ATTENUATION OF HIGH-FREQUENCY GROUND WAVES OVER AN INHOMOGENEOUS EARTH

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The primary objective of this study was to obtain theoretical estimates of the effect of inhomogeneities in the earth's surface upon ground-wave propagation along a particular set of land-sea paths.

The results were obtained by numerically solving the integral equation

$$f^*(d) = f(d) + \int g(d, \theta)\, f(-d \cos \theta)\, f^* (d \cos \theta)\, d\theta,$$

where f^* is the attenuation function for the inhomogeneous case, f is the Sommerfeld function, and g is obtained essentially from the spherical Green's function by a steepest-descent type integration.

Sharp phase and amplitude changes in the attenuation function occur when crossing an "island" or inhomogeneity in the paths. The greater the difference in conductivity and dielectric constant between the island and the rest of the path, the greater are these changes. Also noted is the "recovery" or "focusing" effect found in the amplitude and phase. The effect of moving the transmitting antenna across a coastline was also studied and the results were quite similar to the above.

Computations were performed for three paths at frequencies of 10, 15, 20, and 25 MHz. The results are displayed in tabular and graphical form.

This report is a revised and updated version of an earlier study (Rosich, 1968) which is now out of print.

Key Words: Electromagnetic waves; ground wave; integral equations; propagation; radio waves; Sommerfeld solution; surface waves

I. INTRODUCTION

This report describes the model used and the results obtained in a study (Rosich, 1968) of the effect that inhomogeneities in the earth's surface have upon the attenuation of the surface-wave component of the ground-wave electric field at high frequencies. Results are also presented from a more recent study (Rosich, 1969) of the effect on the attenuation of the placement of antennas near a coastline. All of these results were obtained from a model (Hufford, 1952; Wait, 1956; King, 1965) where the attenuation function is given by an integral equation that is solved numerically. The computations presented here were made for a particular set of land-sea paths (fig. 1) to aid in the design and evaluation of high-frequency ground-wave paths from the Naval Research Laboratory's Chesapeake Bay installation. Since the time of publication of the original report (Rosich, 1968), however, a number of things have come to pass: (1) the report has come into great demand, presumably because it indicates the type of behavior to be expected at high frequencies, (2) the report has gone out of print, and (3) better models (see for example, Ott and Berry, 1970) have been developed than that used in the report. In order to solve the problem caused by (1) and (2) and to direct further attention to the significant advances embodied in (3), this report is being issued. With particular regard to (3), we shall display some comparisons of the model presented here with that developed by Ott and Berry (1970) and Ott (1971a, b). These comparisons will also help to point out the accuracy and limitations of the model presented in this report.

Since this is an updating and improvement of the original report (Rosich, 1968), and since it contains all of the material there and more, it is intended to supplant the earlier report.

