal{E} N_I)/2G_B,\ \hat{A}_\alpha = A_\alpha/2^\alpha G_B^\alpha, \qquad (0 \leq \mathcal{E} \leq \mathcal{E}_B), \end{aligned}} \tag{4.3}$$

and, formally, for large (but not too large) $\mathcal{E}$; from (3.15):

$$\hat{w}_1(\hat{\mathcal{E}})_{B-I} \simeq 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \hat{A}_\alpha^n \frac{\Gamma(1+\frac{n\alpha}{2})}{\Gamma(-\frac{n\alpha}{2})}\, \hat{\mathcal{E}}^{-n\alpha-1}\left\{1 + \frac{(1+n\alpha/2)(1-n\alpha/2)}{1!\hat{\mathcal{E}}^2} + \ldots\right\}$$

$$(0 \ll \mathcal{E} \leq \mathcal{E}_B). \tag{4.3a}$$

When $\mathcal{E} > \mathcal{E}_B$ we obtain at once from (3.17) in (4.1)

$$\boxed{w_1(\mathcal{E})_{B-II} \simeq \frac{e^{-A_B}}{4G_B^2} \sum_{m=0}^{\infty} \frac{A_B^m}{m!} \frac{\mathcal{E} e^{-\mathcal{E}^2/2\hat{\sigma}_{mB}^2}}{\hat{\sigma}_{mB}^2} \,, \quad (\mathcal{E}_B \le \mathcal{E} < \infty),} \tag{4.4}$$

analogous to (4.2) in the Class A cases. Observe from (ii),(iv) of (3.18), and [(ii), (iv) of (3.19)], that $w_1(\mathcal{E})_B$, viz,:

$$\left.\begin{aligned} w_1(\mathcal{E})_B &= w_1(\mathcal{E})_{B-I} \,, \qquad 0 \le \mathcal{E} \le \mathcal{E}_B \\ &= w_1(\mathcal{E})_{B-II} \,, \qquad \mathcal{E}_B \le \mathcal{E} \end{aligned}\right\} , \tag{4.5}$$

is continuous at $\mathcal{E} = \mathcal{E}_B$, with continuous first derivative, so that $w_1(\mathcal{E})_B$, as well as $P(\mathcal{E})_B$, has no break or "jump" at the bend-over point $\mathcal{E}_B$, where the two approximations are joined. Furthermore, unlike the Class A interference, when $\Gamma_B' = 0$ there are no "gaps in time", cf. (4.2a) vs. (4.3): there is always a non-zero probability (density) for $\mathcal{E} = 0$. Figs.(4.3)II,(4.3)II show typical curves of $w_1(\mathcal{E})_B$, analogous to Figs. (3.6)II, (3.7)II for the P.D.

