NTIA Technical Memorandum TM-10-469

Derivations of Relationships among Field Strength, Power in Transmitter-Receiver Circuits and Radiation Hazard Limits

Frank H. Sanders

report series

U.S. DEPARTMENT OF COMMERCE • National Telecommunications and Information Administration

NTIA Technical Memorandum TM-10-469

Derivations of Relationships among Field Strength, Power in Transmitter-Receiver Circuits and Radiation Hazard Limits

Frank H. Sanders

U.S. DEPARTMENT OF COMMERCE

June 2010

CONTENTS

Page

ABBREVIATIONS/ACRONYMS

ACF	antenna correction factor
ANSI	American National Standards Institute
EIRP	effective isotropic radiated power
FS	field strength
HERF	hazards of electromagnetic radiation to fuel
HERO	hazards of electromagnetic radiation to ordnance
HERP	hazards of electromagnetic radiation to personnel
IEEE	Institute of Electrical and Electronic Engineers
IF	intermediate frequency
MILSTD	military standard
RADHAZ	radiation hazard
RF	radiofrequency

DERIVATIONS OF RELATIONSHIPS AMONG FIELD STRENGTH, POWER IN TRANSMITTER-RECEIVER CIRCUITS AND RADIATION HAZARD LIMITS

Frank H. Sanders[1]

This NTIA Technical Memorandum provides a single, comprehensive set of derivations of the mathematical relationships among power in a radio transmitter or measurement circuit, the associated incident field strength in space, transmitter-receiver antenna characteristics, free space propagation equation variables and radiation hazard limits.

Key words: antenna correction factor; effective isotropic radiated power; field strength; hazards of electromagnetic radiation to fuel; hazards of electromagnetic radiation to ordnance; hazards of electromagnetic radiation to personnel; measured power; military standard 461; power in circuits; radiation hazard; RF measurement

1 DIRECTIVITY, GAIN, EFFECTIVE ANTENNA APERTURE AND ANTENNA CORRECTION FACTOR DERIVATIONS

1.1 Introduction

Power in space couples into circuits, and vice versa, via antennas which couple energy between these two media by matching their (generally different) impedances. The relationships among the power in a circuit, the incident or radiated power in space around the circuit, and the characteristics of an antenna that couples the two are completely deterministic and are well-understood. Unfortunately, these relationships are often presented in engineering texts, data reports, data analyses and automated conversion calculators merely as formulae without references to sources of their derivations. While the use of formulae is an expedient way to perform calculations, a formula-based approach to an analysis can lead to calculation errors when the numerical inputs may not be legitimately used in that formula. This problem occurs, for example, when the origins of data inserted into a formula do not correlate to the assumptions behind the derivation of the formula.

This publication seeks to address the lack of a published comprehensive set of derivations of the relationships among power in space, power in circuits, and the characteristics of antennas that couple between them. It provides, in a form that may be easily referenced, derivations of the relationships between these physical quantities.

[1] The author is with the Institute for Telecommunication Sciences, National Telecommunications and Information Administration, U.S. Department of Commerce, Boulder, Colorado 80305.

1.2 Directivity and Gain of Power in Space

The derivations begin with *directivity* and *gain*, which are measures of the directive concentration of power in space by an antenna. *Directivity*, d, is the ratio of power density in some direction, p_{den}, to the power density that would be produced if the power were radiated isotropically at p_{den_iso}:[2]

$$d = \frac{p_{den}}{p_{den\_iso}}. \tag{1}$$

Directivity makes reference only to power in space around an antenna; it is unrelated to power in a circuit that is connected to the antenna terminals. There is loss between the terminals and free space. Gain, g, includes these antenna losses:

$$g = \frac{4\pi r^2 \cdot p_{den}}{p_{in\_out}}, \tag{2}$$

where:

p_{den} = power density in space;
p_{in_out} = power at the circuit input-output of the antenna;
r = distance from the antenna.

Gain and directivity are related by efficiency, ε:

$$g = d \cdot \varepsilon \tag{3}$$

1.3 Effective Aperture of Antennas

Effective antenna aperture, a_e, is unrelated to the physical aperture of an antenna. It is defined as:

$$a_e = \frac{\lambda^2 g}{4\pi} \tag{4}$$

where λ is the free-space wavelength of the radiation. Note that a_e has the units of area.

---

[2] In this document, linear quantities are written in lower case italic (except for voltage, V) and decibel quantities (defined as decibels=$10\cdot\log_{10}(p_2/p_1)$, where p_1 and p_2 are linear power values) are written in upper case italic. For example, $P(dBW) = 10\log_{10}\cdot(p, watts)$.

1.4 Relationship Between Power in a Circuit and Power Density in Space

For an antenna matched to a load, the power in the load, p_{load}, is related to the free-space power density by a_e:

$$p_{load} = p_{den} \cdot a_e \tag{5}$$

If p_{load} is in a 50-ohm circuit and p_{den} is in a free-space impedance of 377 ohms, then the following relations apply:

$$p_{load} = \frac{V_{load}^2}{50} \tag{6a}$$

and

$$p_{den} = \frac{V_{space}^2}{377}. \tag{6b}$$

Rewriting Eq. (5) using Eqs. (6a) and (6b) gives:

$$\frac{V_{load}^2}{50} = a_e \cdot \left(\frac{V_{space}^2}{377} \right). \tag{7}$$

Note that the voltage in the circuit, V^2_{load}, is in units of volts, but that the free-space field strength, V^2_{space}, is in units of volts/m. The effective aperture, in units of m^2, converts the free-space power density on the right side to the power in a circuit on the left side.