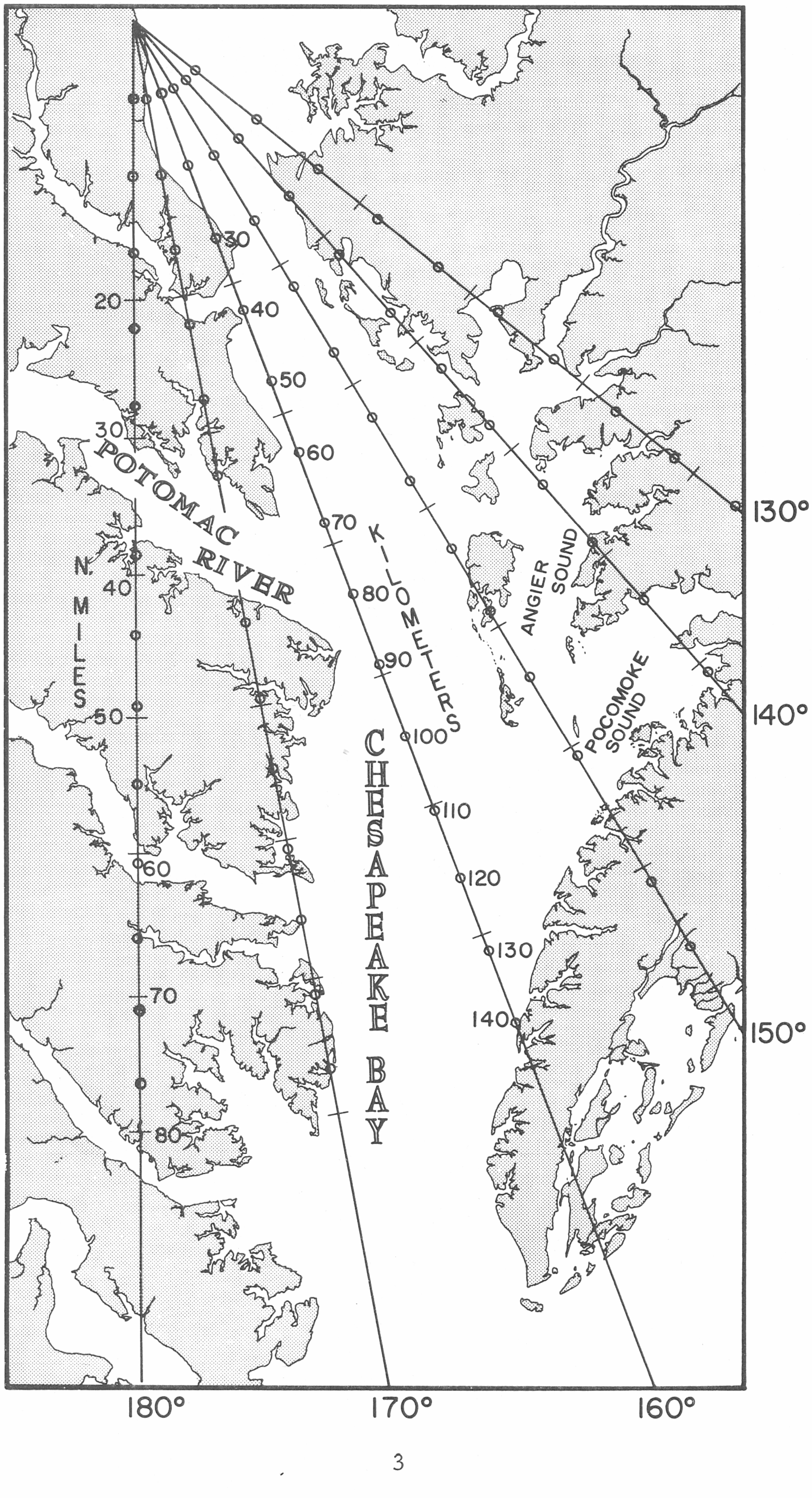

Figure 1. Map of the propagation paths on the Chesapeake Bay.

2. THEORY AND COMPUTER PROGRAM

The problem of ground-wave propagation has a long history and has been covered extensively in the literature. A rather complete list of this literature can be found in a paper by Wait (1964), which includes a very good discussion of the problem.

To make the results given in this report more meaningful, an outline of the method of solution will be presented. It is meant only as a heuristic discussion and not as a rigorous treatment; quite interesting and rigorous treatments can be found in papers by Hufford (1952), Wait (1964), Ott and Berry (1970), and Ott (1971a, b).

We begin by considering Maxwell's equations. It can be shown (Stratton, 1941, ch. 1) for a linear isotropic medium of permittivity ϵ, permeability μ, and conductivity σ, that Maxwell's equations can be expressed in terms of the Hertzian potential $\vec{\Pi}$, which must satisfy the Helmholtz equation

$$\nabla^2 \vec{\Pi} + k^2 \vec{\Pi} = 0 \tag{1}$$

at all ordinary points in space, and which has the following relation to the $\vec{E}$ and $\vec{H}$ fields:

$$\vec{H} = (1/\mu)\, \vec{\nabla} \times (i\omega\mu\epsilon\, \vec{\Pi}) \,, \tag{2}$$

$$\vec{E} = \vec{\nabla}(\vec{\nabla} \cdot \vec{\Pi}) + \omega^2 \mu\epsilon\, \vec{\Pi} \,. \tag{3}$$

In (1), k is the complex wave number

$$k^2 = k_o^2\, \mu_r\, \epsilon_c \,, \tag{4}$$

where

$$\epsilon_c = \epsilon_r - i\,\sigma/\omega\,\epsilon_o \,, \quad \mu_r = \mu/\mu_o \,, \quad \epsilon_r = \epsilon/\epsilon_o \,, \tag{5}$$

and where μ_0 and ϵ_0 are the free-space values of μ and ϵ; k_0 is the free-space wave number (ω/c); ω is the angular wave frequency; c is the velocity of light; an ordinary point is one in whose neighborhood the physical and electrical properties of the medium vary in a smooth and continuous manner; and $\vec{\Pi}$ is assumed to contain an implicit time dependence of $\exp(i\omega t)$. Since the transmitting and receiving antennas and the earth's surface mark points where the properties of the medium change abruptly, they are not ordinary points and, hence, must be excluded from the space where (1) is valid. This space is contained within the surface S shown in figure 2. This surface consists of a half-sphere at infinity S_∞, the earth's surface S_e, and the small sphere and half-sphere around the singular points. The antennas are assumed to be <u>short</u> vertical electric dipoles, in which case $\vec{\Pi}$ for the radiation field in the vicinity of the transmitting antenna takes the particularly simple form (Stratton, 1941, sec. 8.4 - 8.7)

$$\vec{\Pi}(\text{W near T}) \cong \Pi_0 \frac{\exp(-i k_0 |\vec{d}|)}{|\vec{d}|} \hat{z} , \tag{6}$$

where

$$\Pi_0 = - \frac{i I_0 \ell}{4\pi \omega \epsilon_0} , \tag{7}$$

and where I_0 is the amplitude of the current in the antenna, ℓ is the effective length of the antenna, and $\hat{z}$ is a unit vector in the z-direction. Using the above assumption, applying Green's theorem (Morse, 1953, sec. 7.2) to (1), and using the fact that the integral vanishes on S_∞, we obtain the following form for the z-component, Π, of $\vec{\Pi}$ in the vicinity of the receiver:

