STATISTICAL-PHYSICAL MODELS OF MAN-MADE AND NATURAL* RADIO NOISE
PART II: FIRST ORDER PROBABILITY MODELS OF THE ENVELOPE AND PHASE

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Most man-made and natural electromagnetic interferences are highly non-gaussian random processes, whose degrading effects on system performance can be severe, particularly on most conventional systems, which are designed for optimal or near optimal performance against normal noise. In addition, the nature, origins, measurement and prediction of the general EM interference environment are a major concern of an adequate spectral management program. Accordingly, this second study in a continuing series [cf. Middleton, 1974] is devoted to the development of analytically tractable, experimentally verifiable, statistical-physical models of such electromagnetic interference.

Here, classification into three major types of noise is made: Class A (narrowband vis-à-vis the receiver), Class B (broadband vis-à-vis the receiver), and Class C (=Class A+Class B). First-order statistical models are constructed for the Class A and Class B cases. In particular, the APD (a posteriori probability distribution) or exceedance probability, PD, viz. $P_1(\mathcal{E}>\mathcal{E}_0)_{A,B}$, and the associated probability densities, pdf's, $w_1(\mathcal{E})_{A,B}$, of the envelope are obtained; [the phase is shown to be uniformly distributed in $(0,2\pi)$]. These results are canonical, i.e., their analytic forms are invariant of the particular noise source and its quantifying parameter values, levels, etc. Class A interference is described by a 3-parameter model, Class B noise by a 6-parameter model. All parameters are deducible from measurement, and like the APD's and pdf's, are also canonical in form: their structure is based on the general physics underlying the propagation and reception processes involved, and they, too, are invariant with respect to form and occurrence of particular interference sources.

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* The title of this and succeeding Reports in this series, is modified slightly, to emphasize the scope of application, which includes natural as well as man-made interference.

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Excellent agreement between theory and experiment is demonstrated, for many types of EM noise, man-made and natural, as shown by a broad spectrum of examples. Results for the moments of these distributions are included, and more precise analytical conditions for distinguishing between Class A,B, and C interference are also given. Methods for estimating the canonical model parameters from experimental data (essentially embodied in the APD) are outlined in some detail, and a program of possible next steps in developing the theory of these highly nongaussian random processes for application to general problems of spectrum management is presented.

Key Words: Man-made radio noise, Radio noise models, Statistical communication theory.

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PART I: INTRODUCTION, RESULTS AND CONCLUSIONS

1. INTRODUCTION

As in previous studies (for example, [Middleton, 1972a, 1972b, 1973, 1974]) our central problem is to construct analytically tractable models of man-made and natural radio noise. This is done for three principal technical purposes:

(i). to provide realistic, quantitative descriptions of man-made and natural electromagnetic (EM) interference environments;

(ii). to specify and guide experiments for measuring such interference environments; and,

(iii). to determine the structure of optimal communication systems and to evaluate and compare their performance with that of specified, suboptimum systems, when operating in these general classes of EM interference.

These three tasks, in turn, are critical elements in any adequate program of spectrum management [for example, [Middleton, 1975a].

Our aim here, then, as earlier in this series [cf. Part I, Middleton 1974] is to provide analytical models (1), which combine the appropriate physical and statistical descriptions of general EM interference

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* The title of this, and succeeding Reports in this series, is modified slightly, to emphasize the scope of application, which includes natural as well as man-made interference. Similar remarks apply to the initial study in this series (Part I: OT Report 74-36, April, 1974).

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environments; (2), which are analytically manageable; (3), which possess general, canonical properties - i.e., are not specialized to individual noise mechanisms, source distributions, and emission waveforms, for example; and most important, (4), which are both experimentally verifiable and predictive. In addition, the basic, or "generic" parameters of such statistical-physical models must be measurable quantitites with specified physical structure and interpretation. To achieve this is clearly a nontrivial problem, mainly because of the inherent, highly nongaussian nature of these random processes, a characteristic which at once predicates complex descriptions, and resulting difficulties for the analysis of system performance. That these difficulties can be effectively overcome for model-building (i), and experimental verification (ii) will be evident from the results and analyses in this report (also [Middleton, 1974]). For receiver design and performance (iii), this has already been established by recent work of Spaulding and Middleton [1975].

