6. DETERMINATION OF THE BASIC FIRST-ORDER PARAMETERS

In this section we outline procedures for determining the basic parameters of Class A and Class B interference models developed in the preceeding sections. A variety of overlapping procedures is available. We shall select what appears at this stage of the study to be the most direct and/or convenient, (later efforts may suggest modifications, for particular situations).

We begin with:

A. Class A Interference:

The first-order PD (and pdf) are governed by three parameters. It is convenient to distinguish two levels of parametric description: the first level, which we shall call "Basic-I", consists of global parameters, which appear directly in the expression for the P.D., cf. Eq. (3.7), and the second, or "Basic-II" level, contains the associated generic parameters, which are defined directly in terms of the underlying statistical-physical model. The two groups, as we shall see, overlap to some extent. Table 6.1 below gives the global and generic parameters of Class A interference:

Table (6.1): Class A Parameters

Basic I:	Global:	$(A_A, \Gamma'_A, \Omega_{2A}) \rightarrow \{\text{Practical Global: } (A_A, \Gamma'_A, K_A)\}$
Basic II:	Generic:	$(A_A, \sigma_G^2, \langle \hat{B}_{oA}^2 \rangle)$

(6.1)

The generic and global parameters are related by

$$\sigma_G^2 = \Omega_{2A}\Gamma'_A \; ; \; \langle \hat{B}_{oA}^2 \rangle = 2\Omega_{2A}/A_A \quad , \quad \text{[Eq. (3.1a)]} \quad . \tag{6.2}$$

Furthermore, the intensity of the independent gaussian component is

$$\sigma_G^2 = \sigma_R^2 + \sigma_E^2 \; , \; \text{[Eqs. (2.47)]}, \tag{6.3}$$

where σ_R^2 is the intensity of receiver noise (at the output of the initial ARI-stages of the receiver) and σ_E^2 is the intensity of the independent external gaussian component, if any, likewise observed at the output of the ARI-stages of the receiver. By blocking the input to the receiver, (i.e., insuring that $\sigma_E^2 = 0$) one obtains σ_R^2, at the ARI output. Consequently, as σ_E^2 is found by actual reception in the (here) Class A noise environment, one then at once determines σ_E^2 from (6.3).

Because we do not a priori know the normalizations (3.1) by which the threshold E_0 and the envelope E are scaled, it is necessary to convert our analytic expressions (3.7) for the PD, for example, into forms more directly conformable to experimental evaluation. For this purpose we write*

$$\mathcal{E}' \equiv E/\sqrt{2\hat{\sigma}_G^2} = \mathcal{E}\sqrt{\Omega_{2A}(1+\Gamma_A')/\hat{\sigma}_G^2} = \mathcal{E}/K_A; \quad \mathcal{E}_0' = \mathcal{E}_0/K_A, \tag{6.4a}$$

with

$$K_A \equiv [\hat{\sigma}_G^2/\Omega_{2A}(1+\Gamma_A')]^{1/2} = [\hat{\sigma}_G^2/(\Omega_{2A}+\sigma_G^2)]^{1/2} , \tag{6.4b}$$

the new conversion factor, between $\mathcal{E}'$ and $\mathcal{E}$, where $\hat{\sigma}_G^2$ is an a priori determined reference quantity, used to scale the (absolute) values E_0, E, etc.

To obtain the generic parameters (A_A, σ_G^2, $\hat{B}_{0A}^2$) we first must determine the global parameters ($A_A,\Gamma_A',\Omega_{2A}$). Practically, this means we must initially find the "practical" global quantities (A_A,Γ_A',K_A), cf. Table (6.1) above, and then use (6.4b) to eliminate the conversion factor K_A. Three relations involving the practical global parameters are needed. Perhaps the simplest are the first and second moments of $\mathcal{E}'$, and the PD of $\mathcal{E}'$ in the rayleigh region ($\mathcal{E}_0'$<<1), where the slope ($dP_{1A}/d\mathcal{E}_0'$) is constant, cf. Figs. (3.1,3.2)II. Accordingly, from the exact expression (5.12b) and (6.4a) we write

$$\langle(\mathcal{E}')^2\rangle_A = \langle\mathcal{E}^2\rangle_A/K_A^2 = 1/K_A^2 = \Omega_{2A}(1+\Gamma_A')/\hat{\sigma}_G^2 , \tag{6.5}$$

which gives us K_A and hence $\Omega_{2A}(1+\Gamma_A')$ in terms of the known $\hat{\sigma}_G^2$ and $\langle(\mathcal{E}')^2\rangle_A$,

* Of course, one can always measure $\langle E^2\rangle_A=2\Omega_{2A}(1+\Gamma_A')$ and then normalize, so that K_A=1.