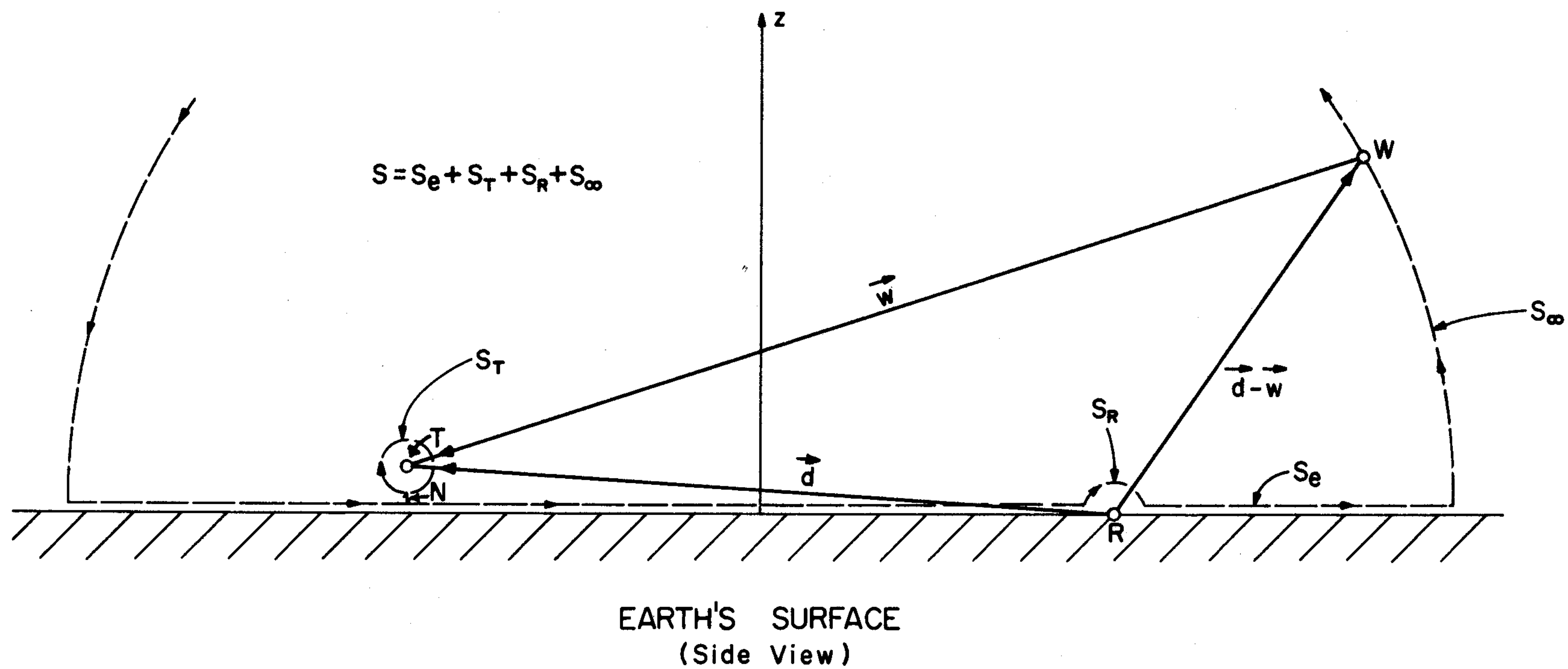

Figure 2. Side view of the path geometry and surfaces of integration.

$$\Pi(R) = 2\Pi_o \frac{\exp(-i k_o |\vec{d}|)}{|\vec{d}|}$$

$$+ \frac{1}{2\pi} \int_{S_e}\int \left[\Pi(W) \frac{\partial}{\partial z} \cdot \left(\frac{\exp(-i k_o |\vec{w}|)}{|\vec{w}|} \right) \right.$$

$$\left. - \frac{\exp(-i k_o |\vec{w}|)}{|\vec{w}|} \cdot \frac{\partial \Pi(W)}{\partial z} \right] dS . \tag{8}$$

All other components of $\vec{\Pi}$ are zero. If we now further assume that the ratio of $(\partial \Pi/\partial z)$ to Π depends only upon the physical and electrical properties of the earth's surface at each point, that is, assume that the concept of surface impedance is valid (Godzinski, 1961), then we have the following relation (Bremmer, 1954; King, 1965; and others),

$$\partial \Pi(W)/\partial z \cong (ik_o/\eta_o)\, Z(W)\, \Pi(W) \tag{9}$$

at the earth's surface, where

$$Z(W) = \eta_o / \sqrt{\epsilon_c(W)} . \tag{10}$$

Equation (8) then takes the form

$$\Pi(R) = 2\Pi_o \frac{\exp(-i k_o |\vec{d}|)}{|\vec{d}|}$$

$$- (ik_o / 2\pi\eta_o) \int_{S_e}\int \frac{\exp(-ik_o |\vec{w}|)}{|\vec{w}|} Z(W)\, \Pi(W)\, dS . \tag{11}$$

Note that the first term under the integral in (8) is zero because the z-derivative of $[\exp(-ik_0|\vec{w}|)/|\vec{w}|]$ is zero on a flat surface (S_e). Equation (11) is the general integral equation for ground-wave propagation over a plane surface whose electrical properties vary as Z(W).

In the case of a homogeneous surface, that is Z(W) constant, we may assume the following form for Π:

$$\Pi(W) = 2\Pi_0 \frac{\exp(-ik_0|\vec{d}-\vec{w}|)}{|\vec{d}-\vec{w}|} F(W, Z) , \tag{12}$$

where Π is a composite of the field due to a vertical electric dipole over an infinitely conducting plane surface and the attenuation, F(W, Z), up to the point W due to the actual plane surface of impedance Z. The substitution of this into (11) produces an equation which is the usual form for the Sommerfeld equation for ground-wave propagation over a plane homogeneous earth, namely,

$$F(R, Z) = 1 - (ik_0 Z/2\pi\eta_0) \; |\vec{d}| \exp(ik_0|\vec{d}|)$$

$$\times \int_{S_e}\int \frac{\exp\left(-ik_0(|\vec{w}| + |\vec{d}-\vec{w}|)\right)}{|\vec{w}| \cdot |\vec{d}-\vec{w}|}$$

$$\times F(W, Z)\, dS . \tag{13}$$

In the inhomogeneous case, that is, Z(W) not constant, we could replace F(W, Z) in (12) by $\tilde{F}(W, Z(W))$ and obtain an equation similar to (13) above with Z(W) under the integral. In the case where the surface S_e is divided into two regions S_e' and S_e'' (refer to fig. 3) with values of Z(W) of Z and Z_1, respectively, it is more convenient to proceed as follows. Supposing S_e' to be the largest of the two regions and S_e'' only a perturbation (an "island"), we can replace (6) by