1.1 Classification of EM Interference:

General EM interference environments can be conveniently classified into three broad categories of interference vis-à-vis any narrow-band* receiver:

Class A Interference: This noise is typically narrower spectrally than the receiver in question, and as such generates ignorable transients in the receiver's front-end (i.e., initial linear stages, viz. aperture-RF-IF) when a source emission terminates;

Class B Interference: Here the bandwidth of the incoming noise is larger than that of the receiver's front-end stages, so that transient effects, both in the build-up and

* This can be broadened to include receivers of arbitrary bandwidth. However, for almost all EM applications narrow band receivers (i.e., those for which the bandwidth of the initial, linear stages is much less than the RF (and IF) central frequencies) are used exclusively. Henceforth here we shall accordingly consider only narrowband receivers. (The IF-stage is regarded as linear, as far as the narrow-band input is concerned, i.e., heterodyning from RF to IF frequency is linear in the input wave.)

decay occur, with the latter predominating. The receiver is to varying degrees "shock-excited", particularly for inputs of very short duration, so that the receiver is said to "ring".

Class C Interference: This is the sum of Class A and Class B interference, which can occur either because of the presence of sources of mixed types (producing Class A, Class B emissions vis-à-vis the receiver), and/or because any received emission is itself strictly Class C: there is always a build-up interval and a decaying transient period in any receiver front-end reaction to an incoming emission. Effective Class C occurs in this latter instance where the build-up and decay times (at comparable levels) are themselves comparable.

For Class A noise the transient decay period is negligible vis-à-vis the emission's duration, while for Class B interference it is highly dominant. See, for example, Fig. (2.1), Part II, following. [More precise, quantitative conditions specifying Class A, or Class B types, vis-à-vis Class C and each other, are derived in Section 7, Part II.]

The above three categories for interference, as it impacts on a typical (narrow-band) receiver, e.g., as (the linear, front-end of) that receiver responds to the EM environment, provide a useful way of describing the different effects which these different categories have on reception. This categorization is useful because receiver response is statistically different for each Class. As will be seen presently, these differences appear most generally and explicitly (as far as first-order statistics are concerned cerned) in the experimentally derived, and theoretically determined exceedance probabilities (PD's) [also often called APD's (a posteriori probability distributions, cf. Spaulding, [1971])], such as $P_1(X>X_0)$, or $P_1(\mathcal{E}>\mathcal{E}_0)$, which are the respective probabilities that the instantaneous amplitude, or instantaneous envelope observed at the receiver's IF output exceed some threshold X_0, or $\mathcal{E}_0$, as these latter are allowed to assume values in the interval $(-\infty,\infty)$, or (o,∞). Furthermore, this categorization is recommended because the conditions governing the various Classes are simple to distinguish, cf. remarks in Section 7 (II). The conditions "spectrally broader than", and "spectrally narrower than", cf. Fig. (1.1), are to be interpreted as "sufficiently broader or narrower", etc., where in any case, care is taken to refer to the definitions of Class A,B, etc., in terms of the residual transients vs. the "on"-time of the input emission which appears at the output of the IF stage of the receiver in question.

It is instructive to extend our schema of classification further, in order to distinguish between man-made and natural interference, and between "intelligent" and "nonintelligent" emissions. Accordingly, we define:

(i). "Intelligent" noise or interference as man-made and intended to convey a message or information of some sort; whereas,

(ii). "Nonintelligent" noise or interference may be attributable to natural phenomena, e.g., atmospheric noise or receiver noise, for example, or may be man-made, but conveys no intended communication, such as automobile ignition, or radiation from power lines, etc.

[We remark again [cf. Middleton, 1960, Sec. 1.3-5] that by definition, "noise" or "interference" is any undesired "signal" at or in the receiver, regardless of origin.] The importance of distinguishing man-made from natural noise lies in the fact that the former is potentially controllable, sometimes to the point of elimination, whereas the latter cannot be eliminated, at the source, and is usually not subject to control: one can seek only to investigate its effects on the communication process. Moreover, the distinction between "intelligent" and "nonintelligent" is always significant with regard to information transfer: the taxonomy of the former can have greatly different implications and consequences from that of the latter.

