A METHOD OF NUMERICAL REPRESENTATION
FOR THE AMPLITUDE-PROBABILITY DISTRIBUTION
OF ATMOSPHERIC RADIO NOISE

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A METHOD OF NUMERICAL REPRESENTATION FOR THE AMPLITUDE-PROBABILITY DISTRIBUTION OF ATMOSPHERIC RADIO NOISE

Hiroshi Akima

It is well known that the amplitude-probability distribution (APD) function of atmospheric radio noise can be closely represented by a curve that one can select from a family of curves by specifying the ratio of rms to average voltage, usually expressed in decibels and denoted by $V_d$. This report presents a method for numerically representing the APD function of atmospheric radio noise for a specified $V_d$ value. It also presents two computer subprograms that implement the method.

Key Words: Amplitude-probability distribution (APD), atmospheric radio noise.

1. INTRODUCTION

To analyze the effects of radio noise on the performance of radio communication systems, we sometimes need knowledge of the amplitude-probability distribution (APD) of the noise, that is, the probability of a voltage level being exceeded by the envelope of the noise. For many applications, the APD function of atmospheric radio noise can be closely approximated by a curve that is selected from a family of curves (Spaulding et al., 1962; CCIR, 1964). One can make this selection by specifying the ratio of rms to average voltage, usually expressed in decibels and denoted by $V_d$.

Although CCIR Report 322 (CCIR, 1964) gives this family of curves, a method is needed for numerically representing the APD function to utilize those curves in computerized analysis of system
performances. In this study we develop such a method. The findings in previous studies (Crichlow et al., 1960; Spaulding et al., 1962) are fully utilized in this study. We also present two computer subprograms that implement this method.

2. DEVELOPMENT OF THE METHOD

2.1. Coordinate System

We denote the instantaneous envelope voltage of atmospheric noise, for which the APD function is desired, by \( v \) and the APD function by \( P(v) \). For the present, the unit of \( v \) is taken arbitrarily.

We use a coordinate system in which the abscissa and the ordinate are defined as

\[
\begin{align*}
x &= -20 \log(-\ln P), \\
y &= 20 \log v,
\end{align*}
\]

(1) (2)

respectively, where \( \log \) is the common logarithm and \( \ln \) the natural logarithm. With \( x \) and \( y \), \( P \) and \( v \) are expressed by

\[
\begin{align*}
P &= \exp\{ -\text{antilog}(-x/20) \}, \\
v &= \text{antilog}(y/20),
\end{align*}
\]

(3) (4)

where \( \text{antilog} \) is the inverse function of the common logarithm.

This coordinate system is shown in figure 1. The origin of the coordinate system corresponds to

\[
\begin{align*}
P &= 1/e = 0.367879, \\
v &= 1.
\end{align*}
\]

As shown in this figure, the Rayleigh distribution that is expressed by

\[
P(v) = \exp(-v^2)
\]

(5)
Figure 1. Coordinate system used.
plots as a straight line through the origin with a slope of -0.5 in this coordinate system. This coordinate system conforms to the custom followed by the previous studies (Crichlow et al., 1960; Spaulding et al., 1962) and the CCIR (1964).

2.2. Basic Assumption of Three-Section Curve

After Crichlow et al. (1960) and Spaulding et al. (1962), we assume that the APD function of atmospheric radio noise can be closely represented by a three-section curve in the coordinate system introduced in the previous section. This curve consists of two straight lines joined tangentially by a circular arc, as shown in figure 2.

For small y values the curve coincides with a straight line, L1, having the same slope as the Rayleigh distribution, i.e., the slope of -0.5. For large y values the curve coincides with another straight line, L2, having a steeper (negative) slope.

2.3. Geometry of the Three-Section Curve

We denote the points where the circular arc is tangent to straight lines L1 and L2 by \((x_1, y_1)\) and \((x_2, y_2)\), respectively, as shown in figure 2. We also denote the center of the circle by \((x_c, y_c)\) and its radius by \(r\). Then, the three-section curve can be expressed by

\[
L_1: \quad y = m_1 x + b_1
\]

for \(x \geq x_1\) and \(y \leq y_1\);

\[
C: \quad (x - x_c)^2 + (y - y_c)^2 = r^2
\]

for \(x_1 \geq x \geq x_2\) and \(y_1 \leq y \leq y_2\);
Figure 2. Three-section curve for the APD function.
\[ L_2: \quad y = m_2 x + b_2 \]  \hspace{1cm} (8)

for \( x \leq x_2 \) and \( y \geq y_2 \).

