APPENDIX B

IF FILTER TRANSFER PROPERTIES

INTRODUCTION

The interfering signal peak power level and time waveform at the radar receiver IF output is a function of the interfering signal emission spectrum, victim receiver IF selectivity characteristics, and the frequency separation between the interfering and victim radars. This appendix discusses the techniques and models used to compute the victim radar receiver IF selectivity characteristics, Frequency-Dependent-Rejection (FDR), interfering signal IF output time waveform, receiver equivalent noise level, and IF filter interference-to-noise ratio (INR) transfer properties.

IF SELECTIVITY

Since a victim radar receiver's IF selectivity characteristic is the principal means by which the receiver discriminates against undesired signals, the receiver's IF selectivity characteristic is required to determine a receiver's Frequency-Dependent-Rejection (FDR) of an undesired signal and IF output undesired signal time waveform.

Receiver spurious responses must also be considered when determining the FDR of a victim radar to an undesired signal. Spurious responses occur when the undesired signal is at a frequency such that it mixes with the local oscillator to produce an output at the receiver IF frequency. Since most radars have relatively clean local oscillator signals or employ balanced mixers, only image responses were investigated. The local oscillator frequency of most radars are tuned to 30 MHz above the receiver RF tuned frequencies in order to obtain an IF frequency of 30 MHz. An RF undesired signal that is 30 MHz above the local oscillator frequency (60 MHz above RF receiver center tuned frequency) will also be down-converted to the 30 MHz IF frequency. Modern radars normally employ an image-rejection mixer or a notch filter at the radar input to suppress image responses. The image response of the AN/CPN-4 and AN/MPN-13 radars are generally only 15 dB down from their center-tuned response, the ASR-4 through ASR-8 have an image response greater than 50 dB down. Reduced FDR due to receiver image response was not incorporated in the FDR model, but was considered in an independent FDR calculation.

The selectivity of a receiver is the composite selectivity of all the tuned circuitry in the receiver prior to detection; however, in a superheterodyne receiver, the selectivity is determined by the IF stages because the preceding mixer and RF circuits are relatively broader band. This is because the required filter characteristics are more physically realizable and less expensive to build at the lower IF frequency.

IF Selectivity Modeling

The radars in the 2.7 to 2.9 GHz band generally employ a combination of synchronously-tuned, staggered-tuned doublet, and stagger-tuned triplet stages of IF amplification. Figure B-1 shows a few stages of the ASR-8 normal channel IF amplifier. Neglecting the bias resistors, since they are usually larger than the source resistor, and also the coupling and bypass capacitors, the equivalent "Y" parameter circuit for one ASR-8 normal channel IF amplifier stage is shown in Figure B-2.

The gain of the equivalent circuit shown in Figure B-2 is:

$$A = \frac{v_2}{v_1} = \frac{Y_{fe}}{Y_{oe}+G+G'} \qquad \text{(B-1)}$$

Where: $Y_{oe} = g_{oe} + j\beta C_{oe}$

Letting: $C = C_{oe} + C_1$

The equivalent impedance of the parallel combination of capacitor $(-j/\beta C)$ and inductor $(R+j\beta L)$ can be expressed as:

$$Z' = \frac{\frac{L}{C}\left(1 - j\frac{R}{\beta L}\right)}{R\left[1 + j\frac{\beta L}{R}\left(1 - \frac{1}{\beta^2 LC}\right)\right]} \qquad \text{(B-2)}$$

Letting: $\beta_o = 1/\sqrt{LC}$ (B-3)

$$\delta = \frac{\beta}{\beta_o} - 1 = \frac{F - F_o}{F_o} \qquad \text{(B-4)}$$

$$Q = \frac{\beta_o L}{R} = \frac{1}{\beta_o CR} \qquad \text{(B-5)}$$

Where: β_o = Receiver tuned IF frequency, in radians per second

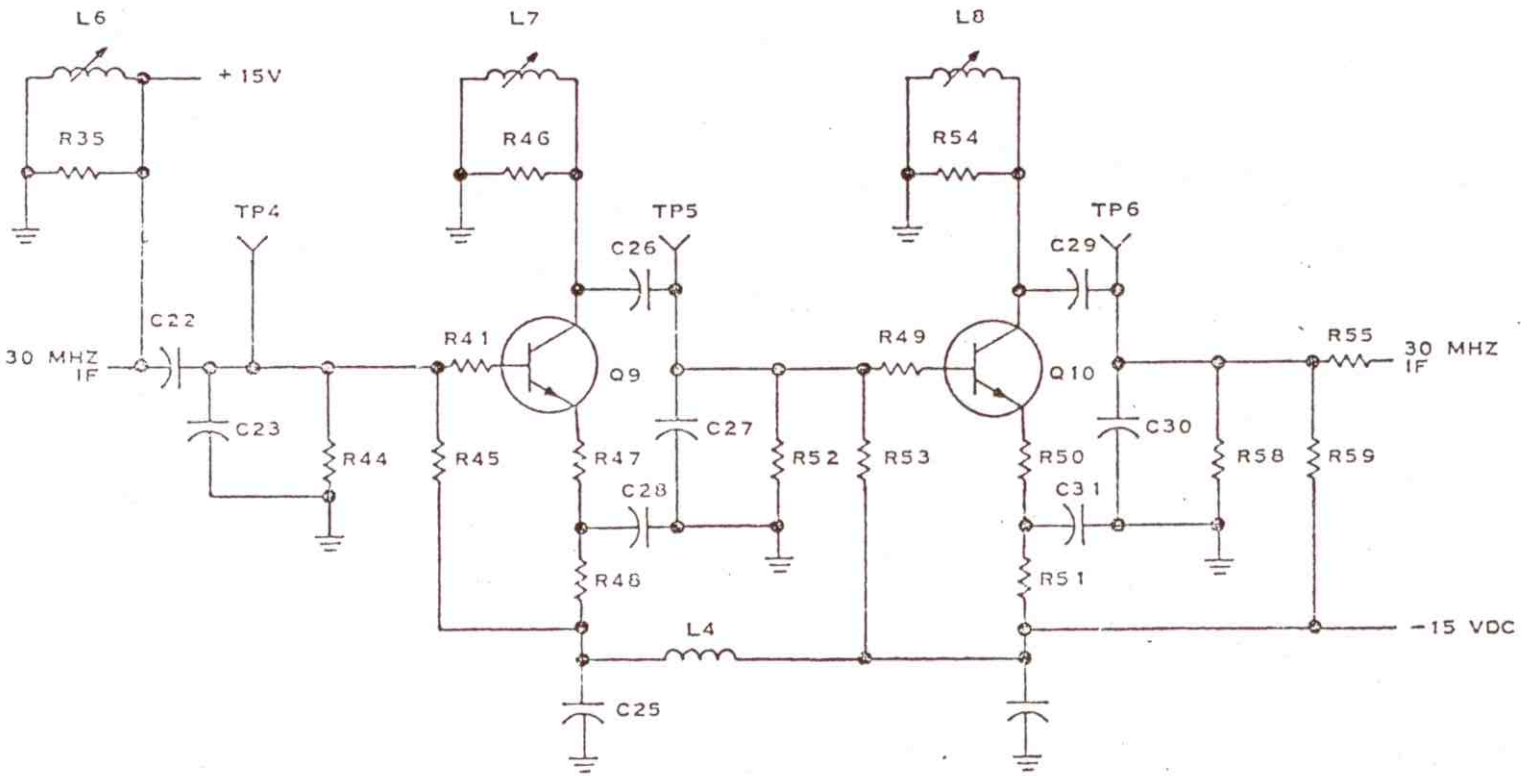

Figure B-1. ASR-8 Normal IF Bandpass Filter Schematic

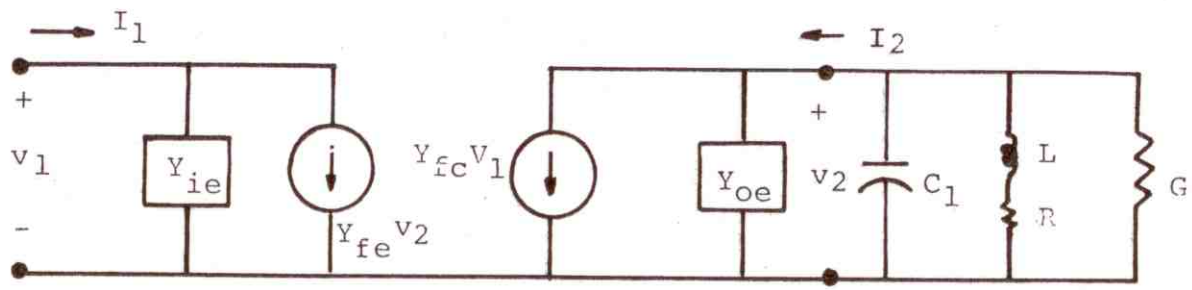

Figure B-2. "Y" Parameter Equivalent Circuit for One IF Amplifier Stage Shown in Figure B-1.

