ABSTRACT

Several examples of the numerical evaluation of an integral equation for the calculation of the attenuation of a radio wave are given. These waves are assumed to be propagated over realistic, smoothly varying irregular, inhomogeneous terrain. Results for propagation over a cylindrical earth show an accuracy to 3-4 significant figures when compared with the classical residue series. Calculations for propagation over smooth mixed land-sea paths agree with classical methods. The applicability of the program to permit computation of propagation over terrain with smooth height variation is demonstrated by calculations of propagation over one and two Gaussian-shaped hills. The ability of the program to allow treatment of variations in both ground conductivity and height combined is illustrated by calculations of propagation from the sea up a sloping beach and by calculations of propagation over an island. This last example illustrates the importance of the terrain profile in mixed path calculations.

1. INTRODUCTION

Despite numerous attempts, a numerically feasible way to calculate the field strength of a radio wave propagating over realistic, smoothly varying, inhomogeneous terrain, has not yet been found. Hufford (1952) developed an integral equation for such propagation by using the free-space Green's function in Green's second identity and showed that his solution yielded the classical result for propagation over a smooth sphere. Berry (1967) succeeded in solving the equation numerically for vertically polarized radio waves, showing sample calculations up to 10 MHz. If the normalized surface impedance is not much smaller than 1, the numerical techniques are very inefficient, however, and round-off errors accumulate so fast that the results are not useful. For normal ground constants, this condition excludes all horizontally polarized waves and all vertically polarized waves above a few megahertz.

The method used in this paper is based on an elementary function that is closely related to the Sommerfeld flat earth attenuation function. This elementary function satisfies a scalar "parabolic" wave equation. The resulting integral equation is numerically feasible for both vertical and horizontal polarization and for normalized surface impedances in the HF band.

The problem to be solved is illustrated in figure 1, which shows a possible propagation path. The signal at the receiver is affected by the mean curvature of the earth, height profile along the path, and the change of surface impedance along the path. The changes may be abrupt (e.g., at a land sea boundary), or gradual (e.g., as the sea state, temperature, or salinity change). The problem of abrupt changes in surface impedance at smooth land-sea boundaries has been solved (Wait, 1964). Numerical results for changes in surface impedance have been calculated by Rosich (1968, 1970).

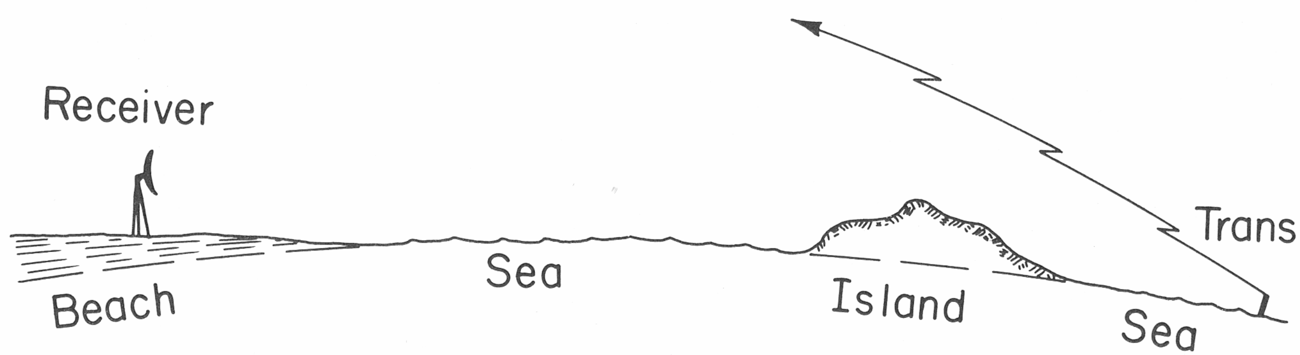

Figure 1. A possible propagation path.

The present work allows the terrain to be represented by a completely arbitrary profile in terms of the elevation versus distance. The hills and valleys themselves are taken to be uniform in the direction transverse to the propagation direction. The terrain may also be characterized by a conductivity and dielectric constant which are functions of distance.