We denote the third straight line that is tangent to the arc at its midpoint by

\[ L_3: \quad y = m_3 x + b_3. \]  \hspace{1cm} (9)

Since \( L_3 \) is parallel to the bisector of the angle between \( L_1 \) and \( L_2 \), \( m_3 \) is related to \( m_1 \) and \( m_2 \) by

\[ m_3 = \tan\left(\frac{\tan^{-1} m_1 + \tan^{-1} m_2}{2}\right). \]  \hspace{1cm} (10)

where a principal value in the range from \(-\pi/2\) to \(\pi/2\) is taken for the arctangent function. If we denote the direction tangent of a bisector of the angle between \( L_1 \) and \( L_3 \) by \( m_4 \), it is given by

\[ m_4 = \tan\left(\frac{\tan^{-1} m_1 + \tan^{-1} m_3}{2}\right). \]  \hspace{1cm} (11)

We further denote the point of intersection between \( L_1 \) and \( L_2 \) by \((x_3, y_3)\), and between \( L_1 \) and \( L_3 \) by \((x_4, y_4)\), as shown in figure 2. Then, these coordinates are given as

\[ x_3 = \frac{b_2 - b_1}{m_1 - m_2}, \]  \hspace{1cm} (12)

\[ y_3 = \frac{m_1 b_2 - m_2 b_1}{m_1 - m_2}, \]  \hspace{1cm} (13)

\[ x_4 = \frac{b_3 - b_1}{m_1 - m_3}, \]  \hspace{1cm} (14)
With these values, the coordinates $x_c$ and $y_c$ are given by

$$\begin{align*}
x_c &= \frac{m_3 (x_4 + m_4 y_4) - m_4 (x_3 + m_3 y_3)}{m_3 - m_4}, \\
y_c &= \frac{(x_3 + m_3 y_3) - (x_4 + m_4 y_4)}{m_3 - m_4}.
\end{align*}$$

The coordinates $x_1$, $y_1$, and $y_2$ are given by

$$\begin{align*}
x_1 &= \frac{x_c + m_1 (y_c - b_1)}{1 + m_1^2}, \\
y_1 &= \frac{b_1 + m_1 x_c + m_1^2 y_c}{1 + m_1^2}, \\
x_2 &= \frac{x_c + m_2 (y_c - b_2)}{1 + m_2^2}, \\
y_2 &= \frac{b_2 + m_2 x_c + m_2^2 y_c}{1 + m_2^2}.
\end{align*}$$

Finally, the radius of the circular arc is expressed by

$$r = \sqrt{(x_c - x_1)^2 + (y_c - y_1)^2}.$$  \hspace{1cm} (22)

It follows from this geometry that, if five constants $m_1$, $m_2$, $b_1$, $b_2$, and $b_3$ are specified, all the necessary values for determining the curve by (6) to (8) can be uniquely determined by following steps from (10) to (22).
2.4. Additional Assumptions

Since straight line $L_1$ has the same slope as that for the Rayleigh distribution (Crichlow et al., 1960; Spaulding et al., 1962), we have

$$m_1 = -0.5.$$ (23)

Spaulding et al. (1962) also showed that the third straight line $L_3$ can be expressed, as an empirical formula, by

$$y - y_3 = m_3(x - x_3) + 1.5(m_2/m_1 - 1).$$

From this, we have

$$b_3 = y_3 - m_3x_3 + 1.5(m_2/m_1 - 1).$$ (24)

Because of these additional conditions, we have now only three parameters, $m_2$, $b_1$, and $b_2$, that we must specify to construct the desired curve.

2.5. Moments of the APD

After Crichlow et al. (1960) and Spaulding et al. (1962), we introduce three moments that are used to specify the APD function of atmospheric radio noise. They are

- rms voltage: $v_{rms} = (v^2)^{1/2}$, (25)
- average voltage: $v_{ave} = \bar{v}$, (26)
- logarithmic average voltage: $v_{log} = \text{antilog}(\log v)$. (27)

These moments can be expressed by

$$v_{rms} = \left\{ \int_{-\infty}^{\infty} [\text{antilog}(y/10)] (-dP/dy) dy \right\}^{1/2},$$ (28)
\[ v_{ave} = \int_{-\infty}^{\infty} \left[ \text{antilog} \left( \frac{y}{20} \right) \right] (-dP/dy) \, dy , \quad (29) \]

\[ v_{log} = \text{antilog} \left[ \int_{-\infty}^{\infty} \left( \frac{y}{20} \right) (-dP/dy) \, dy \right] , \quad (30) \]

where \((-dP/dy)\) is the probability density function. Since \(P\) is a function of \(x\) as given in (3), the probability density function can be expressed by

\[ (-dP/dy) = (dP/dx)(-dx/dy) \]

\[ = \left[ \exp\{\text{antilog}(-x/20)\} \right] \left[ \text{antilog}(-x/20) \right] \left( \frac{1}{20M} \right) (-dx/dy) , \quad (31) \]

where \(M\) is a constant defined by

\[ M = \log_{10} e = 0.434294 \quad (32) \]

Therefore, if \(x = x(y)\) is given as a function of \(y\) in our coordinate system, we can calculate the three moments by numerical integration from (28) through (30).