1.5 Antenna Correction Factor and its Relationship to Gain

At this point, we introduce the *antenna correction factor, acf,* which is defined as:

$$acf = \frac{V_{space}^2}{V_{load}^2}. \tag{8}$$

Note that *acf* has units of m^{-2}. Rewriting Eq. (7) with the substitution of Eq. (8) gives:

$$acf = \frac{377}{50} \cdot \frac{1}{a_e}. \tag{9}$$

Because a_e is dependent upon both gain and frequency, so is *acf*. Substituting Eq. (4) into Eq. (9) gives:

$$acf = \left(\frac{377}{50}\right)\left(\frac{4\pi}{\lambda^2 g}\right). \tag{10}$$

If the frequency, f, is in Megahertz, and the speed of light, c, is taken to be $3 \cdot 10^8$ m/s, then the substitution $\lambda = c/(f \cdot 10^6) = 3 \cdot 10^8/(f \cdot 10^6)$ gives:

$$acf = \left(\frac{377}{50}\right)(4\pi)\left[\left(\frac{10^6}{3 \cdot 10^8}\right)^2 (f, MHz)^2 \cdot \left(\frac{1}{g}\right)\right] \tag{11}$$

which, calculating the constant term values, gives:

$$acf = \left(1.05 \cdot 10^{-3}\right) \cdot (f, MHz)^2 \cdot \left(\frac{1}{g}\right). \tag{12}$$

Expressing acf in decibel terms (ACF = $10 \cdot \log(acf)$) gives[3]:

$$(ACF, dB) = -29.8 + 20\log(f, MHz) - 10\log(g). \tag{13}$$

The quantity $10 \cdot \log(g)$ is the directional gain, G, of an antenna in decibels relative to isotropic (dBi):

$$(G, dBi) = (10\log(g)) = (20\log(f, MHz) - 29.8 - ACF). \tag{14}$$

[3] All log operations in this document are to the base 10.

2 FREE SPACE FIELD STRENGTH CONVERSION DERIVATIONS

2.1 Introduction

Signals are commonly measured in circuits as voltages either linear or log-detected, proportional to input power. The voltage induced in a measurement circuit by an input signal is usually converted to equivalent power by the circuit impedance. This conversion is usually accomplished automatically within the measurement device (e.g., a spectrum analyzer) after which the displayed power in a circuit must be converted into the field strength in free space that generated the voltage.

2.2 Incident Field Strength Related to Power in a Circuit

This power induced into a circuit, p_{load}, is related to incident field strength through either the antenna correction factor or, equivalently, the antenna gain relative to isotropic; this is done as follows. Writing Eq. (5) with a substitution for a_e from Eq. (4) gives:

$$p_{load} = \left(\frac{\lambda^2 g V_{space}^2}{4\pi \cdot 377} \right) \tag{15}$$

and substituting $\lambda = c/f$ gives:

$$p_{load} = \left[\frac{\left(3 \cdot 10^8\right)^2 g V_{space}^2}{(f, MHz)^2 \cdot \left(10^6\right)^2 \cdot 4\pi \cdot 377} \right]. \tag{16}$$

For power in milliwatts and field strength in microvolts/meter, the conversions (power, mW) = 1000·(W) and (field strength, V/m) = 10^{-6}·(μV/m) are used:

$$(p_{load}, mW) = \left[\frac{1000 \cdot \left(3 \cdot 10^8\right)^2 \cdot g \cdot \left(10^{-6}\right)^2 \left(V_{space}, \mu V / m\right)^2}{(f, MHz)^2 \cdot \left(10^6\right)^2 \cdot 4\pi \cdot 377} \right] \tag{17}$$

which, upon combining the constant terms, becomes:

$$(p_{load}, mW) = \left[\frac{1.90 \cdot 10^{-8} \cdot g \cdot \left(V_{space}, \mu V / m\right)^2}{(f, MHz)^2} \right]. \tag{18}$$

Taking 10log of both sides gives (where 10log(p_{load}, mW) = (P_{load}, dBm)):

$$\left(P_{load}, dBm\right) = -77.2 + G + 20\log\left(V_{space}, \mu V/m\right) - 20\log\left(f, MHz\right). \quad (19)$$

Rearrangement of terms yields:

$$\left(FS, dB\mu V/m\right) = \left(P_{load}, dBm\right) + 77.2 - G + 20\log\left(f, MHz\right), \quad (20)$$

where FS rather than V_{space} is now used to denote incident field strength, still in units of dBμV/m. Note that P_{load} is related to the power measured within a circuit (e.g., a spectrum analyzer) by the total decibel correction, C, for all gains and losses between the antenna and the analyzer: $P_{load} = P_{meas} - C$. This changes Eq. (20) to:

$$\left(FS, dB\mu V/m\right) = \left(P_{meas}, dBm\right) - C + 77.2 - G + 20\log\left(f, MHz\right) \quad (21)$$

Eq. (21) is key for the conversion of measured power in a circuit into incident field strength in units of dBμV/m. For example, suppose that an antenna has gain G = 16.9 dBi, at a frequency of 2300 MHz, with 31 dB of amplifier gain and 3 dB of loss between the measurement antenna terminal and the spectrum analyzer input. The measured power is taken to be -12.1 dBm on the spectrum analyzer display. Then the field strength at the measurement antenna must have been:

FS = (-12.1 dBm) – (31-3) + 77.2 – 16.9 + 20log(2300) = +87.4 dBμV/m.