Using Equations B-3 through B-5, Equation B-2 becomes:

$$Z' = \frac{RQ^2 \left(1 - j\frac{1}{Q}\frac{\beta_o}{\beta}\right)}{1 + jQ\left(\frac{\beta}{\beta_o} - \frac{\beta_o}{\beta}\right)} \qquad \text{(B-6a)}$$

$$= RQ^2 \left[\frac{(1 + \delta) - j(1/Q)}{(1 + \delta) + jQ\delta(2 + \delta)}\right] \qquad \text{(B-6b)}$$

At resonance $\beta = \beta_o$, and $\delta = 0$, then

$$Z' = RQ^2 \left(1 - j\frac{1}{Q}\right) \qquad \text{(B-7)}$$

Since the Q for the circuit used is usually high, $Q \geq 10$, then with good approximation

$$Z' = RQ^2 \qquad \text{(B-8)}$$

Therefore, the gain at resonance combining Equations B-1 and B-8 is:

$$A_{RES} = \frac{Y_{fe}}{g_{oe} + G + \frac{1}{RQ^2}} \qquad \text{(B-9a)}$$

$$A_{RES} \quad \frac{-Y_{fe}\,\beta_o LQ}{\beta_o LQg_{oe} + \beta_o LQG + 1} \qquad \text{(B-9b)}$$

Equation B-9b may be written as:

$$A_{RES} = -Y_{fe}\,\beta_o LQ_e \qquad \text{(B-10)}$$

Where Q_e, the effective "Q" of the amplifier, is:

$$Q_e = \frac{Q}{1 + \beta_o LQg_{oe} + \beta_o LQG} \qquad \text{(B-11a)}$$

$$= \frac{F_o}{B_s} \qquad \text{(B-11b)}$$

Where: B_s = The amplifier stage 3 dB bandwidth, in Hz

The impedance, Z', of the single-tuned amplifier at some off-tuned frequency, β, where $\beta \simeq \beta_o$ and δ is small is:

$$Z' = \frac{RQ^2}{1 + j2\delta Q} \tag{B-12}$$

Therefore, the off-tuned gain is:

$$A = \frac{-Y_{fe}}{g_{oe} + G + \frac{1 + j2\delta Q}{RLQ^2}} \tag{B-13}$$

$$= \frac{-Y_{fe}\beta_o L Q_e}{1 + j2\delta Q} \tag{B-13a}$$

Thus the gain ratio can be obtained from Equations B-10 and B-13a as:

$$\frac{A}{A_{RES}} = A(F) = \frac{1}{1 + j2\delta Q} \tag{B-14}$$

For "m" cascaded synchronous single-tuned stages, the gain ratio becomes:

$$A(F) = \frac{1}{[1 + j2\delta Q]^m} \tag{B-15}$$

The gain ratio for a single-tune stage using an electron tube is identical to Equation B-14, and is derived by Seeley (1958).

The IF bandwidth of "m" cascaded synchronous-tuned stages is given by:

$$B_{IF} = B_S \sqrt{2^{1/m} - 1} \tag{B-16}$$

Where: B_{IF} = Receiver IF 3 dB bandwidth, in Hz

Substituting Equations B-4 and B-11a into Equation B-14, the amplitude characteristic in dB form of a single-tuned stage is:

$$A_{dB}(F) = -20 \log_{10}(1 + X^2)^{\frac{1}{2}} \tag{B-17}$$

Where:

$$X = \frac{2\,(F-F_o)}{B_S} \tag{B-18}$$

The IF selectivity characteristics, $A_{dB}(F)$ for "m" cascaded synchronous single-tuned stages is given by:

$$A_{dB}(F) = -20 \log_{10}(1 + X^2)^{m/2} \qquad \text{(B-19)}$$

For "m" cascaded amplifiers consisting of "n" staggered-tuned stages, the IF filter amplitude characteristics are given by Seeley (1958).

$$A_{dB}(F) = -20 \log_{10}(1 + X^{2n})^{m/2} \qquad \text{(B-20)}$$

RECEIVER FREQUENCY-DEPENDENT-REJECTION CHARACTERISTICS

A necessary component in predicting radar to radar interference is the victim radar receiver's Frequency Dependent Rejection (FDR) characteristic to cochannel and adjacent channel undesired signals. The victim receiver's IF selectivity characteristic and undesired signal emission spectrum characteristic determine the victim receiver's rejection (peak power loss) of the undesired signal which is dependent on the frequency separation (ΔF) between the victim receiver RF tuned frequency (ω_o), and the undesired signal RF carrier frequency. The following is a discussion of the technique and model used to determine a radar receiver's FDR of an undesired signal.

FDR Model

FDR is the sum of attenuation of the undesired signal due to Off-Frequency Rejection (OFR) and the On-Tune Rejection (OTR) in dB.

$$FDR(dB) = OFR(dB) + OTR(dB) \qquad \text{(B-21)}$$

Off-Frequency Rejection (OFR) is defined by Fleck (1967) as:

$$OFR = \frac{\int \frac{P(F)}{A(F + \Delta F)} \, dF}{\int \frac{P(F)}{A(F)} \, dF} \qquad \text{(B-22)}$$

Where:

$P(F)$ = Transmitter relative power density

$A(F)$ = Relative receiver selectivity

ΔF = Frequency separation between interfering signal carrier frequency and receiver tuned frequency, in Hz

Absolute transmitter power and receiver sensitivity does not enter into the calculation because FDR depends only on the shape of the victim receiver IF

selectivity and undesired signal emission spectrum.

Since the undesired signal emission spectrum and victim receiver selectivity curves are usually given in dB units, a more practical form of Equation B-22 in dB is:

$$OFR = 10 \log \int 10^{[P_{dB}(F) - A_{dB}(F + \Delta F)]/10} dF - 10 \log \int 10^{[P_{dB}(F) - A_{dB}(F)]/10} dF \qquad \text{(B-23)}$$

Where:

$P_{dB}(F)$ = Emission spectrum, in dB

$A_{dB}(F)$ = Receiver IF selectivity curve, from Equations B-17 through B-20, in dB.

Approximations to Equation B-23 can be obtained using the following equations (Newhouse):

Case I, $\tau B_{IF} > 1$ and ΔF = any value

$$OFR = 20 \log \tau B_{IF} + (F_{\Delta F}) \qquad \text{(B-24)}$$

Case II, $\tau B_{IF} > 1$ and $\Delta F \leq \frac{1}{\Pi\tau} + \frac{B_{IF}}{2}$

$$OFR = 0 \text{ dB} \qquad \text{(B-25)}$$

Case III, $\tau B_{IF} > 1$ and $\Delta F > \frac{1}{\Pi\tau} + \frac{B_{IF}}{2}$

$$OFR = 20 \log \tau B_{IF} + (F_{\Delta F}) - 6_{dB} \qquad \text{(B-26)}$$

Where:

τ = Interfering signal pulsewidth, in seconds

$F_{\Delta F}$ = Interfering signal emission spectrum level relative to fundamental frequency, in dB.

The victim receiver On-Tuned Rejection (OTR) factor is derived by White

(1972). In summary, the OTR factor can be obtained by considering the Laplace transforms of a single pulse of amplitude "A" and width τ:

$$\mathcal{L}\begin{bmatrix}\text{SINGLE}\\ \text{PULSE}\end{bmatrix} = A\tau \left(\frac{\sin \pi F\tau}{\pi F\tau}\right) e^{-j\pi F\tau} \qquad \text{(B-27)}$$

Where the Laplace transform parameter, s, has been replaced by $j2\pi F$. The phase given by the exponential factor indicates that when the receiver bandwidth B_{IF} is less than the emission bandwidth $B_T = 1/\tau$, the phase variation over the receiver bandwidth is small. This means that the voltage must be summed first and then squared instead of summing the square of the voltages as many documents imply. For a relatively constant emission spectrum over the IF pass band, the received peak power is therefore proportional to the ratio of the receiver to transmitter bandwidths and the OTR factor in dB is given by:

$$\text{OTR} = \begin{cases} 20 \log_{10} B_{IF}/B_T & B_{IF} < B_T \\ 0 & B_{IF} > B_T \end{cases} \qquad \text{(B-28)}$$

Where:

B_{IF} = Receiver IF 3 dB bandwidth, in Hz

B_T = Emission 3 dB bandwidth, in Hz

IF OUTPUT TIME WAVEFORM

The following analysis considers the tranformation of pulsed signals through an effective linear IF filter. In general, the IF amplifier output for a pulsed input signal can be expressed as the sum of a steady state term and a transient term. The transient term represents a distortion term and includes the amplitude and phase modulation produced in the IF amplifier. The transient term arises because the system response is unable to build up and decay as fast as the input signal.