The main body of the report describes the results of calculations for several examples including paths similar to that in figure 1. The appendices contain the derivation of the integral equation, the necessary numerical analysis, and a listing of the Fortran computer program.

2. THE INTEGRAL EQUATION

The derivation of the integral equation is given in appendix A. The details will not be reiterated here, but the final result is (Ott, 1971)

$$f(x) = g(x, y)\, W(x, 0) - \sqrt{\frac{i}{\lambda}} \int_0^x f(\xi)\, e^{-ik\omega(x, \xi)} \left\{ y'(\xi)\, W(x, \xi) - \frac{y(x) - y(\xi)}{x - \xi} + (\Delta - \Delta_r) \right\} \left[\frac{x}{\xi (x - \xi)} \right]^{\frac{1}{2}} d\xi \, , \quad (1)$$

where x, ξ, $y(x)$ and $y(\xi)$ are defined in figure 2. The factor $(\Delta - \Delta_r)$ arises in mixed-path problems. That is, substituting Δ_r for Δ in (A-2) and (A-9) will yield the difference $(\Delta - \Delta_r)$. The factor Δ_r is constant with distance and is the relative value of the normalized surface impedance. This factor is computed using the values for σ and ε_r for the first section of a mixed path. The factor Δ varies with distance in a mixed path problem. The variation of Δ with x may be continuous or contain abrupt changes. The factor $(\Delta - \Delta_r)$ is zero for a single section path. The remaining factors in (1) are defined as