If an equation is required to convert field strength in dBμV/m to the *ACF,* then Eqs. (7) and (9) are used to convert *acf* to voltage in a circuit and free-space field strength:

$$\frac{V_{load}^2}{50} = \left(p_{load}\right) = \frac{V_{space}^2}{50 \cdot acf}. \quad (22)$$

Converting power in watts to power in milliwatts, and converting field strength in volts/meter to microvolts/meter, gives

$$\left(p_{load}, mW\right) = \left[\frac{1000 \cdot \left(10^{-6}\right)^2 \cdot \left(FS, \mu V/m\right)^2}{50 \cdot acf}\right] \quad (23)$$

which, combining the constants, gives:

$$\left(p_{load}, mW\right) = \frac{2 \cdot 10^{-11} \cdot \left(FS, \mu V/m\right)^2}{acf}. \quad (24)$$

Taking 10log of both sides, the incident field strength and the measurement antenna's *ACF* is converted to the power coupled into the measurement circuit:

$$\left(P_{load}, dBm\right) = -107 + \left(FS, dB\mu V / m\right) - ACF\ . \tag{25}$$

Rearranging terms and taking into account the gains and losses, *C*, between the measurement antenna terminals and the final power measurement point, the measured power and the decibel antenna correction factor is converted to the field strength:

$$\left(FS, dB\mu V / m\right) = P_{meas} - C + 107 + ACF\ . \tag{26}$$

For example, suppose a measurement of -12.1 dBm is taken on a spectrum analyzer, with 28 dB of net gain in the path between the antenna and the analyzer, with a measurement antenna *acf* of 113 (corresponding to G = 16.9 dBi at 2300 MHz, as in the example above). Then ACF = 20.5 dB, and the corresponding free-space field strength is again computed using Eq. (26) to be +87.4 dBμV/m.

2.3 Incident Power Per Unit Area Related to Field Strength

Next, the conversion between field strength and incident power per unit area is considered. This incident power density, p_{den}, in W/m^2, is equal to the incident field strength squared (units of (V/m)^2), divided by the impedance of free space (377 ohms):

$$\left(p_{den},\ {}^{W}\!/_{m^2}\right) = \frac{\left(fs,\ {}^{V}\!/_{m}\right)^2}{377} \tag{27a}$$

and

$$\left(fs,\ {}^{V}\!/_{m}\right) = \sqrt{377 \cdot \left(p_{den},\ {}^{W}\!/_{m^2}\right)} = 19.4 \cdot \sqrt{\left(p_{den},\ {}^{W}\!/_{m^2}\right)} \tag{27b}$$

where *fs* is the field strength in linear units of (V/m). If field strength is expressed in μV/m (where 1 V/m = 10^6 μV/m), then

$$\left(p_{den},\ {}^{W}\!/_{m^2}\right) = \frac{\left(fs,\ {}^{\mu V}\!/_{m}\right)^2}{377 \cdot 10^{12}} = 2.65 \cdot 10^{-15} \cdot \left(fs,\ {}^{\mu V}\!/_{m}\right)^2 . \tag{28}$$

Using more common units for field strength (dBμ*V/m*) and incident power (μ*W/cm*^{2}), where 1 W/m^2 = [(10^6 μW)/(10^4 cm^2)] =10^2 μW/cm^2, the relation for incident power density is:

$$\left(p_{den},\ {}^{\mu W}\!/_{cm^2}\right) = \frac{1}{377 \cdot 10^{10}} \cdot \left(fs,\ {}^{\mu V}\!/_{m}\right)^2 = 2.65 \cdot 10^{-13} \cdot \left(fs,\ {}^{\mu V}\!/_{m}\right)^2 \tag{29}$$

and the conversion to mW/cm^2 changes the conversion by another factor of 10^3:

$$\left(p_{den},\ {}^{mW}\!/_{cm^2}\right) = \frac{1}{377\cdot 10^{13}}\cdot\left(fs,\ {}^{\mu V}\!/_{m}\right)^2 = 2.65\cdot 10^{-16}\cdot\left(fs,\ {}^{\mu V}\!/_{m}\right)^2. \tag{30}$$