We can readily tabulate these different varieties of interference,

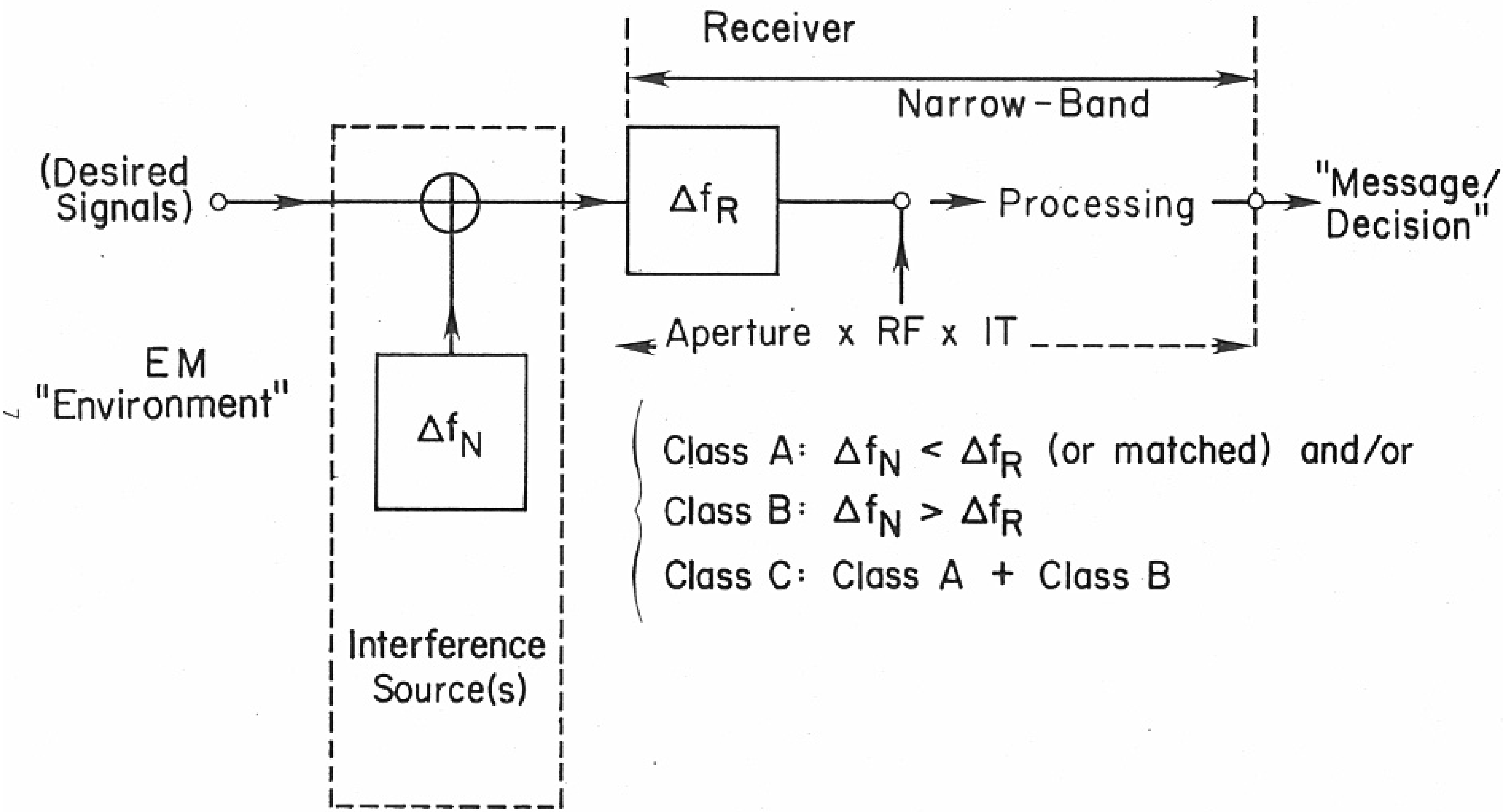

Figure 1.1. Schema of the EM interference and desired signal environment vis-à-vis a typical narrow-band receiver.

in a concise way as suggested in Table (1.1) below:

Table 1.1 Interference Categories* and Classes

Type	"Intelligent"	Class	"Nonintelligent"	Class
Man-Made	1). Compatible 2). Incompatible (Communication) [3). Extra-terrestrial (Communication)	A A,B,C A,B,C]	1). Automobile ignition 2). Other EM emissions: power lines, electric tools, etc.	B A,B,C
Natural			1). Atmospheric 2). Extra-terrestrial solar, galactic, cosmic radiation, etc.	B [A], B,C

* The listing here is not intended to be exhaustive.