It is necessary to have an IF filter model to predict these transient and steady state terms. One method of modeling the IF filter function is in terms of cascaded tuned amplifiers. Both single-tuned and stagger-tuned amplifiers are used in radar receivers. For this analysis, eight cascaded single-tuned amplifier stages were used to model the radar receiver IF filter characteristics. The transfer function for eight cascaded synchronous tuned amplifiers is given by Equation B-15 as:

$$A(F) = \frac{1}{[1 + j2\delta Q]^8} \qquad \text{(B-29)}$$

If the IF filter has an impulse response a(t), the IF filter output response to an input signal, $x(\tau)$, is given by the convolution integral:

$$h(t) = \int_{-\infty}^{\infty} x(\tau)a(t-\tau)d\tau \qquad \text{(B-30)}$$

Since convolution in the time domain is equivalent to multiplication in the frequency domain, it will be more convenient to use the frequency domain expressions. If the input signal, $x(\tau)$, has a Fourier transform, X(F), and the transfer function of the IF filter is A(F), then the output spectrum is given by:

$$H(F) = X(F)A(F) \qquad \text{(B-31)}$$

The interfering signal spectrum for a periodic trapezoidal pulse is given by Meyers (1971).

$$X(F) = \frac{1}{Td\pi^2\tau F_x^{\,2}} \sin\ [\pi\tau F_x(1 + d)]\ \sin\ (\pi\tau dF_x) \qquad \text{(B-32)}$$

Where:

T = Radar pulse period, 1/PRF

τ = Interfering signal pulse width, in seconds

d = Rise time/τ = Fall time/ τ

$F_x = F_o - \Delta F$

The interfering signal output spectrum H(F), after being band limited by the IF filter, is the product of the filter transfer function and the pulse spectrum. The inverse Fourier Transform of H(F) will be the IF filter output time waveform h(t). For modeling purposes, the carrier frequency can, without loss of generality, be assigned the value of the center frequency, β_o, of the IF filter. The interfering signal at the IF output can then be expressed as:

$$v_{IFo}(t) = Bp(t')\cos[\beta_o t + \phi_o + \phi(t')] \qquad \text{(B-33)}$$

Where:

B = Interfering signal voltage amplitude

p(t') = Interfering signal amplitude modulation after IF filtering, value between 0 and 1

ϕ_o = Interfering signal carrier phase angle

t' = $t - t_o$ where t_o is the delay time of the IF

filter

$\phi(t')$ = Interfering signal phase modulation after IF filtering

Typical IF filter bandwidths of radars in the 2.7 to 2.9 GHz band are 1.2 MHz to 5.0 MHz. The following is a discussion of the IF output time waveform as a function of the interfering signal pulsewidth, τ, and off-tuning (ΔF). The simulated output time waveforms were obtained using the RWS Model, Meyers (1971).

When the pulse bandwidth is much greater than the bandwidth of the IF filter, $\tau B_{IF} \ll 1$, the spectrum of the pulse within the IF passband for the on-tune, $\Delta F = 0$, case is approximately flat and has even symmetry about the carrier frequency. The Fourier transform of the resulting spectrum will approximate the impulse response of the network with some ringing on the trailing edge. The output time waveform will be much longer than the input pulse time waveform with the peak amplitude of the output pulse reduced approximately by the ratio of the pulse bandwidth to the filter bandwidth. Since the bandwidth of the pulse is much greater than the IF bandwidth, the shape of the spectrum within the IF passband as the pulse is off-tuned will be approximately the same except near the nulls of the input spectrum. Therefore, as the pulse spectrum is off-tuned the IF output time waveform will remain approximately the same as the on-tune case with the inband power determining the peak level of the output pulse. In the vicinity of the null point the spectrum will no longer be flat and it will approach odd symmetry when centered about the null point. The Fourier transform of an unsymmetrical spectrum will produce both amplitude and phase modulation as shown in Equation B-33. Simulated IF output time waveforms for an IF bandwidth of 1.2 MHz and a 5 μs pulse ($\tau B_{IF} \ll 1$) for ΔF = 0, 5.0 MHz and 20.0 MHz off-tune are shown in Figures B-3, B-4 and B-5. The input pulse of width τ and unity amplitude is assumed to be symmetrical about time t = 0. This is true for all pulses discussed in this section.

When the pulse bandwidth is approximately equal to the IF bandwidth $\tau B_{IF} \simeq 1$, the spectrum of the pulse within the IF passband for the on-tune, ΔF = 0, case will be symmetrical and even. The sidelobes will be attenuated by the filter function causing a more rapid fall-off of the pulse spectrum. The Fourier transform of the resulting spectrum will approximate the impulse response, with the amount of energy within the passband again determining the peak level of the output pulse. As the pulse is off-tuned the pulse spectrum out of the IF filter will be unsymmetrical, thereby producing both amplitude and phase modulation as shown in Equation B-33. The Fourier transform of the output pulse spectrum produces time waveforms that appear to contain a separate response for the leading and trailing edge of the IF input pulse. As the off-tuning is increased the double response becomes more pronounced due to the attenuation of the steady state portion of the pulse by the filter characteristics. Simulated IF output time waveforms for an IF bandwidth of 2.7 MHz and a .83 μs pulse for ($\tau B_{IF} \simeq 1$) for ΔF = 0, 5.0 MHz and 20.0 MHz are shown in Figures B-6, B-7, and B-8, respectively.