2.4 Field Strength Units: Volts per Meter Related to Amperes per Meter

Returning to Eq. (27a), the power density in W/m^2 may also be related to field strength expressed as amperes/meter. The basis of the conversion is the equivalence of a watt to a volt-amp. This can be used as a substitution in Eq. (27a) to yield:

$$\left(p_{den},\ {}^{W}\!/_{m^2}\right) = \left(p_{den},\ {}^{(V\cdot A)}\!/_{m^2}\right) = \frac{\left(fs,\ {}^{V}\!/_{m}\right)^2}{377}, \tag{31a}$$

which reduces to:

$$(V\cdot A) = \frac{V^2}{377} \quad\rightarrow\quad (V = 377\cdot A) \quad\rightarrow\quad \frac{\left(fs,\ {}^{V}\!/_{m}\right)^2}{377} = 377\left(fs,\ {}^{A}\!/_{m}\right)^2, \tag{31b}$$

so that

$$\left(p_{den},\ {}^{W}\!/_{m^2}\right) = 377\left(fs,\ {}^{A}\!/_{m}\right)^2 \tag{31c}$$

and reciprocally

$$\left(fs,\ {}^{A}\!/_{m}\right) = \sqrt{\frac{\left(p_{den},\ {}^{W}\!/_{m^2}\right)}{377}} = 5.15\cdot 10^{-2}\cdot\sqrt{\left(p_{den},\ {}^{W}\!/_{m^2}\right)}. \tag{31d}$$

Eq. (31c) may be combined with Eq. (27a) to relate field strength in V/m to field strength in A/m:

$$\left(p_{den},\ {}^{W}\!/_{m^2}\right) = \frac{\left(fs,\ {}^{V}\!/_{m}\right)^2}{377} = 377\left(fs,\ {}^{A}\!/_{m}\right)^2 \tag{32a}$$

which reduces to:

$$\left(fs,\ {}^{A}\!/\!{}_{m} \right)^2 = \frac{\left(fs,\ {}^{V}\!/\!{}_{m} \right)^2}{377^2}. \qquad (32b)$$

Field strength is correctly rendered in unit terms of $(V/m)^2$ or $(A/m)^2$ but in reality, since all of the terms in Eq. (32b) are squared, field strength is sometimes conveniently but incorrectly rendered in un-squared units as "V/m" or "A/m". *If this technically incorrect but widely applied convention is used, the proportionality constant that must be used between V/m and A/m should be 377 rather than* 377^2.

2.5 Field Strength Units: Decibel Volts per Meter and Amperes per Meter

Just as field strength may be expressed in dBμV/m, so it may also be expressed in dBμA/m by rendering Eq. (32b) in decibel terms:

$$(FS, dB\mu A/m) = (FS, dB\mu V/m) - 20\log(377) = (FS, dB\mu V/m) - 51.5, \qquad (33a)$$

where, as always,

$$(FS, dB\mu V/m) = 10\log\left(fs, \frac{\mu V}{m} \right)^2 = 20\log\left(fs, \frac{\mu V}{m} \right) \qquad (33b)$$

and

$$(FS, dB\mu A/m) = 10\log\left(fs, \frac{\mu A}{m} \right)^2 = 20\log\left(fs, \frac{\mu A}{m} \right). \qquad (33c)$$

(Note that, because field strength goes as $(A/m)^2$, 1 dBA/m = 120 dBμA/m, just as 1 dBV/m = 120 dBμV/m.) Incident power density in W/m^2 is related to field strength in μA/m via Eq. (31c):

$$\left(p_{den},\ {}^{W}\!/\!{}_{m^2} \right) = 377 \frac{\left(fs,\ {}^{\mu A}\!/\!{}_{m} \right)^2}{10^{12}} = 3.77 \cdot 10^{-10} \cdot \left(fs,\ {}^{\mu A}\!/\!{}_{m} \right)^2, \qquad (34a)$$

and thus

$$\left(fs,\ {}^{\mu A}\!/\!{}_{m} \right)^2 = \frac{10^{12}}{377}\left(p_{den},\ {}^{W}\!/\!{}_{m^2} \right) = 2.65 \cdot 10^9 \cdot \left(p_{den},\ {}^{W}\!/\!{}_{m^2} \right). \qquad (34b)$$

Converting Eqs. (34a) and (34b) into field strength in decibel terms (dBμA/m):

$$10\log\left(p_{den},\ {}^{W}\!/_{m^2}\right) = 10\log\left(\left(1.94\cdot 10^{-5}\right)^2 \cdot \left(fs,\ {}^{\mu A}\!/_{m}\right)^2\right) = \left(FS, dB\mu A/m\right) - 94.2$$

(35a)

and reciprocally

$$\left(FS, dB\mu A/m\right) = 94.2 + 10\log\left(p_{den},\ {}^{W}\!/_{m^2}\right) = 94.2 + \left(P_{den}, dBW/m^2\right). \qquad (35b)$$

2.6 Field Strength Units: Amperes per Meter, Magnetic Field Intensity and Flux Density

Sometimes there is a requirement to convert between a magnetic field intensity (H-field, in units of A/m) and a magnetic flux density (B-field, in units of Tesla (T) or Gauss (G), where 1 T = 10^4 G). In free space, B = μ_0H, where μ_0, the permeability of free space, is defined as $4\pi\cdot 10^{-7}$ N/A^2, where force is expressed in MKS units of Newtons (kg·m/s^2). Thus,

$$\left(mfd, T\right) = 4\pi\cdot 10^{-7}\cdot\left(fs, A/m\right) = 1.257\cdot 10^{-6}\cdot\left(mfi, A/m\right) \qquad (36)$$

where *mfd* and *mfi* are magnetic flux density and magnetic field intensity, respectively.