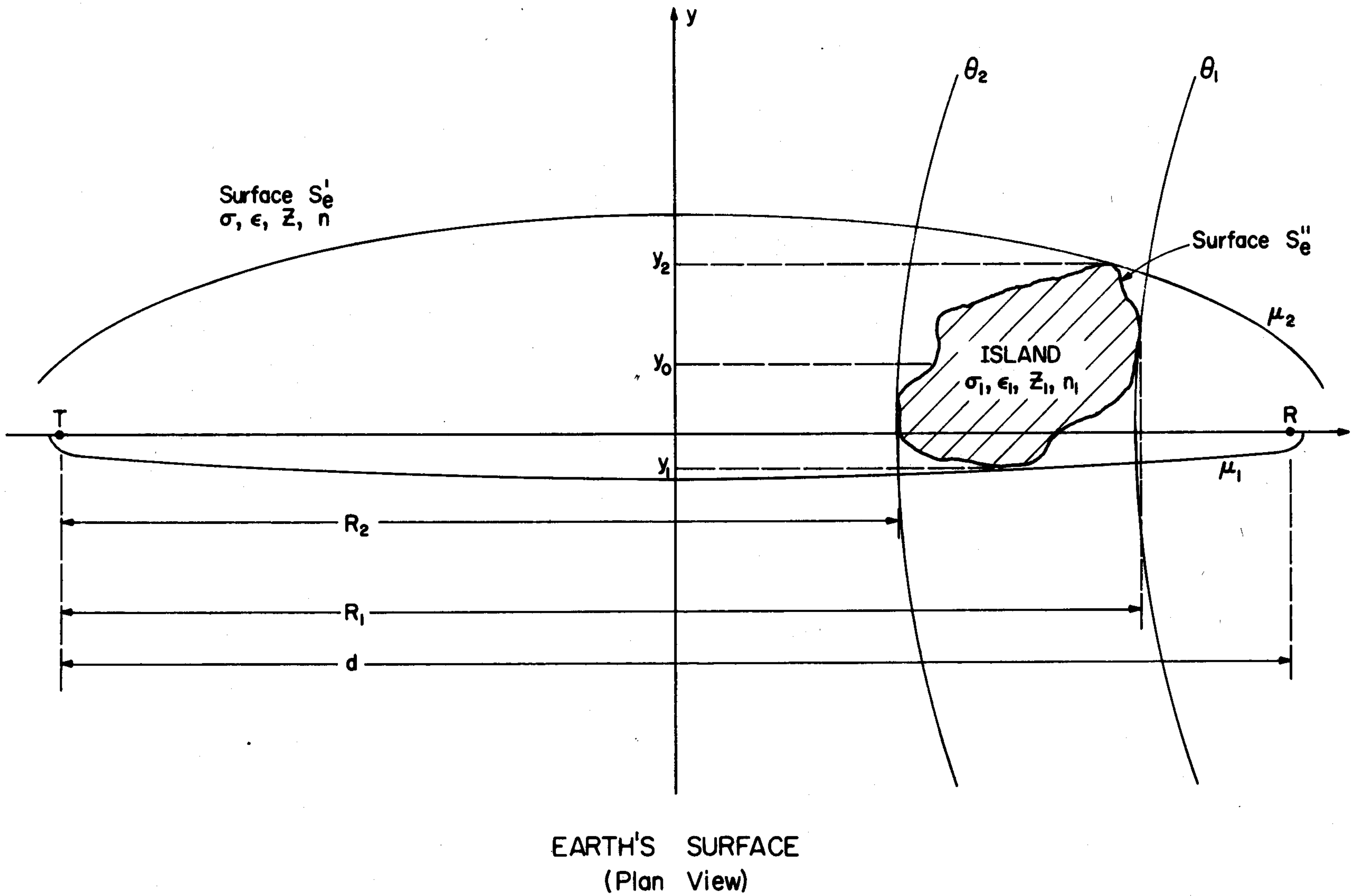

Figure 3. Top view of the path geometry.

$$\vec{\Pi}(W \text{ near } T) \cong \Pi_o \frac{\exp(-i k_o |\vec{d}|)}{|\vec{d}|} F(R, Z)\, \hat{z}\,, \tag{14}$$

(12) by

$$\Pi(W) = 2\Pi_o \frac{\exp(-i k_o |\vec{d} - \vec{w}|)}{|\vec{d} - \vec{w}|} F^*(W, Z, Z_1)\,, \tag{15}$$

$Z(W)$ by $(Z - Z_1)$, and S_e by S_e'', proceed as above, and obtain the following result:

$$F^*(R, Z, Z_1) = F(R, Z) - [i k_0 (Z - Z_1)/2\pi \eta_0] |\vec{d}| \exp(i k_0 |\vec{d}|)$$

$$\times \iint_{S_e''} \frac{\exp\left(-i k_o(|\vec{w}| + |\vec{d} - \vec{w}|)\right)}{|\vec{w}| \cdot |\vec{d} - \vec{w}|}$$

$$\times F(W, Z)\, F^*(W, Z, Z_1)\, dS\,. \tag{16}$$

If one now transforms this equation into elliptical coordinates, (μ, θ), and applies the method of steepest descent to evaluate the μ-integral, (16) becomes (King, 1965; Tsukamoto et al., 1966) for an "island" inhomogeneity