We use \(v_{\text{rms}}\) to represent the intensity of atmospheric radio noise, and the ratios of \(v_{\text{rms}}\) to the other two moments to represent the statistical characteristics of the atmospheric radio noise or the shape of its APD function. These ratios are expressed in decibels, denoted by \(V_d\) and \(L_d\), and defined as

\[ V_d = 20 \log \left( \frac{v_{\text{rms}}}{v_{ave}} \right) , \quad (33) \]

\[ L_d = 20 \log \left( \frac{v_{\text{rms}}}{v_{log}} \right) . \quad (34) \]

If we move the curve vertically without deforming it, the magnitudes of \(v_{\text{rms}}\), \(v_{ave}\), and \(v_{log}\) vary by a common factor and neither \(V_d\) nor \(L_d\) will vary, because the ordinate of our coordinate system is envelope voltage expressed in decibels. Since moving the curve vertically without deforming it corresponds to changing \(b_1\) and \(b_2\) without
changing $m_2$ and $(b_1 - b_2)$, we can see that both $V_d$ and $L_d$ are functions of $m_2$ and $(b_1 - b_2)$ and are independent of $b_1$ or $b_2$ itself.

Setting $b_1 = 0$ momentarily, we determined the APD function for each set of $m_2$ and $(b_1 - b_2)$ values from (10) through (24), calculated $v_{\text{rms}}$, $v_{\text{ave}}$, and $v_{\text{log}}$ by numerical integration from (28) through (30), and calculated $V_d$ and $L_d$ from (33) and (34), respectively. The results are shown in figures 3 and 4 as their respective contour maps. Note that, for the Rayleigh distribution, $V_d = 1.05$ dB and $L_d = 2.51$ dB. We can use these figures to determine the set of necessary values of $m_2$ and $(b_1 - b_2)$ for a set of specified values of $V_d$ and $L_d$ by overlapping two respective contours on a sheet and determining the point of intersection of the two contours.

2.6. Correlation of the Two Moments

Spaulding et al. (1962) showed that observed values of $V_d$ and $L_d$ are very highly correlated. This fact enables us to simplify the procedure of determining the APD function and to determine both $m_2$ and $(b_1 - b_2)$ by specifying only one parameter.

From figure 1 of their paper, we determined the average relation between observed $V_d$ and $L_d$ as

$$L_d = 1.697 V_d + 0.7265$$  

(35)

Using now the calculated values of $V_d$ and $L_d$ given by (28) - (34), we calculated

$$\Delta = L_d - (1.697 V_d + 0.7265)$$  

(36)

as a function of $m_2$ and $(b_1 - b_2)$. The result is shown in figure 5 as a contour map. This figure indicates that the contour for $\Delta = 0$ dB behaves in a very strange manner. But, if we relax (35) and allow a
Figure 3. Contour map of $V_d$. 
Figure 4. Contour map of $L_d$. 
Figure 5. Contour map of $\Delta$. 
deviation from it by a small amount that does not exceed 0.3 dB, we can
draw a smooth and monotonic increasing curve, as shown in this figure.
We adopt this curve as a basic relation between $m_2$ and $(b_1 - b_2)$.

We could draw another smooth and monotonic increasing curve
that has a steeper slope than the adopted curve. For example, the con­tour for a small value of $V_d$ in figure 3 gives a small deviation from
(35). Such a curve, however, does not lead to actually observed APD
function of atmospheric radio noise (Spaulding, private communication).
Moreover, it cannot cover large values of $V_d$, say 5 dB or greater.
Therefore, we disregard such a curve as an extraneous solution.

2.7. Determination of Necessary Parameters

Once the numerical relation between $m_2$ and $(b_1 - b_2)$ is deter­mined, we can establish a method that allows us to construct a curve
for the APD function by specifying only one parameter, $V_d$. All nec­essary values of parameters are determined in the following manner.

We first assign a value to $m_2$ and next read the value of $(b_1 - b_2)$
for this assigned $m_2$ value from figure 5. Setting $b_1 = 0$ momentarily,
we calculate the necessary $b$ and $m$ constants, determine the APD func­tion from (10) through (24), calculate $v_{rms}$ and $v_{ave}$ by numerical in­tegration from (28) and (29), and calculate $V_d$ from (33). Although $b_1$
could take any value, it is convenient to assign a value of $-20 \log v_{rms}$
to $b_1$, where $v_{rms}$ is the value calculated with $b_1 = 0$. By this option,
we can make the value of $v_{rms}$ calculated with the new $b_1$ value to be
unity (or 0 dB) and let the variable $v$ denote the envelope voltage nor­malized with its rms value as a unit.