3 EFFECTIVE ISOTROPIC RADIATED POWER CONVERSION DERIVATIONS

3.1 Introduction

It may be necessary to know the effective isotropic radiated power (*eirp* linear; *EIRP* decibels) that a device transmits. The conversion from measured power in a circuit to *EIRP* is described in this section.

3.2 Power in a Circuit Related to Free-Space Loss and Transmitted EIRP

First, free space loss must be determined. From transmit power (p_t in watts), transmit antenna gain relative to isotropic (g_t), receive antenna gain relative to isotropic (g_r), receive power (p_r in watts) and receive antenna effective aperture (a_e) the effective isotropic radiated power is:

$$eirp = p_t \cdot g_t \tag{37}$$

and

$$p_r = a_e \cdot \left(\frac{eirp}{4\pi r^2} \right) \tag{38}$$

with r being the distance between the transmit and receive antennas.

The effective aperture (Eq. (4)) of an antenna is the effective aperture of a (theoretical) isotropic antenna multiplied by the (actual) antenna gain (here a receiving antenna, g_r) over isotropic:[4]

$$a_e = \left(\frac{\lambda^2}{4\pi} \right) \cdot g_r . \tag{39}$$

A change to decibel units makes Eq. (38):

$$P_r = EIRP + G_r + 20\log(\lambda) - 20\log(4\pi) - 20\log(r). \tag{40}$$

In Eq. (40) and the following equations the term *EIRP* can also be written, from Eq. (37), as the decibel sum of the transmitted power, P_t, and the gain of the transmitter antenna, G_t, relative to isotropic:

$$EIRP = P_t + G_t . \tag{41}$$

[4] By symmetry, the same relationships will hold for transmitting antenna gain, g_t.

Substituting c/f for λ,

$$P_r = EIRP + G_r + 20\log(c) - 20\log(f) - 20\log(4\pi) - 20\log(r). \tag{42a}$$

Gathering the constants, this becomes:

$$P_r = EIRP + G_r + 20\log\left(\frac{c}{4\pi f}\right) - 20\log(r), \tag{42b}$$

which can be written in terms of propagation loss, L_p, in space between the transmitter and receiver antennas:

$$P_r = EIRP - L_p + G_r, \tag{42c}$$

where

$$L_p = 20\log(r) - 20\log\left(\frac{c}{4\pi f}\right). \tag{42d}$$

Note that the frequency dependence of the term L_p for so-called propagation path loss does *not* arise from any frequency-dependent loss of radio power through space. All frequencies of electromagnetic radiation propagate through free space equally, with only a geometric $1/r^2$ decrease in power density occurring as the distance increases from the point of transmission. The frequency dependence in the path-loss term is due to the wavelength dependence (and therefore frequency dependence) in the definition of effective aperture, a_e, of an isotropic antenna (Eq. (4)). For algebraic convenience, when the gain values, g_t and g_r, of the antennas relative to the gain of an isotropic antenna are converted to the decibel variables G_t and G_r, the wavelength term in the effective aperture of an isotropic antenna is separated from the gain values and is gathered with other constants into the propagation path-loss term. While this gathering of multiple unrelated constants into the path-loss term provides convenience and efficiency for performing calculations, it contributes to a common misconception that propagation path-loss in free space is somehow frequency-dependent; it is not.

3.3 Received Power as a Function of EIRP for Convenient Units of Distance

If frequency and distance are expressed in megahertz and meters, then Eq. (42a) becomes

$$P_r = EIRP + G_r + 20\log\left(3\cdot10^8\right) - 20\log\left(f, MHz\cdot10^6\right) - 20\log(4\pi) - 20\log(r), \tag{43}$$

which yields

$$P_r = EIRP + G_r + 169.5 - 20\log(f, MHz) - 120 - 22 - 20\log(r, meters) \quad (44a)$$

or

$$P_r = EIRP + G_r + 27.5 - 20\log(f, MHz) - 20\log(r, meters). \quad (44b)$$

Similarly, for *r* in kilometers, Eq. (44b) becomes:

$$P_r = EIRP + G_r - 32.5 - 20\log(f, MHz) - 20\log(r, km), \quad (45)$$

for *r* in statute miles:

$$P_r = EIRP + G_r - 36.5 - 20\log(f, MHz) - 20\log(r, statute\_miles) \quad (46)$$

and for *r* in nautical miles:

$$P_r = EIRP + G_r - 37.7 - 20\log(f, MHz) - 20\log(r, nautical\_miles). \quad (47)$$

3.4 EIRP Conversions Between Convenient Engineering Units of Power

For measurements of transmitters that are regulated by Part 15 or Part 18 of Federal Communication Commission (FCC) rules, Eq. (44b) is convenient. If the gain of the receive antenna is known and the received power has been measured at a known distance from the emitter, then Eq. (44b) can be rearranged to yield *EIRP* in decibels relative to a watt (dBW):

$$(EIRP, dBW) = (P_r, dBW) - G_r - 27.5 + 20\log(f, MHz) + 20\log(r, meters). \quad (48)$$