$$F^*(d, Z, Z_1) = F(d, Z) - \left(K(\mu_1, \mu_2)(Z - Z_1)/2\, \eta_0\right) \left(\frac{id}{\lambda}\right)^{\frac{1}{2}}$$

$$\times \int_{\theta_1}^{\theta_2} F^*\left(\tfrac{1}{2} d(\cosh \mu_o + \cos \theta),\, Z,\, Z_1\right)$$

$$\times F\left(\tfrac{1}{2} d(\cosh \mu_o - \cos \theta),\, Z\right)$$

$$\times (\sinh^2 \mu_o - \sin^2 \theta) / (\cosh^2 \mu_o - \cos^2 \theta)\, d\theta\,, \tag{17}$$

where d is the distance between the transmitter and receiver, μ_1, μ_2, θ_1, and θ_2 define the boundaries of the "island" and μ_0 its "center-line" in the elliptical coordinate system (see fig. 3), and,

$$K(\mu_1, \mu_2) = \operatorname{erfc}\left(\sqrt{i\, k_0\, d/2}\, \mu_1\right) - \operatorname{erfc}\left(\sqrt{i\, k_0\, d/2}\, \mu_2\right) . \qquad (18)$$

The "center-line" μ_0 refers to the center line of the steepest descent integral and in our case is the value of μ ($=\mu_0$) corresponding to the value of y ($=y_0$) on the "island" which is closest to the line $\overline{TR}$ (the x-axis). In figure 3, therefore, y_0 should be zero. The y_0-line in figure 3 is displaced from this value only for the sake of illustrating its presence and is thereby unfortunately somewhat misleading. As a further matter of practical use of the model, the "steepest-descent-center-line" role of y_0 implies that R_1 and R_2 should be chosen as the "front" and "rear" edges of the "island" along the y_0-line when $\overline{TR}$ crosses the "island." If $\overline{TR}$ does not cross the "island," the largest and smallest values of R (projected onto the x-axis) should be used.

Equation (17) must now be solved to obtain the solution to the inhomogeneous case. An iterative technique can be used to obtain a numerical solution. An initial approximation for F* is substituted into the right side of (17) and this whole expression is then evaluated. From the left side of (17), we see that this result is the next approximation to F* and hence can itself be inserted on the right side. Iteration proceeds in this manner until the desired accuracy is attained. The convergence of this method (the Neumann method) has been studied extensively (Lovitt, 1950, pp. 7, 110, 114); suffice it to remark (King, 1965) that it converges in our case. Following Tsukamoto et al. (1966), we chose as the initial approximation to F*, the attenuation function for perpendicular propagation across a straight boundary, namely,

$$F_o^*(d, Z, Z_1) = F(d, Z) + \left((Z - Z_1)/\eta_0\right)\left(\frac{id}{\lambda}\right)^{\frac{1}{2}}$$

$$\times \int_0^{\theta_2} F\left(\tfrac{1}{2}\, d(\cosh \mu_o + \cos\theta), Z\right)$$

$$\times\ F\left(\tfrac{1}{2} d(\cosh \mu_o - \cos\theta), Z_1\right) d\theta\ , \qquad (19)$$

where F is the Sommerfeld attenuation function and is given by

$$F(r, Z) \cong 1 - i\sqrt{\pi p}\ \exp(-p)\ \mathrm{erfc}(i\sqrt{p})\ , \qquad (20)$$

with

$$p = -(i k_o\, r/2)\ (Z/\eta_o)^2\ , \qquad (21)$$

and

$$\mathrm{erfc}(x) = (2/\sqrt{\pi}) \int_x^{\infty} e^{-t^2}\, dt\ . \qquad (22)$$

The results presented in section 7 of this report were obtained from a computer program that calculates F and F^* by implementing the method of solution described in the paragraph above. The program is an adapted and modified version of a program used by King (1965) in his dissertation, and a much more complete description of the program and the problem can be found there.

3. CALCULATIONS

The results of the calculations based on the method and program described above are contained in tables 1 through 12 and graphs 1 through 24. The odd-numbered graphs contain plots of the amplitude of F and F^* versus the distance from the transmitter on the bearing indicated, while the even-numbered graphs contain plots of the phase of F and F^* versus distance. Recall that F and F^* are the attenuation functions in the homogeneous and inhomogeneous cases, respectively.

To obtain the electric field from these results, one need only combine (12) and (15) with (3). The results are

$$\vec{E}(d) = i(I_o \ell \mu_o \omega/2\pi)\left(\exp(-i k_o |\vec{d}|)/|\vec{d}|\right) F(d, Z)\hat{z} , \tag{23}$$

$$\vec{E}^*(d) = i(I_o \ell \mu_o \omega/2\pi)\left(\exp(-i k_o |\vec{d}|)/|\vec{d}|\right) F^*(d, Z, Z_1)\hat{z} , \tag{24}$$

where $\vec{E}(d)$ is the electric field at a distance d over a homogeneous plane earth of impedance Z, while $\vec{E}^*(d)$ is that over an inhomogeneous earth. We can therefore write (23) and (24) approximately as

$$|E(d)| = \left(|E_o|/d\right) |F(d, Z)| , \tag{25}$$

$$\mathrm{Arg}\left(E(d)\right) = \mathrm{Arg}(E_o) + (\pi/2) - k_o d + \mathrm{Arg}\left(F(d, Z)\right) , \tag{26}$$

$$|E^*(d)| = \left(|E_o|/d\right) |F^*(d, Z, Z_1)| , \tag{27}$$

$$\mathrm{Arg}\left(E^*(d)\right) = \mathrm{Arg}(E_o) + (\pi/2) - k_o d + \mathrm{Arg}\left(F^*(d, Z, Z_1)\right) , \tag{28}$$

where E and E^* are the z-components of $\vec{E}$ and $\vec{E}^*$, respectively, Arg denotes the phase of a quantity, and E_o can be taken to be approximately the electric (radiation) field (times $d \cong \lambda$, of course) generated by the actual antenna in its proximity ($d \cong \lambda$) on the desired bearing.