this last by measurement in practice. From the approximate expression for $\langle \mathcal{E}' \rangle_A$, viz. (5.3) with (6.4a), we obtain

$$\langle \mathcal{E}' \rangle_A = \langle \mathcal{E} \rangle_A / K_A \cong K_A^{-1} \left[e^{-A_A} \frac{\sqrt{\pi}}{2} \sum_{m=0}^{\infty} \frac{(m/A_A + \Gamma_A')^{1/2}}{(1+\Gamma_A')^{1/2}} \frac{A_A^m}{m!} \right] , \tag{6.6}$$

and from (3.7b) specialized to the rayleigh region, e.g. (3.8), we have

$$P_1(\mathcal{E}' > \mathcal{E}_o')_A \cong 1 - \left[e^{-A_A} \sum_{m=0}^{\infty} \frac{(1+\Gamma_A')A_A^m}{(\Gamma_A' + m/A_A)m!} \right] (\mathcal{E}_o' K_A)^2 \quad . \tag{6.7}$$

In practice, of course, $\langle \mathcal{E}' \rangle_A$, $\langle (\mathcal{E}')^2 \rangle_A$, and $P_1(\mathcal{E}' > \mathcal{E}_o')$ are estimated from the experimentally derived data, i.e. $\langle \mathcal{E}' \rangle_A$, $\langle (\mathcal{E}')^2 \rangle_A$, and $P_1(\mathcal{E}' > \mathcal{E}_o')_A$ are respectively replaced by their estimates from the necessarily finite empirical data, so that (6.5)-(6.7) are three relations for joint estimation of the "practical" global parameters (A_A, Γ_A', K_A), a proceedure requiring a modest amount of computational assistance, particularly when the expressions in brackets [ ] have been programmed. With the help of (6.4b) for K_A involving Ω_{2A}, Γ_A' we next get directly the (estimates of the) global quantities $(A_A, \Gamma_A', \Omega_{2A})$. Then it is a simple matter to use (6.2) to obtain finally the (estimates of the) desired generic parameters $(A_A, \sigma_G^2, \langle \hat{B}_{oA}^2 \rangle)$. The desired estimates to be used in (6.5)-(6.7) are

$$\langle \mathcal{E}' \rangle_A \to \frac{1}{n} \sum_{i=1}^{n} \mathcal{E}_i' \; ; \quad \langle (\mathcal{E}')^2 \rangle_A \to \frac{1}{n} \sum_{i=1}^{n} (\mathcal{E}_i')^2 \; ; \quad P(\mathcal{E}' > \mathcal{E}_o')_A \to P_1(\mathcal{E}' > \mathcal{E}_o')_{A\text{-expt'l}} . \tag{6.8}$$

B. Class B Interference:

Here we have a six-parameter model for the Class B cases. Table (6.2) summarizes the global and generic parameters involved:

Table 6.2: Class B Parameters

Basic I:	Global:	$(A_\alpha, \alpha, A_B, \Gamma_B', \Omega_{2B}, N_I)$
		$\rightarrow$ [Practical Global: $(A_\alpha, \alpha, A_B, \Gamma_B', K_B; N_I)$]
Basic II:	Generic:	$(A_B, \alpha, \sigma_G^2, \langle \hat{B}_{oB}^2 \rangle, \langle B_{oB}^\alpha \rangle; N_I)$

The global and generic parameters are related by (6.9)

$$\left.\begin{array}{l} \left.\begin{array}{ll} \sigma_G^2 = \Omega_{2B}\Gamma_B', \text{ [Eq. (3.2a)]} & \\ \quad = \sigma_R^2+\sigma_E^2 \text{ , [Eq. (2.47)]} & \end{array}\right\} ; \\ \\ \left\langle \hat{B}_{oB}^2 \right\rangle = 2\Omega_{2B}\, A_B^{-1} \text{ , \quad , [Eq. (3.2a)]} \\ \\ \left\langle \hat{B}_{oB}^\alpha \right\rangle = (2\Omega_{2B}(1+\Gamma_B'))^{\alpha/2} \dfrac{\Gamma(1+\alpha/2)A_\alpha}{2\Gamma(1-\alpha/2)A_B} \quad \text{[Eq. (3.12a)]} \end{array}\right\}$$

and (6.10)

$$\mathcal{E}_B = K_B \mathcal{E}_B', \quad \text{[cf. (6.11) below] .}$$

The common global and generic parameters are clearly (A_B, α).