If the received power is measured in dBm rather than dBW, Eq. (48) becomes

$$(EIRP, dBW) = (P_r, dBm) - G_r - 57.5 + 20\log(f, MHz) + 20\log(r, meters). \quad (49)$$

If *EIRP* in decibels relative to a picowatt (dBpW) is required, then Eq. (48) becomes:

$$(EIRP, dBpW) = (P_r, dBm) - G_r + 62.5 + 20\log(f, MHz) + 20\log(r, meters). \quad (50)$$

For example, if -10 dBm of received power is measured at a frequency of 2450 MHz, with an antenna gain of +16.9 dBi at a distance of 3 meters, then *EIRP* = +113 dBpW.

Effective radiated power relative to a dipole (ERP_{dipole}) is sometimes required. The conversion is accomplished by applying the fact that *EIRP* is 2.1 dB higher than ERP_{dipole}, since by definition and theory the gain of a dipole antenna is 2.1 dBi.

4 CONVERSION DERIVATIONS FOR RADIO RADIATION HAZARD LIMITS

4.1 Introduction

Radiation hazard (radhaz) limits have been developed for non-ionizing (radio, or radiofrequency (RF)) energy by the American National Standards Institute (ANSI) and the Institute of Electrical and Electronic Engineers (IEEE) [1]. Another source for radhaz limits is the U.S. Department of Defense (DoD) Military Standard 461 (MILSTD-461) [2], which includes limits for hazards of electromagnetic radiation to fuel (HERF), ordnance (HERO), and personnel (HERP).

All of these limits are generally expressed in units of average incident power per unit area for any given frequency. For example, a hypothetical limit might be specified as 10 mW/cm^2 for frequencies between 1-10 GHz.[5]

This section addresses the problem of computing the maximum permissible power level that a transmitter may emit in order to comply with a stated radhaz limit. It is assumed in this derivation that all transmitted and incident power levels are averaged in accordance with the definitions of [1]. This is a necessary step to take prior to operating any transmitter that may potentially produce a power level in excess of a radhaz limit at some point in space.

4.2 Relationship Between EIRP and the Critical Separation Distance from a Transmitter Required to Comply with Radiation Hazard Limits

Radhaz limits are not limits on transmitter power; they are limits on the incident power density (power per unit area) at a point in space. Ordinarily, they are not limits on the peak power at a location, but rather on the average incident power, with the averaging interval being defined in existing documentation [1, 2].[6] Therefore, the designer of a transmitter system needs to be able to determine the critical distance, r_{crit}, at which the power from a transmitter with a given *eirp* will generate an incident power level that is equal to the critical radhaz limit. When the mathematical relationship between the *eirp* and r_{crit} established, then either parameter may be used to control the other.

The derivation assumes that radio power radiates from an antenna that behaves as a point source and that the power density in space will be assessed in the far field of the radiating antenna. This means that the power passes through a location of interest as a plane wave,

[5] Units of incident power per unit area are commonly used because they may be related to heating effects of radiofrequency energy.

[6] In practice, this is equivalent to verifying the level of the power at a critical point in space by measuring it with a spectrum analyzer or RF power meter in a root-mean-square (RMS) detection mode, using measurement intermediate frequency (IF) and video bandwidths that are each equal to or greater than the 10-dB bandwidth of the transmitter's emission spectrum.

and that nominal far-field antenna gain factors may be used in all calculations of power levels in space.

Since surface area multiplied by the power density per unit area yields incident power, the fundamental relationship between *eirp*, power density ρ at a point in space, and the distance r between the radiating antenna and the point in space is:

$$eirp = \rho\left(4\pi r^2\right). \tag{51}$$

Eq. (51) assumes that power radiating from a point in space will be distributed on a spherically shaped front (or a portion of such a front) of radius r. The density of power on the surface (or a portion of the surface) multiplied by the total area of the hypothetical spherical surface ($4\pi r^2$) gives the total *equivalent* power that would be emitted from an isotropic source to produce that power density over an entire sphere of radius r.[7]

A radiation hazard power density is a *maximum* specified power density, ρ_{max}. Eq. (51) is converted to a radiation hazard relationship by giving subscripts to the variables to indicate that they have become critical:

$$eirp_{crit} = \rho_{\max}\left(4\pi r_{crit}^2\right). \tag{52}$$

Since *eirp* is the product of transmitter power, p_t, and transmitter antenna gain, g_t, Eq. (52) can be written as:

$$\left(p_t g_t\right)_{crit} = \rho_{\max}\left(4\pi r_{crit}^2\right). \tag{53}$$

Suppose, for example, that a clear distance of 12 m needs to be maintained from a transmitter that has a linear gain of 10, and that the critical radhaz power density limit is 30 W/m^2. Then Eq. (53) becomes:

$$\left(p_t \cdot 10\right)_{crit} = 30\,\tfrac{W}{m^2} \cdot \left(4\pi \cdot \left(12\,meters\right)^2\right) = 54287\,watts\,.$$

Dividing out the gain factor, g_t, of 10 gives p_t = 5429 W = 5.43 kW.