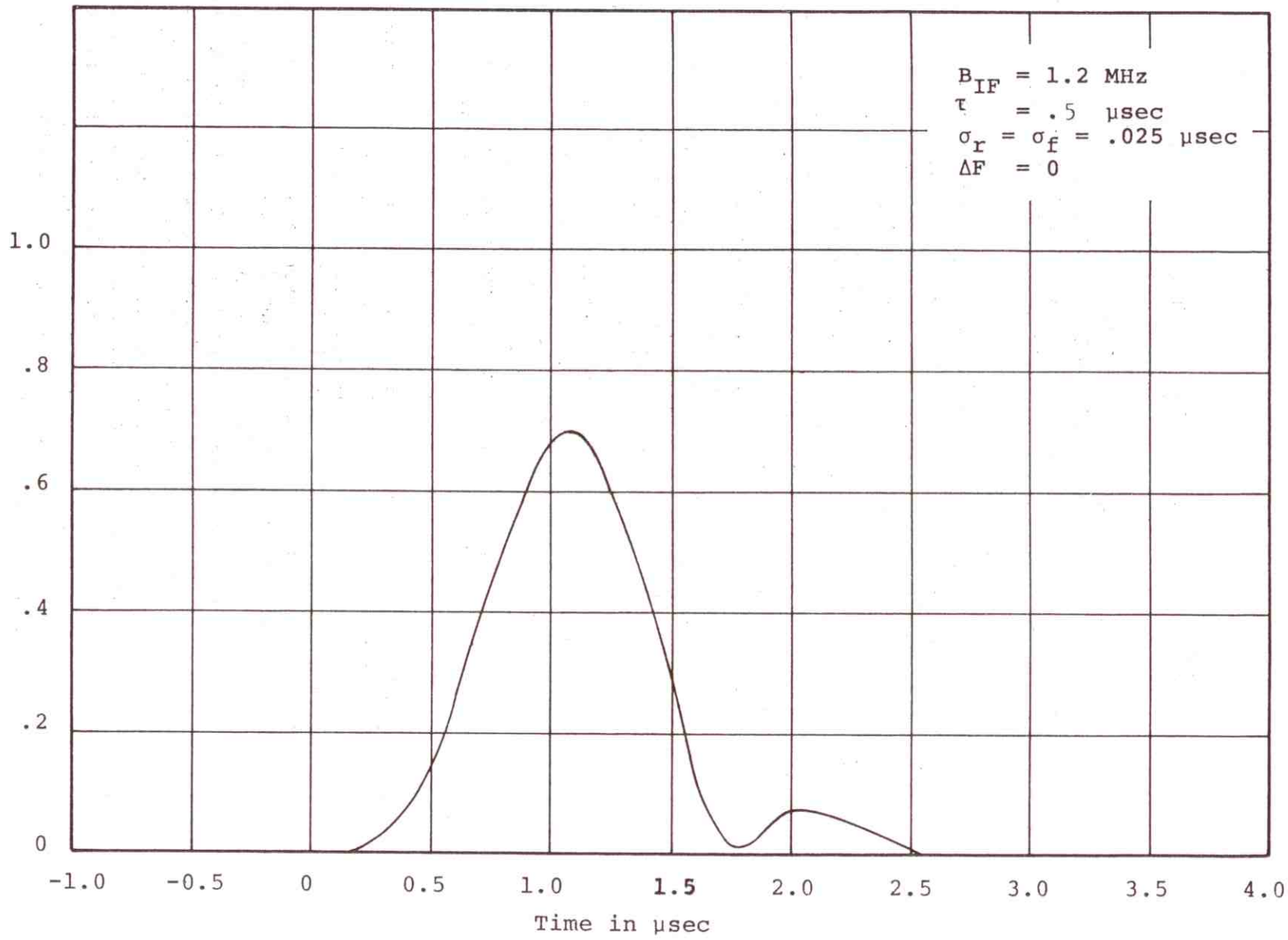

Figure B-3. Simulated IF Output Time Waveform Envelope

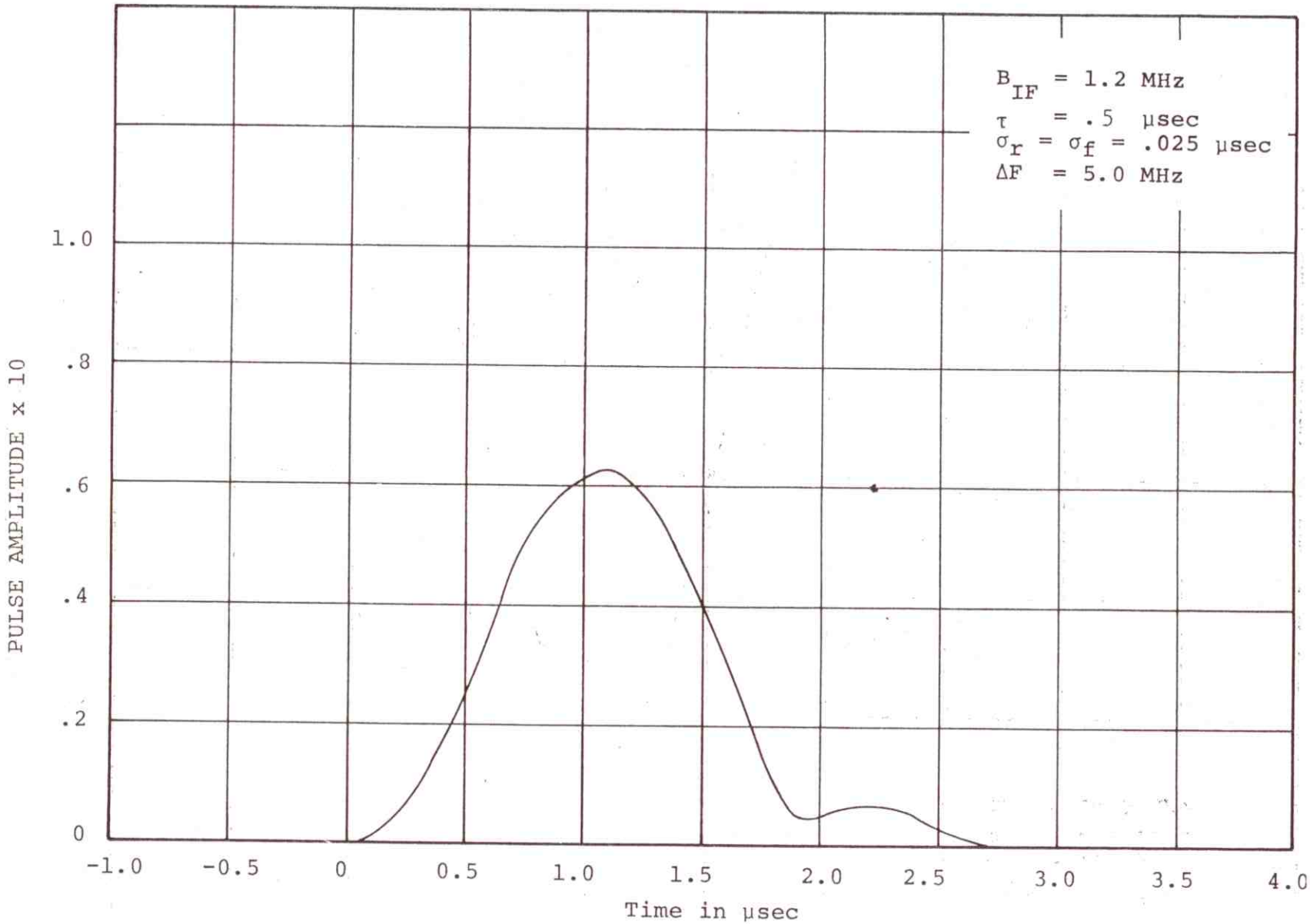

Figure B-4. Simulated IF Output Time Waveform Envelope

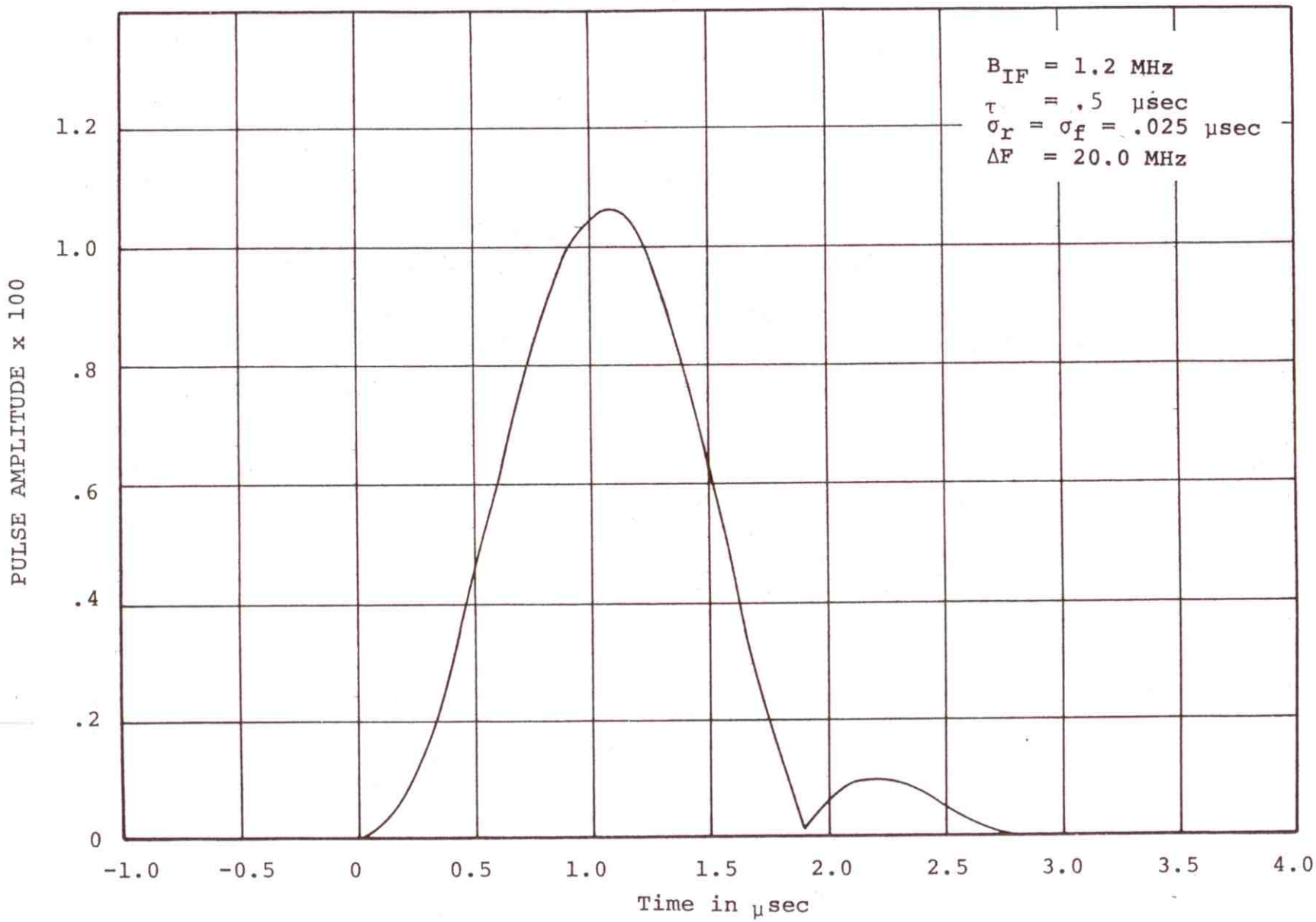

Figure B-5. Simulated IF Output Time Waveform Envelope

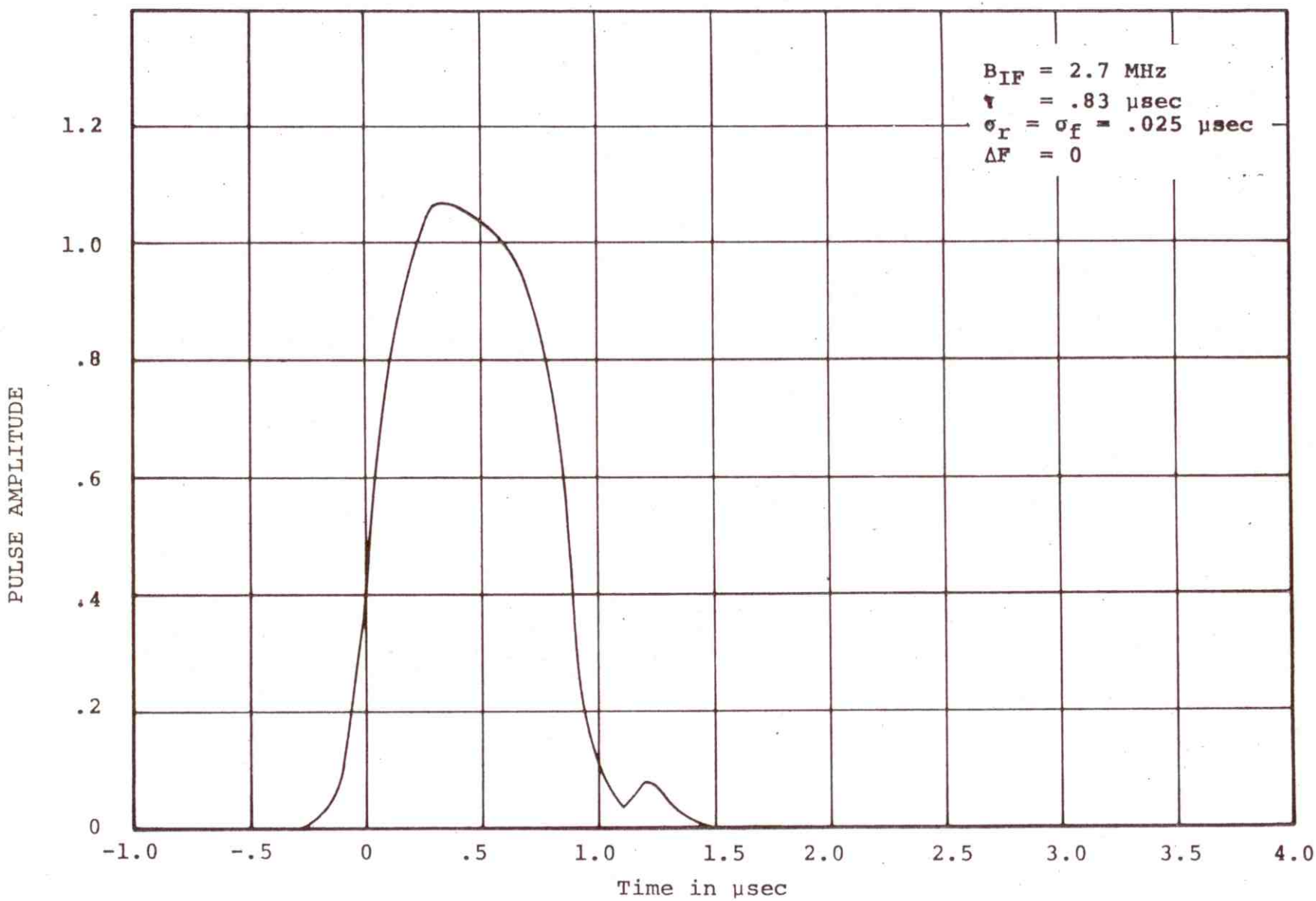

Figure B-6. Simulated IF Output Time Waveform Envelope

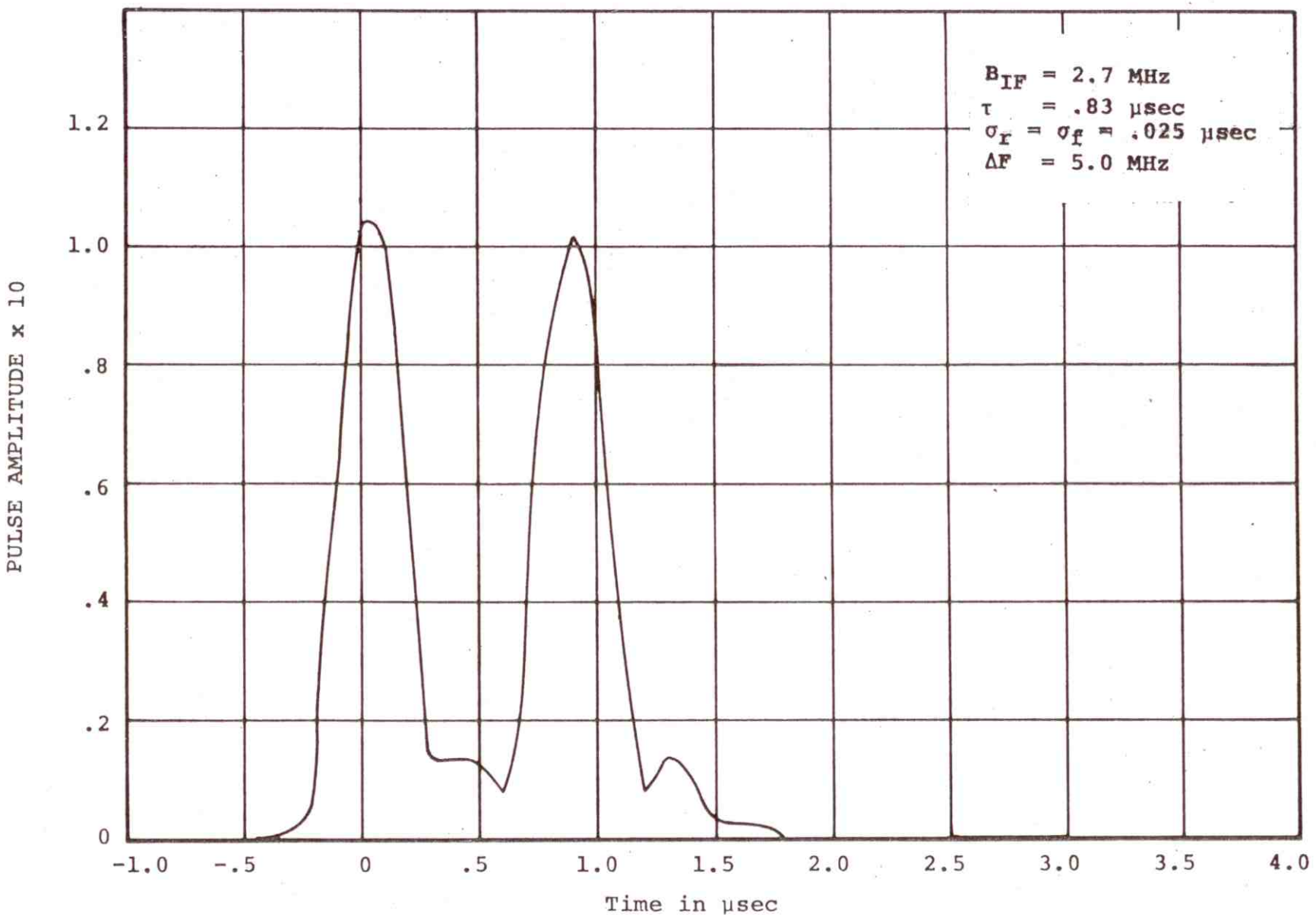

Figure B-7. Simulated IF Output Time Waveform Envelope

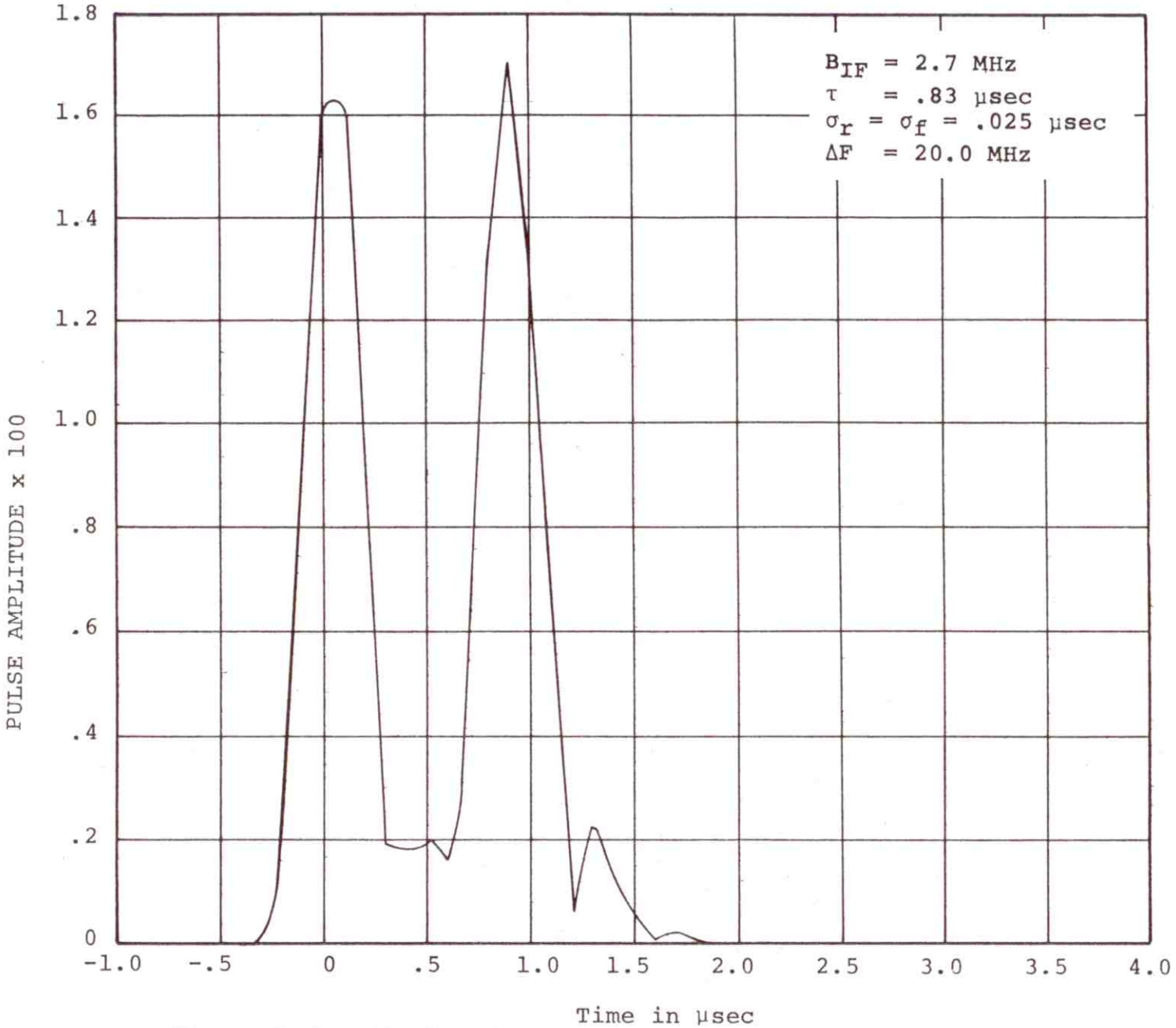

Figure B-8. Simulated IF Output Time Waveform Envelope

When the pulse bandwidth is much narrower than the IF filter bandwidth, $\tau B_{IF} >> 1$, and ΔF less than one half the IF bandwidth, the input time characteristics are produced at the filter output along with some ringing or overshoot on the leading and trailing edges. As the pulse is further off-tuned, the pulse spectrum out of the filter will be unsymmetrical, thereby producing amplitude and phase modulation. The steady state portion of the output pulse will again be attenuated by the off-tuned characteristics of the filter