Returning to the results contained in section 7, we note that they represent the calculations for two distinct paths on bearings of 150° and 160° from a fixed transmitting site. In all cases the transmitter is taken to be at the U.S. Naval Reservation (38° 39' 17'' N, 76° 31' 40'' W) just north of Locust Grove Beach, Maryland, on the shore of the Chesapeake Bay. In what follows, reference to the map in figure 1 is suggested, and the parameter 'd' is the distance from the transmitter.

Graphs 1 through 8 and tables 1 through 4 are for a bearing of 160° E of N from the transmitter. The four cases contained in this set are for frequencies of 10, 15, 20, and 25 MHz, in that order. On this path, Chesapeake Bay is assumed to be homogeneous with electrical parameters of

$$\sigma = 2.0 \text{ mho/m}, \quad \epsilon = 81.0\, \epsilon_0 ,$$

while the perturbing inhomogeneity is a section of land 6.85 km long (28.3 km $\le d \le 35.15$ km; 38° 24' 55'' N, 76° 25' 00'' W to 38° 21' 28'' N, 76° 23' 24'' W) across the Cove Point, Maryland area, which is assumed to have the electrical parameters of

$$\sigma_1 = 0.002 \text{ mho/m}, \quad \epsilon_1 = 15.0\, \epsilon_0 .$$

This path ends in the Church Neck, Virginia area ($d \cong 142.57$ km; 37° 26' 51'' N, 75° 58' 31'' W).

Graphs 9 through 16 and tables 5 through 8 are for a bearing of 150° from the transmitter. Again the four cases are for frequencies of 10, 15, 20, and 25 MHz. Chesapeake Bay has the same parameters as above, while the perturbing inhomogeieity is a section of an island

4.32 km long (84.18 km $\leq d \leq$ 88.5 km; 37° 59' 51' N, 76° 02' 50'' W to 37° 57' 50'' N, 76° 01' 23'' W) across the Smith Island, Maryland area which is assumed to have the electrical parameters of (marsh)

$$\sigma_1 = 1.0 \text{ mho/m}, \quad \epsilon_1 = 48.0\,\epsilon_0 .$$

This path ends in the Matcholank Creek, Virginia area ($d \cong 121.3$ km; 37° 42' 32'' N, 75° 50' 17'' W). Because of the possibility of poor earth on this island in addition to marsh, the above calculations were repeated with values of the electrical parameters of

$$\sigma_1 = 0.002 \text{ mho/m}, \quad \epsilon_1 = 15.0\,\epsilon_0 .$$

The results are contained in graphs 17 through 24 and tables 9 through 12.

Let us now note some of the rather prominent features of the results. In each case F and F* naturally agree up to the "island," but in crossing the "island," which is a poorer conductor and dielectric than the surrounding medium, the amplitude falls off very sharply and a large change of phase occurs. After having crossed the "island," the phase and amplitude begin to recover and seem to approach the homogeneous values asymptotically. Note (see, for example, graph 3) that in the cases where the amplitude change is most drastic, the amplitude rises rapidly in the region after the "island" and "peaks up" before beginning to decrease again and asymptotically approach the homogeneous values. This is the so-called "recovery" or "focusing" effect. A similar phenomenon is exhibited in the phase (see, for example, graph 4).

That the asymptotic approach to the homogeneous values is at least <u>qualitatively</u> what should be expected can be seen by an analogy. One can visualize this system as a transmission line of impedance Z_a with a section of line of impedance Z_b inserted somewhere in its length.

If both $|Z_b|$ and $ARG(Z_b)$ are larger than $|Z_a|$ and $ARG(Z_a)$, respectively, as is the case here, then one can easily see that the amplitude should drop and an added phase change should occur in crossing the Z_b section. Asymptotic recovery should occur as the Z_b section becomes a very small perturbation on the system. This latter condition is met when its length is much smaller than the distance between the Z_b section and the point of observation. Since ground-wave propagation can be considered in terms of a transmission line in that it transports energy from one point to another, the above considerations tend to confirm the qualitative features of the results.

The "recovery" or "focusing" effect seen in the amplitude of F^* in the region immediately following the "island" was first discovered and experimentally verified by Millington (1949a; 1949b) for propagation across a coastline. A theoretical explanation of the recovery of the amplitude was also given by Millington (1949c), though the question of the phase change was left open. Both the amplitude and phase recoveries have been treated theoretically by a number of investigators; some discussion of these effects can be found in a paper by Wait (1964). The phase recovery was finally confirmed by Pressey et al. in 1956. Elson (1949) remarks that the recovery phenomenon owes its existence to a vertical redistribution of energy near the boundaries between media, and that this redistribution is inevitable because the field must vary with height differently on either side of the boundary because of the differing electrical parameters. The height-gain function will therefore have a different form depending upon whether the ground is primarily a conductor or primarily a dielectric. For the frequency range considered in this report, sea water has a fairly constant nature as a quasi-conductor, while land changes from a poor conductor at the lower frequencies to a poor dielectric at the higher ones. A very rough measure of the redistribution due to these effects can be seen as follows. According to Wait (1964, p. 199), for low heights and sufficiently far from the coast line, the height-gain function has the approximate form

$$h(z) \cong 1 + ikz(Z/\eta_o) , \tag{29}$$

where Z is the surface impedance, $\eta_o = 120\pi\,\Omega$, $k = 2\pi/\lambda$, and z is the height above the surface. Letting

$$Z = \mathrm{Re}(Z) + i\,\mathrm{Im}(Z) , \tag{30}$$

we obtain

$$h(z) \cong \left(1 - kz\,\mathrm{Im}(Z)/\eta_o\right) + ikz\,\mathrm{Re}(Z)/\eta_o . \tag{31}$$