Consider another example in which the transmitter power and antenna gain are already fixed and the minimum safety distance must be adjusted accordingly. If a power density of, for example, 10 W/m^2 is not to be exceeded, and the transmitter in question is going to produce 10,000 W that will be radiated from an antenna with a linear gain factor of 2, then Eq. (53) becomes:

((10,000 W)*2) = (10 W/m^2)*($4\pi(r_{crit})^2$),

---

[7] This argument is true even if the radiated power is concentrated in some direction; that is why the dependent variable has the word *equivalent* in its name.

or

$$r_{crit} = \sqrt{(10{,}000 \times 2)/(10 \times 4\pi)} = \sqrt{159\,\mathrm{m}^2} = 12.6\,\mathrm{m}\,.$$

4.3 Determining the Measured Power in a Circuit that Corresponds to a Radiation Hazard Limit in Space

The next problem to be considered is that of verifying through a direct measurement that the critical power limit is not being exceeded for an existing emitter. The average (RMS) power level that should not be exceeded can be written as (from Eq. (38)):

$$p_{meas\_crit} = a_e\left(\frac{eirp_{crit}}{4\pi r_{crit}^2}\right), \tag{54}$$

where

$p_{meas\ crit}$ = average power measured at the critical radhaz distance limit;
a_e = effective aperture of the measurement antenna.

The quantity inside the brackets in Eq. (54) is the total *eirp* divided by the surface area of a sphere of radius r_{crit}, and thus is the critical power density at distance r_{crit} from the transmitter. Multiplied by the effective aperture (which has units of area) of the measurement antenna, this yields the power that is coupled into the measurement circuit at r_{crit}.

The effective aperture of the measurement antenna is given by Eq. (39). Substituting the terms of Eq. (52) into Eq. (54) yields:

$$p_{meas\_crit} = \left(\frac{\lambda^2}{4\pi} g_r\right)\cdot\left(\frac{\rho_{\max}\left(4\pi r_{crit}^2\right)}{4\pi r_{crit}^2}\right) = \frac{\lambda^2}{4\pi} g_r \rho_{\max}\,. \tag{55}$$

At this point it is useful to convert the expression into decibel units by taking $10\log_{10}$ of both sides:

$$10\log(p_{meas\_crit}) = 20\log(\lambda) - 10\log(4\pi) + 10\log(g_r) + 10\log(\rho_{\max})\,. \tag{56a}$$

Writing decibel quantities in upper case and solving for the constant term we obtain:

$$P_{meas\_crit} = 20\log(\lambda) - 11 + G_r + 10\log(\rho_{\max})\,. \tag{56b}$$

It is convenient to use engineering units of dBm for P_{meas_crit} and MHz for frequency instead of the wavelength λ. The unit conversions are performed as follows:

$$\left(P_{meas\_crit}, dBm\right) = \begin{matrix} 20\log\left(\dfrac{2.99\cdot 10^8 m/s}{10^6 Hz/MHz}\right) - 20\log(f, MHz) \\ -11 + G_r + 10\log\left(\rho_{\max}, \dfrac{W}{m^2}\right) + 30 \end{matrix} \tag{57}$$

where the quantity of 30 dB is added to adjust the equation units from dBW to dBm. Simplifying:

$$\left(P_{meas\_crit}, dBm\right) = 49.5 - 20\log(f, MHz) - 11 + G_r + 10\log\left(\rho_{\max}, \frac{W}{m^2}\right) + 30 \tag{58a}$$

and combining constants and rendering the radiation density in the linear units that it would have in a look-up table such as is found in [1] result in:

$$\left(P_{meas\_crit}, dBm\right) = 68.5 - 20\log(f, MHz) + G_r + 10\log\left(\rho_{\max}, \frac{W}{m^2}\right). \tag{58b}$$

For example, [1] gives a radhaz limit for members of the general public of 10 W/m^2 for frequencies between 2000 MHz to 100 GHz. If we want to verify that this limit is being met using a measurement antenna with a gain of 3 dBi at a frequency of 5700 MHz, then the critical power level that we would measure with that antenna connected to a spectrum analyzer would be:

$P_{meas\ crit}$ = 68.5 – 75.1 + 3 + 10log(10 W/m^2) = +6.4 dBm.

4.4 Limiting EIRP of a Transmitter Based on Measured Power and a Critical Distance where a Radiation Hazard Limit is Reached

How would this critical measured power limit relate to the *eirp* of the transmitter and some distance, r_{crit}, in meters from the transmitter? For this, Eq. (49) is used:

$$\left(EIRP_{crit}, dBW\right) = \left(P_{meas\_crit}, dBm\right) - 57.5 + 20\log(f, MHz) + 20\log\left(r_{crit}\right) - G_r.$$

To continue the example above, if the critical power value that must be measured is +6.4 dBm and r_{crit} = 3 meters, with a frequency of 5700 MHz and a measurement antenna gain of +3 dBi, then the maximum $EIRP_{crit}$ power level of the transmitter and antenna that is connected to it must not exceed:

$EIRP_{crit}$ = (+6.4 dBm -57.5 + 75.1 + 9.5 – 3) = +30.5 dBW = 1133 W = 1.13 kW.