and, therefore, the shape and magnitude of the output pulses will be determined by the power and shape of the spectrum within the filter passband. Simulated IF out time waveform for an IF bandwidth of 5.0 MHz and a .83 μs pulse ($\tau B >> 1$) for ΔF = 0, 5.0 MHz and 20.0 MHz are shown in Figures B-9, B-10, and B-11, respectively. Photographs of measured IF output time waveforms on an ASR-8 (Figures B-12 through B-14) are shown for identical parameter conditions as the simulated waveforms shown in Figures B-9 through B-11, respectively.

Figure B-15 is a plot of phase angle as a function of time at the output of a 5.0 MHz bandwidth IF filter for a .83 μs pulse off-tuned 5.0 MHz. The plot shows the phase modulation produced by an off-tuned pulse during the leading edge, steady state and trailing edge of the pulse. The 5.0 MHz beat tone phase modulation during the steady state portion of the pulse is equal to the 5.0 MHz off-tuning of the pulse. The effect of this phase modulation produced by off-tuned pulsed interference on the MTI channel phase detector is discussed in Appendix C.

In summary, the transformation of pulsed signals through a radar linear IF filter has been discussed in terms of τB_{IF} categories and off-tuning. An understanding of the transfer properties of radar IF filters is required since the IF output time waveform determines the waveform at the normal channel detector output, and the phase modulation produced by the IF filter determines the MTI channel phase detector output waveform.

IF OUTPUT NOISE LEVEL

The receiver equivalent noise level at the radar input is essentially determined by the radar receiver RF stages since the noise contribution due to the IF stages is small because of the high gain of the IF and previous receiver stages. Since the radar receiver can be modeled as a linear receiver with 0 dB gain up to the receiver IF output, which allows the signal and noise to be treated separately, the noise level at the radar IF output can be treated as being equal to the receiver RF input equivalent noise level N_I. Thus, the receiver equivalent noise level at the receiver input is given by:

$$N_I = kT_O B_{IF} F \qquad \text{(B-34)}$$

Where:

N_I = Receiver noise (referenced to RF input)