For $|z| << 1$, we then obtain the following approximate forms for the magnitude and phase of the height-gain function:

$$|h(z)| \cong 1 - \alpha z , \tag{32}$$

$$\mathrm{Arg}\left(h(z)\right) \cong \beta z , \tag{33}$$

where

$$\alpha = k\,\mathrm{Im}(Z)/\eta_o , \quad \beta = k\,\mathrm{Re}(Z)/\eta_o . \tag{34}$$

For Z_1 one obtains, similarly, h_1 depending upon α_1 and β_1. From the headings of the tabular results, λ, Z, and Z_1 can be obtained for each case. The results can be found in the table below.

	All paths		Paths 1 and 3		Path 2	
Frequency	α	β	α_1	β_1	α_1	β_1
10 MHz	2.4	2.5	6.3	53	3.4	3.5
15 MHz	4.5	4.6	6.4	80	6.3	6.5
20 MHz	6.8	7.1	6.4	108	9.6	10.0
25 MHz	9.5	10.0	6.5	134	13.0	14.0

A comparison of these magnitudes with the graphical results shows that the relative magnitude of the recovery of the phase and amplitude follows roughly the above pattern. This lends some credence to the redistribution explanation of the recovery effect. A more detailed analysis and proof is beyond the scope of this report, however.

Next, we wish to consider the effects of moving the transmitting antenna from an assumed site over water, across a coastline, to land. This part of the study (Rosich, 1969) was prompted because of the "island inhomogeneity" restriction of the model used. This restriction coupled with the desire to investigate the attenuation beyond the "island" forced the assumption that transmitting antenna was over water in the Chesapeake Bay. The reasons for this should be clear from the fore going discussion of the model. The actual situation, however, was that the transmitting antenna was behind the coastline on land. If we now are willing to give up values of the attenuation beyond what was previously our "island" on path 1, then we can again use the model to investigate the case where the transmitting antenna is behind the coastline on land. In this latter case, our "island" of inhomogeneity becomes the water between the two sections of land: that at the transmitter and that at the Cove Point, Maryland area. The results for 10 MHz for this path are shown in graphs 25 and 26. In these graphs, the horizontal axis is the distance of the receiver from the coastline, not from the transmitter as in the earlier graphs, and Δ is the distance of the transmitting antenna behind the coastline. Therefore, Δ plus the value on the horizontal axis is equal to the distance from the transmitter to the receiver. Graphs 27 through 30 present the percent changes in the amplitude and phase (relative to the values at $\Delta=0$) at a given distance D (from the coastline) before and across the "island" of our previous discussion (the Cove Point, Maryland area). One can easily see from these graphs that the magnitudes of the changes are largest before the "island," while the percentage change is significant in both regions. In graphs 29 and 30 note particularly the peaks in the curves which are suggestive of some type of focusing effect. This focusing effect points up the fact that for each receiver location, D, there is an optional location, Δ, for the transmitting antenna. As noted above, unfortunately nothing can be said about the region following the "island" (of our previous discussion), as the model only allows a three-section path, where the first and last sections have the same electrical properties. This is a shortcoming in the model for the present case, but the results presented above should outline the importance of the actual antenna sites for high-frequency ground-wave propagation and permit some estimate of the quantitative nature of the effect.

Lastly, in order to prevent misunderstanding, a few questions should be anticipated. They are: (1) How valid are the results for $|\Delta|$ less than a few wavelengths, since the model ignores the static and induction fields of the antennas? (2) Is the case labeled "Δ=0" really this case, since the model ignores the land behind the coastline in this case? In answer to (1), the results of Wait (1963) show that the results should not be altered significantly except in a region $|\Delta| << 1$ (actually a skin depth or so) around the coastline. The only change even then is the removal of a singularity (already removed from the graphs) in the field at the boundaries of the media. Thus, for small values of $|\Delta|$ the results presented here are approximately correct or at least indicative of the behavior. Because of this, and the fact that the model ignores reflections from boundaries and the effect of any media "behind" the transmitter or receiver, the case labeled "Δ=0" should really be labeled "Δ slightly greater than a skin depth in front of the coastline." Since the skin depth here is between

1×10^{-5} and 1×10^{-6} m, "$\Delta=0$" should convey the correct meaning, however. Also, Wait (1963) showed that the reflection effects are relatively small, thus the results are good approximations. This answers objection (2).

4. RECOMMENDATIONS AND CONCLUSIONS

Since many of the recommendations concern the approximations made in the course of obtaining the solutions presented here, we shall first list these approximations, then discuss their advantages, validity, and limitations; finally we shall put forth a number of recommendations for possible improvements to this model. In making these approximations, we (1) assumed that the earth is essentially flat over the distances involved; (2) assumed that the antennas are short vertical electric dipoles; (3) ignored the static and induction fields of the antennas; (4) used the surface impedance concept; (5) assumed that there is only one "island" of the inhomogeneity imbedded in an otherwise homogeneous medium and that within each of these two regions the electrical parameters are constants.