The fact that there are six generic parameters for our statistical-physical model of Class B interference stems directly from our pair of approximations $\hat{F}_{1-I}, \hat{F}_{1-II}$, Eqs. (2.90), (2.93), to the exact cf. (2.87): (i), the Impulsive Index A_B [(2.38), (2.39) in (2.51); (ii), the spatial density-propagation parameter α, (2.82); (iii), the independent gaussian component σ_G^2, [(2.47), (2.88c)]; (iv). the α-moment of the generic, filtered envelope waveform $\langle \hat{B}_{oB}^\alpha \rangle$, cf. (2.87a) , (2.87d); (v), the mean-square of this generic waveform, $\langle \hat{B}_{oB}^2 \rangle$; and finally, (vi), the scaling factor

$N_I(A_\alpha,\alpha,A_B,\Omega_{2B},\Gamma_B'/\mathcal{E}_B)$, cf. remarks in Sec. 3.2-A. This factor N_I is functionally involved with but not solely determined by the other global (and generic parameters, through the APD form, <u>and is independent of</u> $\mathcal{E}_B$ [cf. remarks below in Sec. 6C]; hence it is regarded as a generic parameter here also. The quantity N_I ranges from 0(10db) to 0(50,60db) in practice. For example, comparison of Fig. (2.4) with Fig. 3.3(II) ($A_\alpha \doteq 1$, $\alpha \doteq 1$), for the same P_1=0.9 gives $N_I \doteq -6-(-44) \doteq 38$db. The point of inflexion, or "bendover" point $\mathcal{E}_B$, cf. Fig. (3.5)II, at which the PD's (and pdf's) corresponding to the two approximating c.f.'s, $\hat{F}_{1-I}$, $\hat{F}_{1-II}$, are joined, to give us the desired composite PD (and pdf), is purely empirical [(vi), (3.18)]. The conversion factor K_B is here

$$K_B \equiv \left\{\hat{\sigma}_G^2/\Omega_{2B}(1+\Gamma_B')\right\}^{1/2} \quad , \tag{6.11}$$

cf. (6.4b), where, again, $\hat{\sigma}_G^2$ is a known (measured) gaussian noise reference level. Also as before, we may obtain the components of σ_G^2 as indicated above, cf. (6.3) et seq. [See, also, footnote, Eq. (6.4a).]

Now to obtain the desired global parameters of our model from observed data we need six convenient nonidentical relations involving these parameters in various, sometimes simple ways. First, we use the exact expression for the mean square envelope (5.14), with the renormalization (6.4), to write

$$\langle(\mathcal{E}')^2\rangle_B = \langle\mathcal{E}^2\rangle_B K_B^{-2} = 1/K_B^2 = \Omega_{2B}(1+\Gamma_B')/\hat{\sigma}_G^2 = \Delta\sigma_G^2/\hat{\sigma}_G^2 \ , \tag{6.12}$$

cf. (6.5). Since the expressions for the Class B moments are analytically quite involved, and because the PD contains all moment information here, we use [(i), (iii), (iv)] of (3.18), (3.19) where P_{1-B} is empirically determined from the data. Accordingly, we have the additional five relations:

<u>rayleigh region</u>: $P_1(\mathcal{E}'\geq\mathcal{E}_0')_B \simeq \dfrac{(\mathcal{E}_0'K_BN_I)^2}{4G_B^2}\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\hat{A}_\alpha}{n!}\,\Gamma(1+\frac{\alpha n}{2})$
[2 relations]

$$\cong (\mathcal{E}_0'K_B)^2\,\frac{e^{-A_B}}{4G_B^2}\sum_{m=0}^{\infty}\frac{A_o^m}{m!}\,(2\hat{\sigma}_{mB}^2)^{-1}; \tag{6.13a}$$

large thresholds: [2 relations]
$$P_1(\mathcal{E}'\geq\mathcal{E}_B')_B \cong \frac{\hat{A}_\alpha\Gamma(1+\frac{\alpha}{2})}{\Gamma(1-\alpha/2)}\left(\frac{\mathcal{E}_B'K_BN_I}{2G_B}\right)^{-\alpha}[1+O_{(iii)}(\mathcal{E}_B'N_I^{-\alpha}K_B^{-\alpha},\mathcal{E}_B'^{-2}K_B^{-2})]$$
$$\cong \frac{e^{-A_B}}{4G_B^2}\sum_{m=0}^{\infty}\frac{A_B^m}{m!}e^{-(\mathcal{E}_B'K_B)^2/2\hat{\sigma}_{mB}^2} \tag{6.13b}$$

at bend-over point:
$$\frac{\hat{A}_\alpha\alpha\Gamma(1+\alpha/2)}{\Gamma(1-\alpha/2)}\frac{(2G_B)^{\alpha+1}}{(\mathcal{E}_BN_I)^{\alpha+1}}[1+O(\ )] = \frac{\mathcal{E}_Be^{-A_B}}{4G_B^2}\sum_{m=0}^{\infty}\frac{A_B^me^{-\mathcal{E}_B^2/2\hat{\sigma}_{mB}^2}}{m!}$$

