7. PRACTICAL CONDITIONS FOR CLASS A AND CLASS B INTERFERENCE

In our preceeding analyses we have postulated limiting forms of interference which are strictly Class A or Class B, neglecting the very small contributions of the associated other components (e.g. Class B where Class A is said to occur, etc.). Here we shall establish quantitative conditions which permit us to neglect these other-component effects and to assert that our analytical forms may be applied to the corresponding physical situation, e.g., essentially only Class A, or Class B noise is present.

To do this we start with the relation (2.52) for the general Class C interference and use the results [Eqs. (2.77), (2.78)] for Class A, [Eqs. (2.90), (2.93)] for Class B, to write

$$\hat{F}_1(ir)_{C+G} = \hat{F}_1(ir)_{A+G}\hat{F}_1(ir)_B \tag{7.1a}$$

$$= e^{-A_A}\sum_{m=0}^{\infty}\frac{A_A^m}{m!}\,e^{-(m/A_A+\Gamma_A')\Omega_{2A}r^2/2}\left\{\begin{array}{ll} e^{-b_{1\alpha}A_B r\,-b_{2\alpha}A_B r^2/2} & :\ (\hat{F}_{1-I})\ ; \\ e^{-A_B}\cdot\exp(A_B e^{-b_{2\alpha}r^2/2}) & :\ (\hat{F}_{1-II}) \end{array}\right\}, \tag{7.1b}$$

where we include σ_G^2, (2.47), in the Class A form $(\sim\Gamma_A')$ here.

<u>When Class A noise heavily predominates</u>, we use the transformation $r=a_A\lambda$, cf. (3.3) et seq., and expand the Class B components of the c.f., to get the pair of approximations

$$\hat{F}_1(ia_A\lambda)_{A+G}\doteq e^{-A_A}\sum_{m=0}^{\infty}\frac{A_A^m}{m!}\cdot\left\{\begin{array}{l} e^{-[(m/A_A+\Gamma_A')\Omega_{2A}+b_{2\alpha}A_B]a_A^2\lambda^2/2}\left\{1-b_{1\alpha}A_\alpha a_A^\alpha\lambda^\alpha+O(\lambda^{-\alpha})\right\} \\ \text{and} \\ e^{-[(m/A_A+\Gamma_A')\Omega_{2A}+b_{2\alpha}A_B]a_A^2\lambda^2/2}\left\{1+b_{2\alpha}^2A_B\,\frac{a_A^4\lambda^4}{8}+\ldots\right\} \end{array}\right. . \tag{7.2}$$

For this to reduce to the principal form (3.5), we have at once the <u>two</u> pairs of conditions

<u>Class A:</u>

$$\left.\begin{array}{l}
\dfrac{b_{2\alpha}A_B}{\Omega_{2A}} = \left(\dfrac{4-\alpha}{2-\alpha}\right)\dfrac{\langle\hat{B}_{oB}^2\rangle A_B}{\langle\hat{B}_{oA}^2\rangle A_A}\left(1+2\sigma_G^2/A_A\langle\hat{B}_{oA}^2\rangle\right)^{-1} \ll \Gamma_A', \qquad (7.3a) \\
\text{with} \\
\left\{\begin{array}{l}
\dfrac{b_{1\alpha}A_\alpha}{[2\Omega_A(1+\Gamma_A')]^{\alpha/2}} = \dfrac{\Gamma(1-\alpha/2)\langle\hat{B}_{oB}^\alpha\rangle A_B}{2^{\alpha-1}\Gamma(1+\alpha/2)[2\Omega_A(1+\Gamma_A')]^{\alpha/2}} \ll 1\,, \quad \text{for } \mathcal{E}<\mathcal{E}_B: \qquad (7.3b) \\
\text{and} \\
\dfrac{b_{2\alpha}^2\ A_B}{[2\Omega_A(1+\Gamma_A')]^2} = \left(\dfrac{4-\alpha}{2-\alpha}\right)^2\dfrac{\langle\hat{B}_{oB}^2\rangle^2 A_B}{[2\Omega_A(1+\Gamma_A')]^2} \ll 1, \text{ for } \mathcal{E}>\mathcal{E}_B \qquad (7.3c)
\end{array}\right.
\end{array}\right\}.$$

From (2.37), (2.38), we also note that $A_{A,B}=\gamma_\infty\bar{T}_{s(A,B)}$. Equation (7.3a) is usually the weaker condition and (7.3b,c) the stronger, with the Impulsive Indexes $A_{A,B}$ not too large. A useful, rough rule for considering the interference to be Class A only is, in effect, the condition $A_B \ll A_A$, or $\bar{T}_{sB} \ll \bar{T}_{sA}$, the latter representing the fact that the amount of time, on the average, that the Class A component is present, is much larger than that for the Class B term, a not at all surprising condition from an intuitive viewpoint. The conditions $(\ll 1, \Gamma_A')$ in (7.3) are, of course, matters of judgment, usually $O(10^{-2},10^{-3})$ is sufficient, unless we are concerned with the very rare events, i.e., extremely large values of $\mathcal{E}$. In general, we shall adopt the stricter conditions (7.3b,c). (We shall pursue the detailed anatomy of these and the Class B conditions further in a later study.)

<u>When Class B noise is heavily dominant,</u> on the other hand, we rewrite (7.1b) to include the independent gaussian component embodied in Γ_A' with $\hat{F}_1(ia_B\lambda)_B$, (2.90) and (2.93), so that we have now for the Class A contribution

$$\hat{F}_1(ia_B\lambda)_A \doteq e^{-A_A a_B^2 \langle \hat{B}_{oA}^2 \rangle \lambda^2/4+O(\lambda^4)} = 1- \frac{A_A\langle \hat{B}_{oA}^2 \rangle \lambda^2}{8\Omega_B(1+\Gamma_B)} + O(\lambda^4) \ . \qquad (7.4)$$

Accordingly, we get

$$\hat{F}_1(ia_B\lambda)_{B+G} \doteq [\text{Eqs. (2.90) and (2.93)}] \ , \qquad (7.5)$$

provided the single condition

<u>Class B:</u>

$$\frac{A_A\langle \hat{B}_{oA}^2 \rangle}{8\Omega_{2B}(1+\Gamma_B')} = \frac{1}{4}\left(\frac{2-\alpha}{4-\alpha}\right)\frac{\langle \hat{B}_{oA}^2 \rangle A_A}{\langle \hat{B}_{oB}^2 \rangle A_B} \cdot \left[1+\sigma_G^2/2\left(\frac{4-\alpha}{2-\alpha}\right)\hat{B}_{oB}^2 A_B\right]^{-1} << 1 \qquad (7.6)$$

is obeyed. Again, the simple, intuitively obvious condition is that $A_A << A_B$ (or $\bar{T}_{sA} << \bar{T}_{sB}$): the Class B interference is "on" for a much longer period than the Class A component. The amount of Class A noise is negligible vis-à-vis the Class B contribution. As expected, this condition is, not unexpectedly, just the reverse of that (roughly) required for Class A dominance. [Note, too, that the more precise condition (7.6) is a kind of inverse of conditions (7.3b,c) above for the Class A cases.]