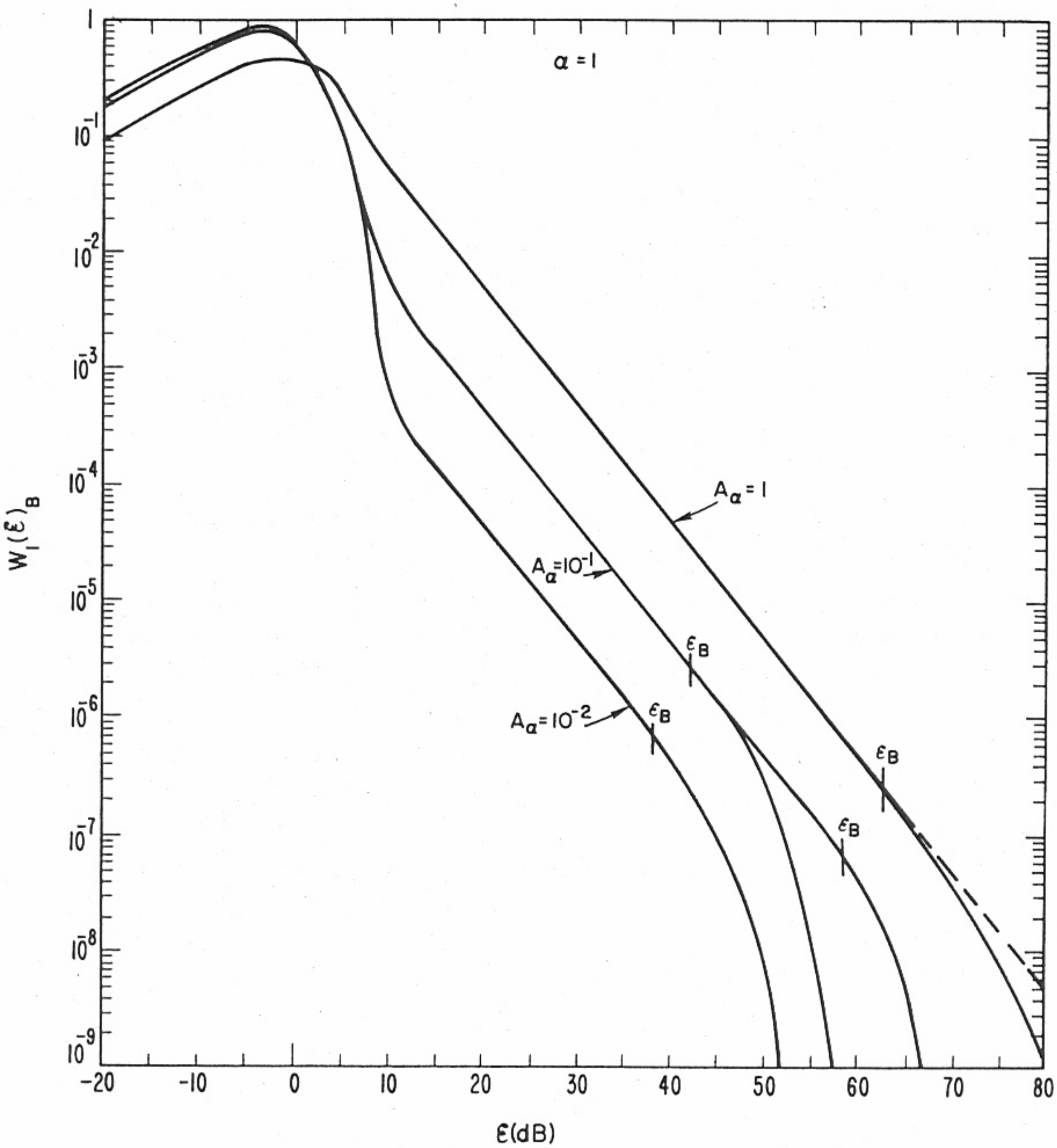

Figure 4.3 (II). The (complete) pdf $w_1(\mathcal{E})_B$, eq. (4.5), of the envelope for Class B interference, calculated from eqs. (4.3, 4.4) for various A_α, given α [cf. (3.19)]. [See fig. 3.6 (II) for the associated $P_1(\mathcal{E} > \mathcal{E}_0)_B$ and parameter values.]

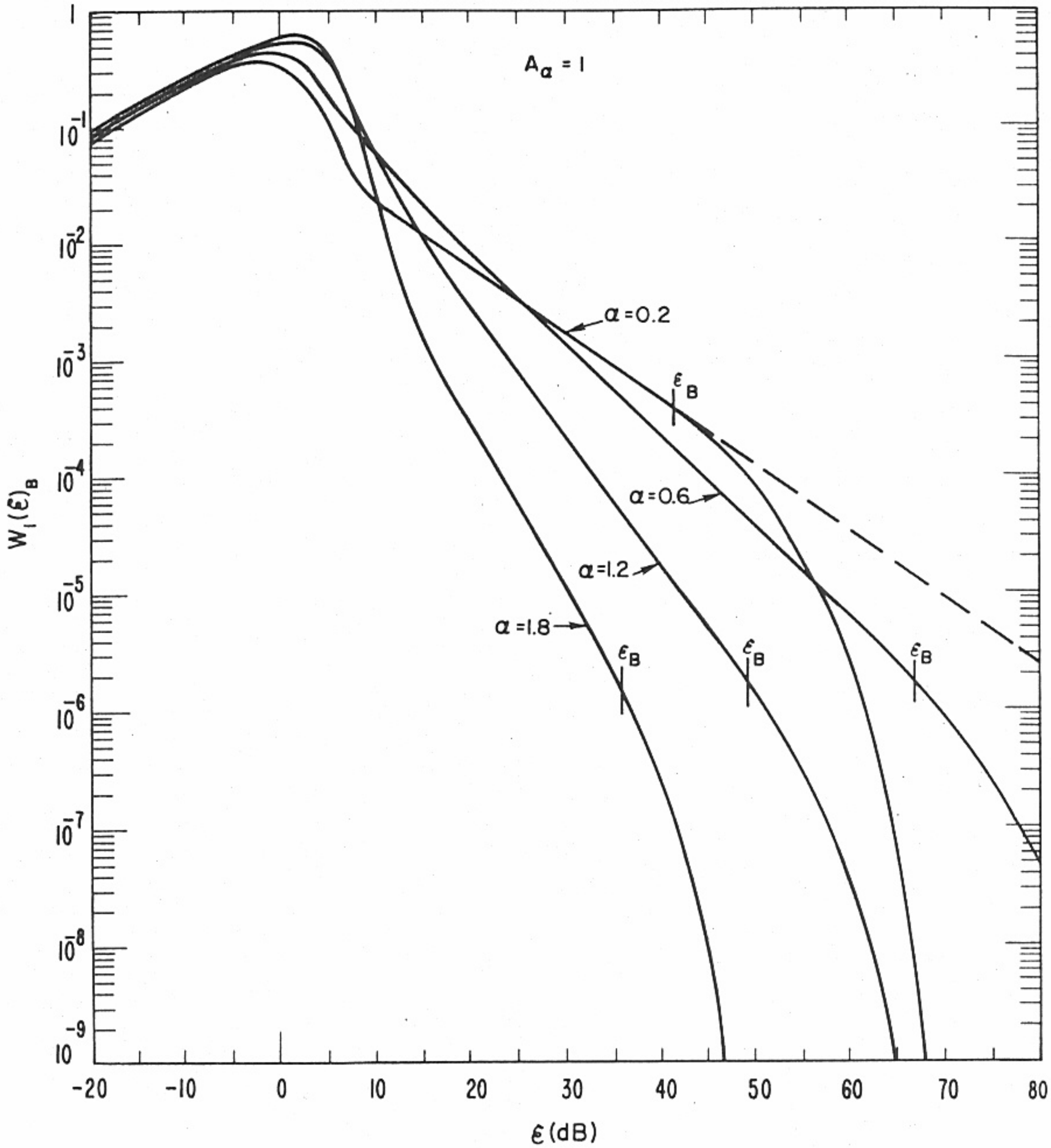

Figure 4.4 (II). The (complete) pdf, $w_1(\mathcal{E})_B$, eq. (4.5), of the envelope Class B interference, calculated from eqs. (4.3, 4.4) for various α, given A_α [cf. (3.19)]. [See fig. 3.7 (II) for the associated $P_1(\mathcal{E} > \mathcal{E}_0)_B$ and parameter values.]