We have included a further refinement through the term "compatible". By definition, compatible interference here is one that is appropriately matched spectrally to the receiver band Δf_{ARI}, in the sense of being equivalent to Class A interference vis-à-vis the receiver and occupying a spectral region in Δf_{ARI}, and such as to produce ignorable transients in the ARI-stages. "Incompatible" may mean that $\Delta f_N > \Delta f_{ARI}$ (Class B), or that only a portion of the incident emission is spectrally available to the receiver: Class A again, e.g. $\Delta f_{N\text{-effective}} < \Delta f_{ARI}$, but now the interference is not wholly in the receiver band Δf_{ARI}. Class C above in the Table reminds us that combinations of Class A and B noise can occur, as well.

1.2 Earlier Work:

For the most part, earlier efforts at modelling man-made and natural noise (principally atmospheric noise) have produced a wide variety of analytical results, often with the virtue of mathematical simplicity, but severely limited in usefulness by lack of generality and physical insight, and a concommitant dependence on local, empirical data and circumstances. Two important exceptions to the above are the work of Furutsu and Ishida [1960] on obtaining the APD's (and associated probability densities [pdf's]) of atmospheric noise under rather broad conditions, and the more recent studies of Giordano [1970], and Giordano and Haber [1972], similarly directed to atmospheric noise. Both sets of intestigations, however, are (necessarily) constrained to Class B types of interference (cf. Sec. (1.1) above), and neither attempts the canonical formulation, which is a key feature of our current efforts [Middleton, 1972, 1973, 1974, and this Report; see also comments, Sec. 5.3 (II)] following. This canonical formulation allows us to apply the new models formally by Class(A,B, etc.) to all types of (EM) interference, unrestricted in general structure by the particular physical mechanism involved. [These latter, of course, determine the generic properties of the model parameters, and must be specifically introduced into model building if the ad hoc and arbitrary empiricism of much of the earlier work is to be avoided.] For a more detailed review of earlier work vis-à-vis this newer approach, see Chapter 2 of Spaulding and Middleton [1975], and references therein.

1.3 New Results:

The principal new results of this study may be briefly introduced here, in contrast to our remarks above on previous work. Here we obtain canonical, analytical, first-order statistical models of both Class A and Class B interference, specifically for the envelope (E) and phase (ψ) of the narrow-band output of the composite aperture-RF-IF stages of a typical receiver. As noted above, these models are based on a general physical mechanism (cf. Section 2, Middleton [1974], for example), providing, among other things, insight into the parameter structure, as well as contributing, in a broad way to the analytical form of the probability distribution (PD's) and probability densities (pdf's) themselves, which are the principal results here. In addition, the general method of approximating the governing (1st-order) characteristic functions (c.f.'s) is described, which enables us to obtain the required canonical structures in tractable analytic forms. These, in turn, give the resulting analytical models their broad applicability, unrestricted by particular physical mechanisms, and, in fact, controlled only by the underlying poissonian postulate of independent source emissions in space and time [cf. Section (2.1) Part II].

Included, also, are specific procedures for determining the model parameters from experimental data, analytical results for the first-order, Class A and B moments of the envelope, and detailed, quantitative conditions for specifying Class A or Class B interference. Excellent agreement with experiment is found, and a variety of comparisons of theory with experiment is included, involving many different physical types of radio interference, not only to illustrate this agreement, but to demonstrate the canonical character of the approach as well, cf. Section (2.4). Finally, the definition of Class A models, and their quantitative identification with observed noise processes are new, although, of course, such interference has been physically present for many years. Class B models are "classical", although not so designated until now, but here, again, our present approach is to a large extent original, particularly with regard to canonical results.

1.4 Organization of the Report:

As one can see from the Table of Contents, this Report is divided into two principal units: Parts I and II. Part I contains introductory, background material (Section 1), and in Section 2 following an extensive summary and discussion of the main results, as well as related matters and next steps in our program of interference modelling. Part II, on the other hand, is devoted to the detailed analytical development of the theory: Section 2(II) describes the canonical approximations of the characteristic function (c.f.) required for the Class A and B envelope distributions. Sections 3(II) and 4(II) are devoted to these distributions and distribution densitees, while Section 5(II) contains results for the (first-order) moments. In Section 6(II) the problem of determining model parameters from experimental data is addressed, again for Class A and B interference. Section

7(II) completes this study with the derivation of analytical conditions which quantitatively determine when a Class A, or Class B model is appropriate.