4.5 Maximum EIRP Related to Radiation Hazard Field Strength Limits

In addition to the physical criteria that have been discussed so far (power density in space, *eirp* from a transmitter, and power measured in a circuit), there is one more factor that is sometimes used in incident radiation criteria and measurements, and that is field strength. For this purpose, we adapt Eq. (21):

$$(P_{meas\_crit}, dBm) = FS_{crit} - 77.2 - 20\log(f, MHz) + G_r$$

where FS_{crit} is the incident field strength, in dBμV/m, at the radhaz limit. Substituting this expression for P_{meas_crit} into Eq. (49) gives the connection between maximum allowable *EIRP* and the maximum allowable field strength at a point in space:

$$(EIRP_{crit}, dBW) = FS_{crit} + 20\log(r, meters) - 134.7. \qquad (59)$$

5 REFERENCES

[1] ANSI/IEEE, “IEEE Standard for Safety Levels with Respect to Human Exposure to Radio Frequency Electromagnetic Fields, 3 kHz to 300 GHz,” ANSI/IEEE document C95.1, 1995. See in particular Tables 8 and 9 for recommended exposure limits.

[2] Military Standard 461, “Electromagnetic emission and susceptibility requirements for the control of electromagnetic interference,” MIL-STD-461C, U.S. Department of Defense, August 1986.

FORM **NTIA-29** U.S. DEPARTMENT OF COMMERCE
(4-80) NATIONAL. TELECOMMUNICATIONS AND INFORMATION ADMINISTRATION

BIBLIOGRAPHIC DATA SHEET

1. PUBLICATION NO. TM-10-469	2. Government Accession No.	3. Recipient's Accession No.
4. TITLE AND SUBTITLE Derivations of Relationships among Field Strength, Power in Transmitter-Receiver Circuits and Radiation Hazard Limits		5. Publication Date June 2010 6. Performing Organization Code NTIA/ITS.T
7. AUTHOR(S) Frank H. Sanders		9. Project/Task/Work Unit No. 6454000-200
8. PERFORMING ORGANIZATION NAME AND ADDRESS Institute for Telecommunication Sciences National Telecommunications & Information Administration U.S. Department of Commerce 325 Broadway Boulder, CO 80305		10. Contract/Grant No.
11. Sponsoring Organization Name and Address National Telecommunications & Information Administration Herbert C. Hoover Building, 14th & Constitution Ave., NW Washington, DC 20230		12. Type of Report and Period Covered Tech Memorandum, FY10
14. SUPPLEMENTARY NOTES		
15. ABSTRACT This NTIA Technical Memorandum provides a single, comprehensive set of derivations of the mathematical relationships among power in a radio transmitter or measurement circuit, the associated incident field strength in space, transmitter-receiver antenna characteristics, free space propagation equation variables and radiation hazard limits.		
16. Key Words (Alphabetical order, separated by semicolons) antenna correction factor; effective isotropic radiated power; field strength; hazards of electromagnetic radiation to fuel; hazards of electromagnetic radiation to ordnance; hazards of electromagnetic radiation to personnel; measured power; military standard 461; power in circuits; radiation hazard; RF measurement		
17. AVAILABILITY STATEMENT ☒ UNLIMITED.	18. Security Class. (This report) Unclassified	20. Number of pages 19
	19. Security Class. (This page) Unclassified	21. Price:

NTIA FORMAL PUBLICATION SERIES

NTIA MONOGRAPH (MG)

A scholarly, professionally oriented publication dealing with state-of-the-art research or an authoritative treatment of a broad area. Expected to have long-lasting value.

NTIA SPECIAL PUBLICATION (SP)

Conference proceedings, bibliographies, selected speeches, course and instructional materials, directories, and major studies mandated by Congress.

NTIA REPORT (TR)

Important contributions to existing knowledge of less breadth than a monograph, such as results of completed projects and major activities. Subsets of this series include:

NTIA RESTRICTED REPORT (RR)

Contributions that are limited in distribution because of national security classification or Departmental constraints.

NTIA CONTRACTOR REPORT (CR)

Information generated under an NTIA contract or grant, written by the contractor, and considered an important contribution to existing knowledge.

JOINT NTIA/OTHER-AGENCY REPORT (JR)

This report receives both local NTIA and other agency review. Both agencies' logos and report series numbering appear on the cover.

NTIA SOFTWARE & DATA PRODUCTS (SD)

Software such as programs, test data, and sound/video files. This series can be used to transfer technology to U.S. industry.

NTIA HANDBOOK (HB)

Information pertaining to technical procedures, reference and data guides, and formal user's manuals that are expected to be pertinent for a long time.

NTIA TECHNICAL MEMORANDUM (TM)

Technical information typically of less breadth than an NTIA Report. The series includes data, preliminary project results, and information for a specific, limited audience.

For information about NTIA publications, contact the NTIA/ITS Technical Publications Office at 325 Broadway, Boulder, CO, 80305 Tel. (303) 497-3572 or e-mail info@its.bldrdoc.gov.

This report is for sale by the National Technical Information Service, 5285 Port Royal Road, Springfield, VA 22161,Tel. (800) 553-6847.