As noted from (3.18),(v),(vi), $\mathcal{E}_B$ is the joining point (or point of continuity (through the second derivative of $P_{1-I,II}$, at least) for the approximations to P_{1-B}. $\mathcal{E}_B$ is the point of inflexion of P_{1-B}, obtained from $(P_{1-B})_{expt}$. Here

$$\mathcal{E}'_0 = E_0/\sqrt{2\hat{\sigma}_G^2} \ . \tag{6.14}$$

In practice, $\mathcal{E}_B'$ is available from inspection of the experimental* PD, $P_{1-B,exp}$, so that in addition to (6.12) only the five relations (6.13a,b,c) are then required for the remaining six global parameters. Once these have been obtained, we may use (6.10) to determine the six ultimately generic parameters of the Class B interference under study. Of course, in practice, our data are finite and P_{1-B} is an empirical function; $\langle(\mathcal{E}')^2\rangle_B$ is an estimate, cf. (6.8), based on sample values, and $\mathcal{E}_B'$ is likewise an estimate by inspection, so that all parameters actually obtained are necessarily themselves estimates. We do not include $\mathcal{E}_B$ in our list above of global parameters, and exclude it from the basic, or generic parameters, cf. (6.9), since it is in effect, an empirical quantity resulting from the procedure of joining P_{I-1}, P_{1-II} in approximation to the true P_{1-B}, at $\mathcal{E} = \mathcal{E}_B$.

C. Degenerate Cases:

When $\mathcal{E}_B$ (or $\mathcal{E}_B'$) is not known -- i.e., is not evident from the empiri-

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* However, see the important situation discussed in C following.

cal PD, P_{1-B} -- we can only work with the P_{1-I} form of the PD, namely the approximation suitable for small and intermediate values of $\mathcal{E}_0$ (and $\mathcal{E}_0'$) $< \mathcal{E}_B(\mathcal{E}_B')$. This is the case, for example, of much of the atmospheric noise data, cf. Fig. 2.4, where no bend-over point is at all evident. The model now reduces from six to a five-parameter approximation, in $(A_\alpha, \alpha, \Gamma_B', \Omega_B, N_I)$ [or $(A_\alpha, \alpha, \Gamma_B', K_B, N_I)$] for the global parameters and in $(\alpha, \sigma_G^2, N_I)$ for the generic parameters, cf. Table (6.2). Because $\mathcal{E}_B$ is not known, we are unable to obtain A_B, and hence we can determine only $\Omega_{2B} = A_B\langle \hat{B}_{oB}^2\rangle/2$, $A_\alpha \sim \langle \hat{B}_{oB}^\alpha\rangle A_B$, and not their individual factors $\langle \hat{B}_{oB}^\alpha\rangle, \langle \hat{B}_{oB}^2\rangle$. For these five global parameters we need accordingly five equations. The conversion factor K_B is again given by (6.12), and for the four other parameters we use

Eq. (3.11b): rayleigh region:

$$P_1(\mathcal{E}' \geq \mathcal{E}_0')_{B-I} \cong \left(\frac{\mathcal{E}_0' K_B N_I}{2G_B}\right)^2 \sum_{n=0}^{\infty} \frac{(-1)^n \hat{A}_\alpha^n}{n!} \Gamma(1+\frac{\alpha n}{2}) = 0.99, \text{ say}; \tag{6.15a}$$

and

P_{1-I} in the "bend-up" region, where P_{1-I} departs from the "straight line" rayleigh form, so that (3.11b) fully applies, and two points $P_{1-I} = P_3, P_4$ with

Eq. (3.15): large $\mathcal{E}_0'$ $(<\mathcal{E}_B')$:

$$P_1(\mathcal{E}' \geq \mathcal{E}_0')_{B-I} \simeq \frac{\hat{A}_\alpha \Gamma(1+\alpha/2)(\mathcal{E}_B' K_B N_I)^{-\alpha}}{\Gamma(1-\alpha/2)\ (2G_B)^{-\alpha}} (1+O_{(iii)}] = P_3, P_4 \tag{6.15b}$$

where the PD's are empirically determined. Without the turnover point $\mathcal{E}_B$ we cannot join the large-threshold approximation P_{1-BII} to P_{1-BI} for $(\mathcal{E}_0 < \mathcal{E}_B)$, and are thus unable to determine the generic parameters, except for α, σ_G^2, N_I. This indicates the importance of obtaining the "rare-event" data $(\mathcal{E} > \mathcal{E}_B)$, so that the fundamental (i.e., generic) parameters of the interference model may be estimated, as the fundamental descriptors of this noise environment, as specified, of course, by our statistical-physical model in this case [cf. Section (2.1) et seq.].