For distances less than 100 km, the flat earth is probably a reasonable approximation, although the matter should be decided by a comparison of the attenuation function for a homogeneous earth in the flat and spherical cases. This would give a good indication of the range of validity of the approximation for the inhomogeneous case also, as it is computed as a perturbation of the homogeneous values.

The antennas in this model have been assumed, for simplicity, to be short vertical electric dipoles, while the actual antenna, as evidenced by its several interacting elements and its radiation pattern, will, in addition, have several higher order multipole terms. If desired, the actual antenna can be modeled in terms of its various multipole moments and the governing equations reformulated and solved. Unless extreme accuracy is desired, however, the use of (25) through (28) with the value of the electric field (times d, of course) generated at the actual antenna for the desired bearing substituted for E_o should suffice.

The third assumption requires that the antennas and all boundaries of the media be sufficiently distant (a few wavelengths) from each other so that only the radiation field interacts. For the physical dimensions and the wavelength ($\lambda < 0.03$ km) considered here, this condition is met; hence, the approximation is valid. Should future work not meet the condition, however, the static and induction terms could be included in the equations in a fashion similar to that for the multipole moments.

The validity of the concept of surface impedance is discussed completely elsewhere (Godzinsky, 1961) and the concept seems reasonable under the conditions imposed here, especially since it simplifies the problem so greatly and since no similar alternative simplifying assumption is known. Of course, this assumption could be eliminated and the resulting boundary-value problem solved exactly, but this greatly complicates the problem, its numerical solution, and the computer program to implement it.

The computer program used at present is constrained as described in approximation (5) above. That this is a severe limitation for some of the ground-wave propagation paths of interest in the Chesapeake Bay area is obvious. At least this scheme should be generalized to allow a number of "islands", and a better decision would be to modify the model to allow for continuous variation of the electrical parameters.

The pragmatic consideration of attempting to get a computer program operational as soon as possible precluded initial inclusion of the refinements outlined above. There was also a desire to verify the model and computer program to be used by experimental or other dependable means. Since a computer program used by a group at the University of Colorado (King, 1965; Tsukamoto et al., 1966) in their comparison of theory and experiment was available, and since its predictions agreed well with experimental results, it was chosen as a starting point for this study.

The numerous modifications (mostly extension of the range of arguments accepted for the various special functions, improved output format, listing of the geographic coordinates for a given prediction, and automatic generation of graphs of the results) were applied to this program to produce the version now in use.

In summary, two recommendations for further extension of this model are to be stressed: (1) comparison of the flat and spherical homogeneous earth attenuation functions in order to adequately decide on the range of validity of the flat-earth approximation; and (2) extension of the program to allow several "islands" or <u>even</u> continuous variation of the electrical parameters. In addition, it is recommended that other, possibly more advantageous, methods of calculating the propagation of an electromagnetic field over an inhomogeneous earth be looked into (Bremmer, 1951; Bremmer, 1954; Berk et al., 1967; Wait, 1967), including a method of calculating the electric field at various heights above the earth's surface instead of just at the earth's surface as was done here. This might introduce height-gain terms that could produce greater field strengths. Some thought should also be given to improving the numeric iteration procedure used for the solution of the integral equation for F*. Although the Neumann method (Lovitt, 1950) is a wellestablished technique and is known to converge in the case considered here, there are newer and more sophisticated techniques (Bekey et al., 1967) that converge much more rapidly and improve the numeric accuracy of the solution. Lastly, some consideration should be given to the effect of the vertical profile of the land and sea upon the attenuation function. The effect of land topography and of waves could be non-negligible.

The above remarks are meant to delineate the limitations of the model used here and to point out reasonable directions for its improvement. At the time the study (Rosich, 1968) was originally performed, these remarks were also a reasonable outline of the state of the art. Since then, however, significant advances have been made and much better models now exist. For example, the model by Ott and Berry (1970) and Ott (1971a, b) allows numerical treatment of quite general situations. In their model, an elementary function that is closely related to the Sommerfeld flat-earth attenuation function is used to derive an integral equation for propagation of radio waves over irregular terrain. The numerical solution of the integral equation yields the attenuation function normalized to the free-space field. The terrain may be represented by a completely arbitrary profile in terms of the elevation versus distance. This allows treatment of flat-earth, curved-earth, and much more general terrain. The hills and valleys themselves are taken to be uniform (cylindrical) in the direction transverse to the propagation direction, but this should pose no serious restriction to the use of the model since these are generally second order effects. The terrain may also be characterized by a conductivity and dielectric constant which are arbitrary functions of distance. Both the transmitting and receiving antennas may assume arbitrary locations on or above the surface. The solution to the integral equation is numerically feasible for both vertical and horizontal polarization up to the present limit of about 50 MHz, depending upon the profile under consideration. It is hoped that future developments will raise the upper limit on the frequency. Graph 31 shows a comparison of the Ott and Berry (1970) and Ott (1971a, b) model with one of Furutsu, Wilkerson, and Hartmann (1964) and with the model presented in this report. The results are for the path and parameters given in graph 1. Note the excellent agreement of the model used here with the other two more sophisticated models. Although the model compares quite well in this case, graphs 32 through 34 illustrate that effects which we ignore, for example, terrain height, can make substantial differences in the attenuation. The latter comments should underline the fact that if the situation one is modelling happens to fit the assumptions we have made, then the results can be expected to be accurate. If, on the other hand, the situation being modelled departs much from these assumptions, the results <u>may not</u> be correct. While the model we have presented here lacks generality (and accuracy in some cases), it does generally provide speed, however, so that in any given instance the optimum trade-off between speed and accuracy will have to be decided. As with all models, this one is useful, but should be used with caution and with full cognizance of its assumptions and limitations.

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