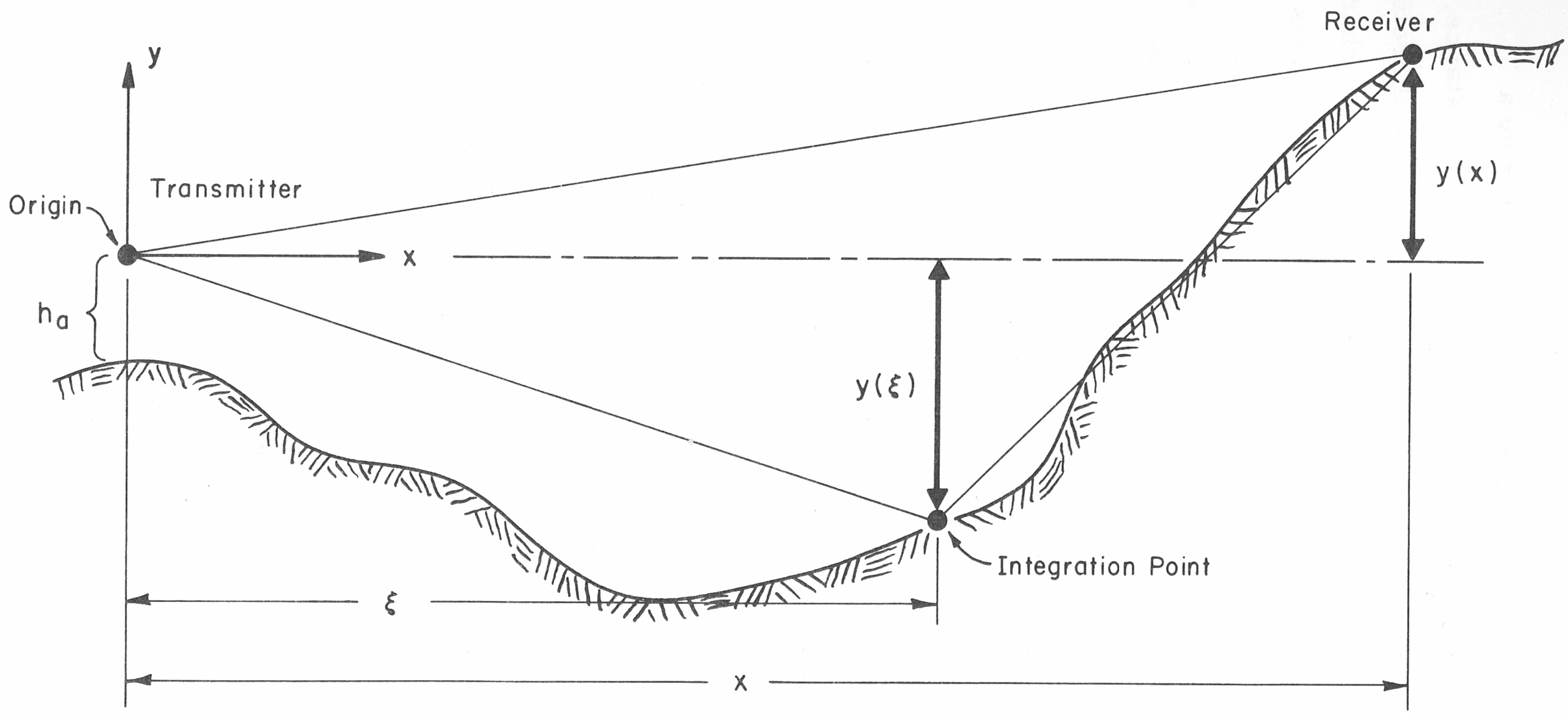

Figure 2. Geometry for integral equation

$$\omega(x, \xi) = \frac{[y(x) - y(\xi)]^2}{2(x - \xi)} + \frac{y^2(\xi)}{2\xi} - \frac{y^2(x)}{2x} ,$$

$$W(x, \xi) = 1 - i\sqrt{\pi p}\; w(-\sqrt{u}) ,$$

$$p = -ik\,\Delta^2(x - \xi)/2 ,$$

$$u = p\left\{1 - \frac{y(x) - y(\xi)}{\Delta(x - \xi)}\right\}^2 , \quad \xi < x$$

$$w(-\sqrt{u}) = e^{-u}\,\mathrm{erfc}\,(i\sqrt{u})$$

$$= \frac{1}{i\pi}\int_{-\infty}^{\infty} \frac{e^{-t^2}\,dt}{\sqrt{u} + t} \quad \text{(Abramowitz and Stegun, 1964)}$$

$$\Delta = \begin{cases} \dfrac{\sqrt{\eta - 1}}{\eta} , & \text{vertical polarization} \\ \sqrt{\eta - 1} , & \text{horizontal polarization} \end{cases}$$

$$\eta = \epsilon_r - \frac{i\,18(10^3)\,\sigma}{f(\mathrm{MHz})}$$

f = frequency, in MHz

σ = ground conductivity

ϵ_r = dielectric constant

$g(x, y)$ = antenna pattern factor.

Equation (1) gives the integral equation for the attenuation function normalized to twice the free-space field. The details of the numerical solution of (1) are given in appendix B. Since the upper limit of integration in (1) is x, the effects of backscatter are excluded. That is, to include the effects of backscatter, the range of integration would have to include the entire terrain. Also, the integral equation in (1) neglects the effects of side-scatter since the derivation of (1) assumed ridges uniform in the direction transverse to the propagation direction. In the case of small slopes and the transmitting antenna near the earth, side-scatter and backscatter are second order effects.

3. EXAMPLES

In this section we examine the behavior of the attenuation function, f(x), for eight terrain profiles, y(x). Comparisons of results from (1) with previous results for a flat earth, a smooth homogeneous cylindrical earth, a smooth sea-land-sea path and a single Gaussian-shaped ridge seem to validate the technique. Its more general applicability is illustrated by calculations for propagation over two Gaussian hills, over an island that rises above sea level, and over a sea-sloping beach with a sand-dune path.

3.1 A Flat Earth

$y(x) = 0$, $y'(x) = 0$. The solution of the integral equation (1) is trivial and is simply

$$f(x) = W(x), \tag{2}$$

where $W(x)$ is the Sommerfeld flat-earth attenuation function (Wait, 1964).

3.2 A Parabodoidal Earth

$y(x) \cong -x^2/2a$, $y'(x) = -x/a$, where a is the radius of the cylinder and is taken to be about 6.37×10^3 kilometers. The frequency of the transmitting antenna is 1 MHz and is vertically polarized. The ground constants are: $\sigma = 0.01$ mho/m and $\epsilon_r = 10$. The magnitude and phase of the attenuation function versus horizontal distance, x are given in table I. These results are compared in table I with those