T_o = Reference Temperature = 290°k

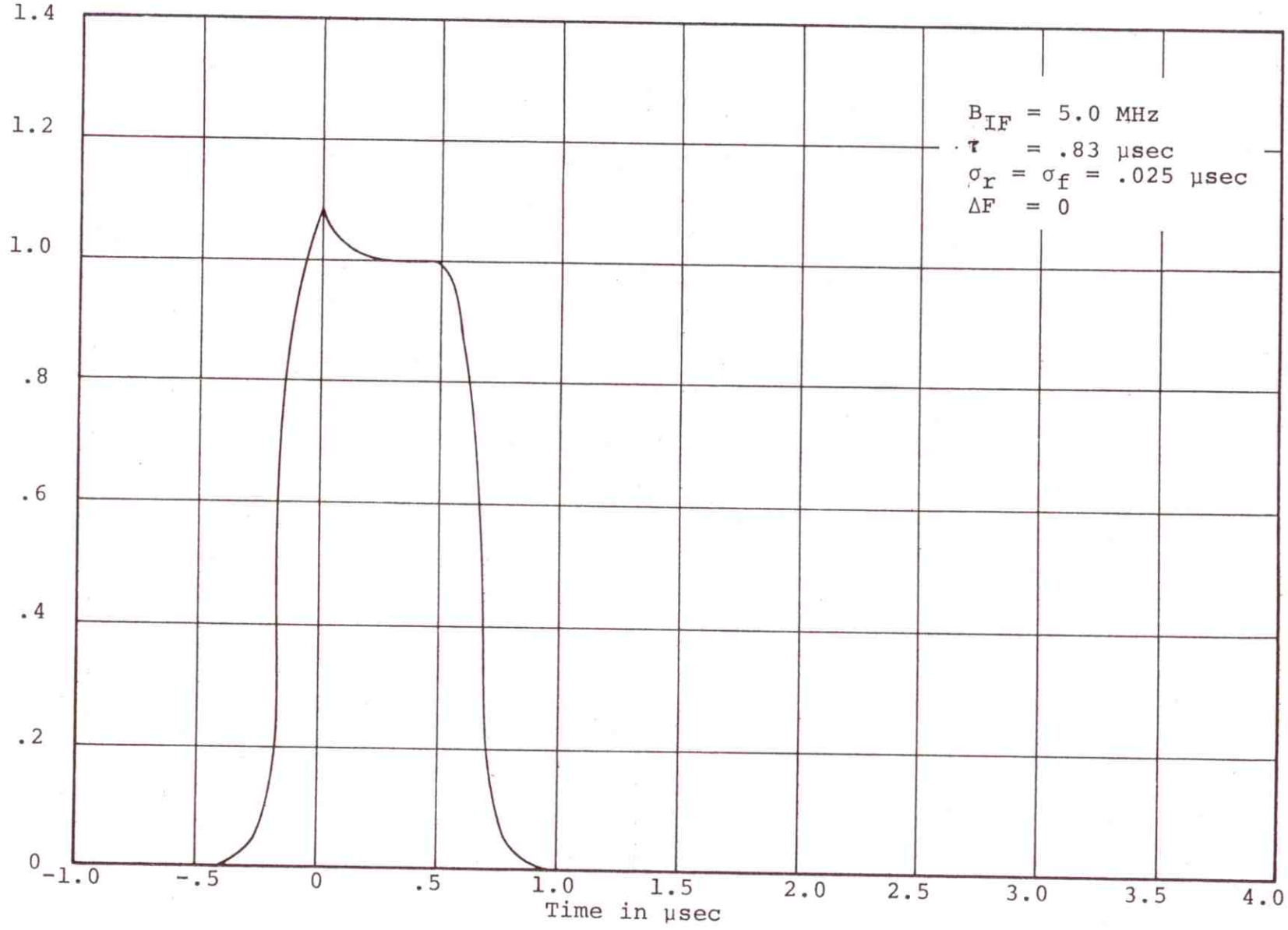

Figure B-9. Simulated IF Output Time Waveform Envelope

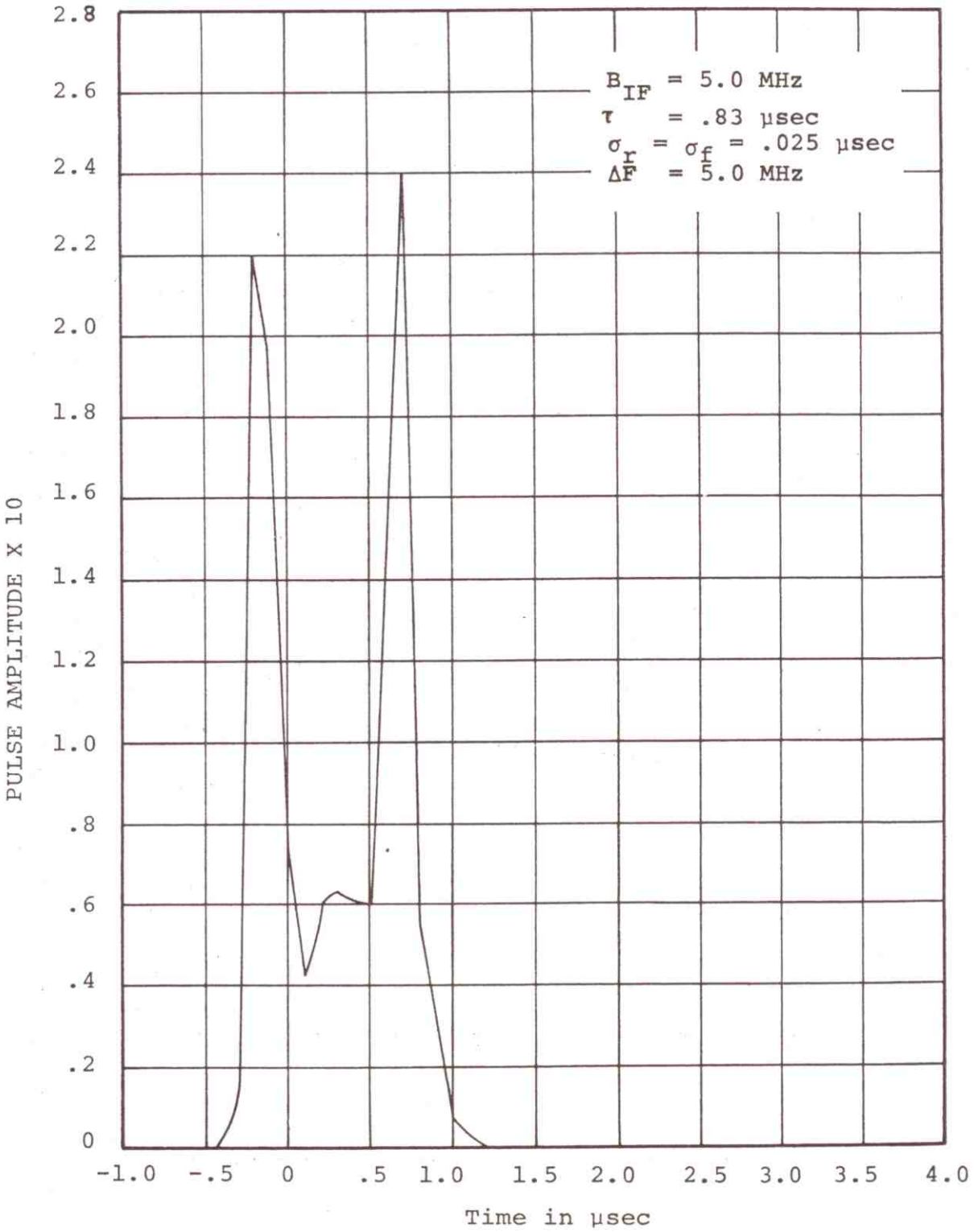

Figure B-10. Simulated IF Output Time Waveform Envelope

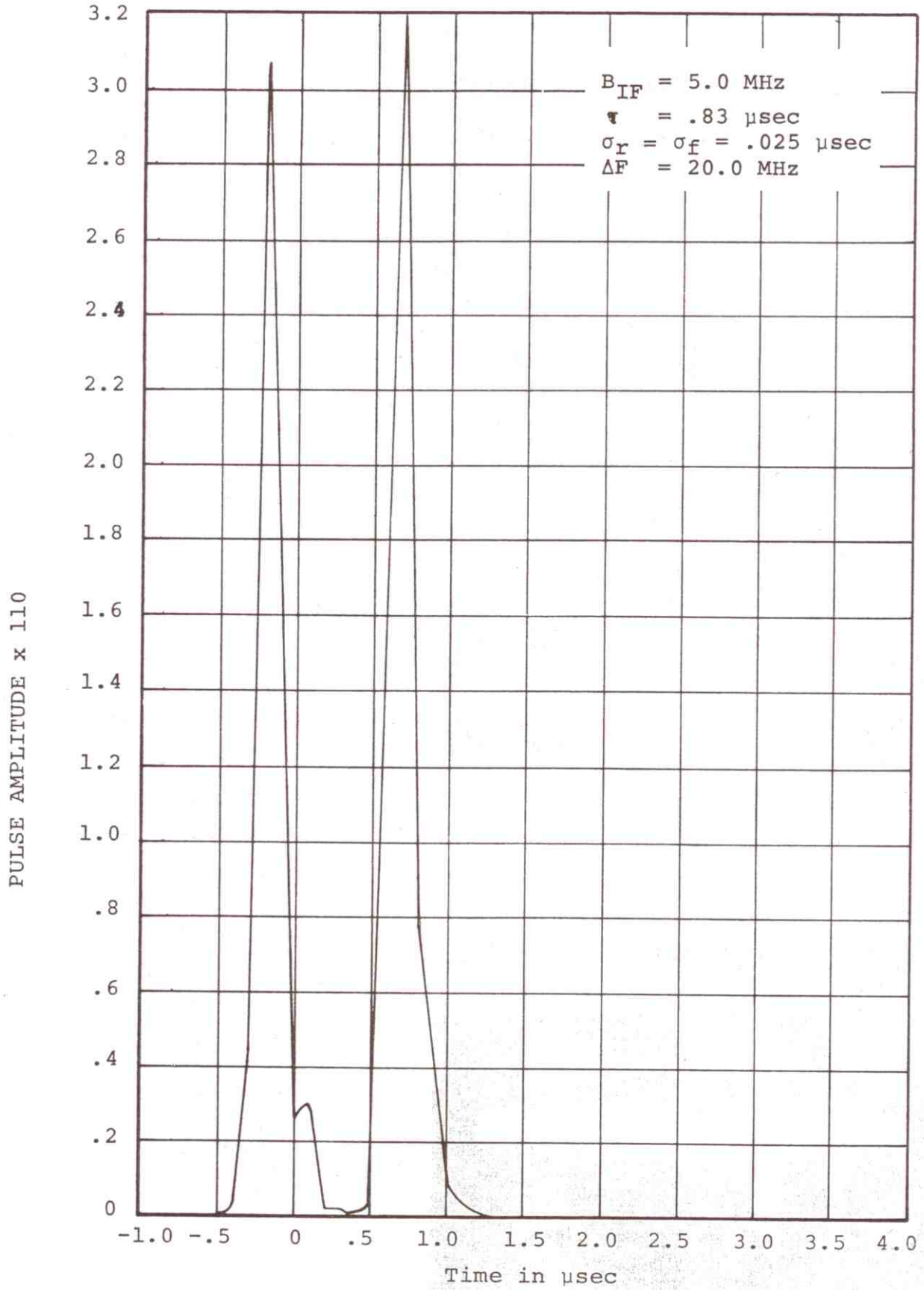

Figure B-11. Simulated IF Output Time Waveform Envelope

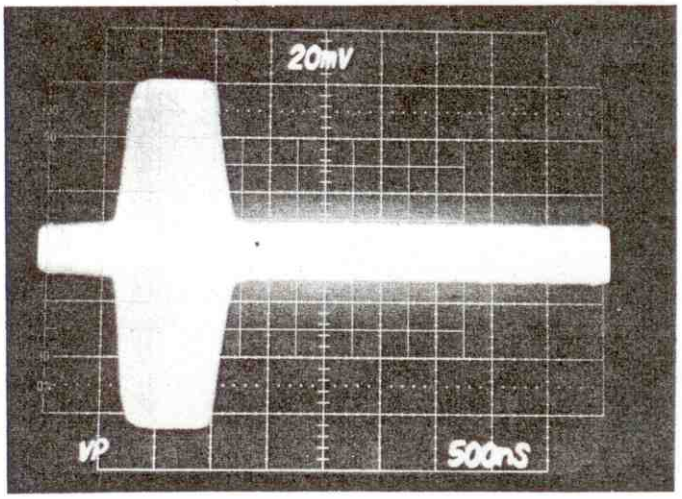

Figure B-12. Measured IF Output Time Waveform

B_{IF} = 5.0 MHz

τ = .83 μsec

ΔF = 0

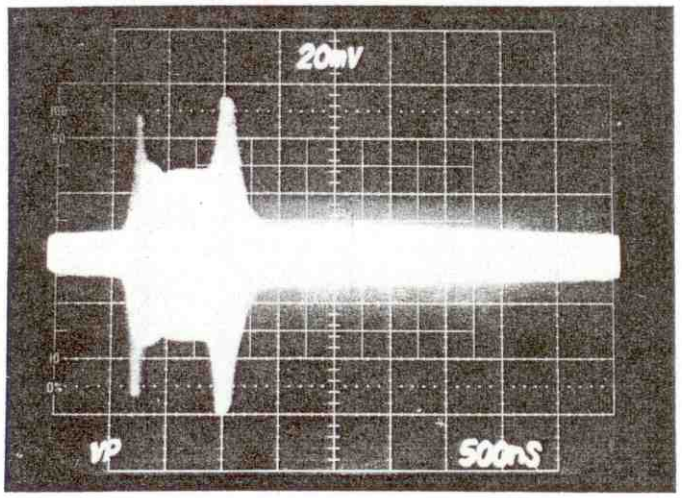

Figure B-13. Measured IF Output Time Waveform

B_{IF} = 5.0 MHz

τ = .83 μsec

ΔF = 5.0 MHz

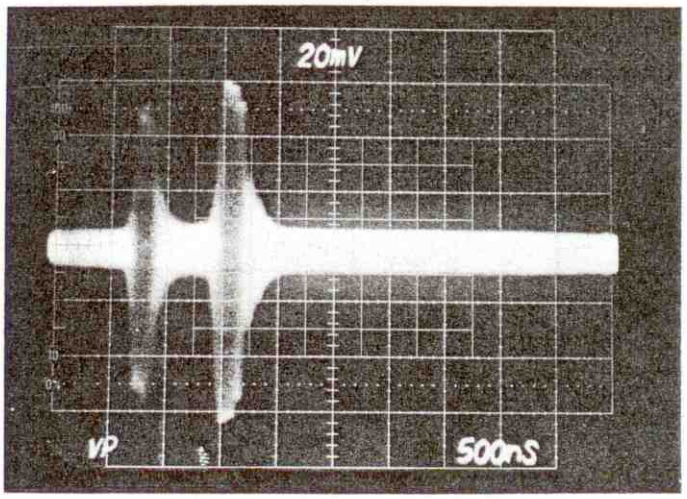

Figure B-14. Measured IF Output Time Waveform

B_{IF} = 5.0 MHz

τ = .83 μsec

ΔF = 20.0 MHz

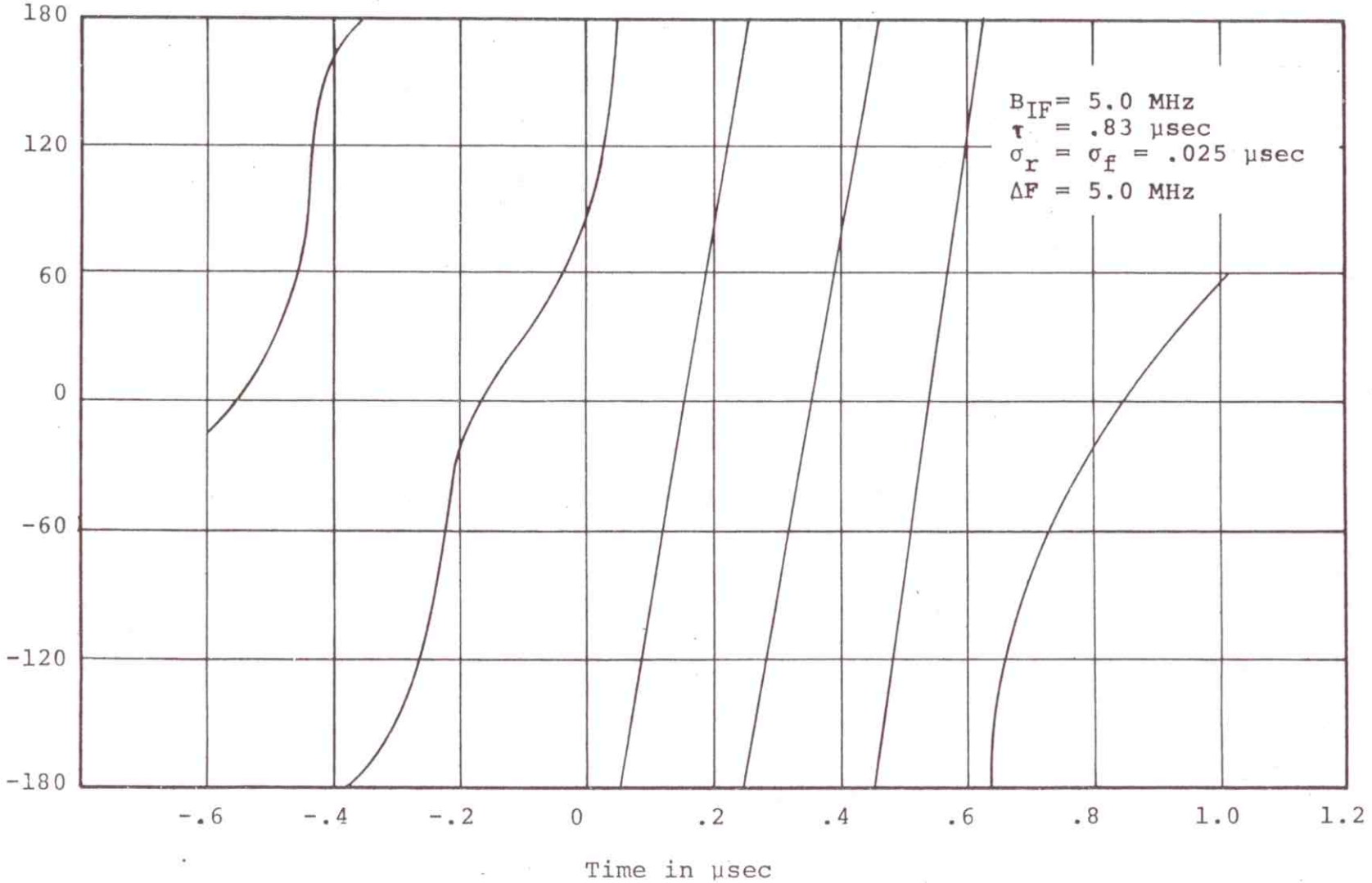

Figure B-15. Simulated IF Output Phase Modulation for an off-tuned Pulse Signal

k = Boltzmann's constant = $1.38 \times 10^{-23} \frac{\text{Watt-sec}}{^{o}k}$

F = Receiver noise figure = 4 dB

B_{IF} = Receiver IF 3 dB bandwidth, in Hz

Expressing Equation B-34 in dBm

$$N_{IdBm} = -174 \text{ dBm} + 10 \log B_{IF} + F \tag{B-35}$$

The interference-to-noise ratio at the receiver RF input can be expressed as:

$$INR_I = I_{I\,dBm} - N_{I\,dBm} \tag{B-36}$$

Where:

$I_{I_{dBm}}$ = Interfering signal peak power level at the receiver input, in dBm

IF FILTER INR TRANSFER PROPERTIES

Since the IF filter has the narrowest bandwidth and sharpest selectivity characteristics in the radar receiver front end, the filtering prior to the receiver IF filter can be neglected for analytical simplicity, and the interference to noise ratio at the IF filters input (INR_{IFi}) can be treated as being equal to the receiver input interference to noise ratio. Thus:

$$INR_{IFi} = INR_I \tag{B-37}$$

$$= I_{I_{dBm}} - N_{I_{dBm}} \tag{B-37a}$$

The receiver IF output interference to noise ratio, INR_{IFo}, is determined by the receiver FDR, and can be expressed as:

$$INR_{IFo} = INR_I - FDR \tag{B-38}$$

Where the FDR factor is given by Equation B-21.