The values of $b_1$, $b_2$, and $V_d$ thus calculated for several $m_2$
values are shown in table 1. Since $b_1$, $b_2$, and $V_d$ are smooth and
monotonic functions of $m_2$, we can interpolate values of $m_2$, $b_1$ and
Table 1. Computed Dependence of $b_1$, $b_2$, and $V_d$ upon $m_2$

<table>
<thead>
<tr>
<th>$m_2$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$V_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.0491</td>
</tr>
<tr>
<td>-0.6</td>
<td>-0.4329</td>
<td>-0.7529</td>
<td>1.1779</td>
</tr>
<tr>
<td>-0.7</td>
<td>-0.8909</td>
<td>-1.5309</td>
<td>1.3215</td>
</tr>
<tr>
<td>-0.8</td>
<td>-1.3751</td>
<td>-2.3351</td>
<td>1.4803</td>
</tr>
<tr>
<td>-0.9</td>
<td>-1.8867</td>
<td>-3.1667</td>
<td>1.6549</td>
</tr>
<tr>
<td>-1.0</td>
<td>-2.4269</td>
<td>-4.0269</td>
<td>1.8466</td>
</tr>
<tr>
<td>-1.2</td>
<td>-3.5983</td>
<td>-5.8383</td>
<td>2.2831</td>
</tr>
<tr>
<td>-1.4</td>
<td>-4.8927</td>
<td>-7.7827</td>
<td>2.7973</td>
</tr>
<tr>
<td>-1.6</td>
<td>-6.3195</td>
<td>-9.8695</td>
<td>3.3941</td>
</tr>
<tr>
<td>-1.8</td>
<td>-7.8868</td>
<td>-12.1068</td>
<td>4.0796</td>
</tr>
<tr>
<td>-2.2</td>
<td>-11.4495</td>
<td>-17.0495</td>
<td>5.7218</td>
</tr>
<tr>
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</tr>
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<td>7.7069</td>
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<tr>
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<td>8.8107</td>
</tr>
<tr>
<td>-3.5</td>
<td>-26.3694</td>
<td>-37.0919</td>
<td>12.9794</td>
</tr>
<tr>
<td>-4.0</td>
<td>-32.6321</td>
<td>-46.0824</td>
<td>16.0528</td>
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<tr>
<td>-5.0</td>
<td>-44.9001</td>
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<td>22.1551</td>
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<tr>
<td>-6.0</td>
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<td>28.2294</td>
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<tr>
<td>-7.0</td>
<td>-69.2146</td>
<td>-109.4042</td>
<td>32.2720</td>
</tr>
<tr>
<td>-8.0</td>
<td>-81.3777</td>
<td>-133.2062</td>
<td>40.2839</td>
</tr>
<tr>
<td>-9.0</td>
<td>-93.6426</td>
<td>-158.0634</td>
<td>46.2711</td>
</tr>
<tr>
<td>-10.0</td>
<td>-105.8298</td>
<td>-183.8612</td>
<td>52.2264</td>
</tr>
</tbody>
</table>
b_2 for a given V_d value. Therefore, we can uniquely construct a curve for the APD function for a given V_d value.

2.8. Comparison With the CCIR Curves

CCIR Report 322 (CCIR, 1964) gives the APD curves parametric in V_d for V_d = 2 to 30 dB at a 2-dB interval. It also gives a curve that corresponds to V_d = 1.05 dB, i.e., the curve for the Rayleigh distribution. We calculated the APD function, P, for the same values of V_d by the method described in this report and compared the result with the CCIR curves.

For V_d ≥ 8 dB, the APD curve generated by this method differs from the respective CCIR curve by less than 1 dB. For V_d = 2 to 6 dB, the APD curve by this method falls within 1 dB of the respective CCIR curve for P ≥ 10^{-3}, but the former is lower than the latter by more than 1 dB for P < 10^{-3}. In other words, the slope of the curve by this method is more gentle than that of the CCIR curve in the range of small P values.

The normalized envelope voltage, v, at P = 10^{-6} calculated by this method is smaller than the CCIR value by as much as 3 dB for V_d = 2 to 6 dB. However, the v values at P = 10^{-6} for equally spaced V_d values calculated by this method are more regularly spaced than the CCIR values. This seems to be a rather desirable feature for the purpose of presenting a family of idealized APD curves of atmospheric radio noise.
3. CONCLUSIONS

We have described a method for numerically representing the APD function of atmospheric radio noise for a specified value of $V_d$, the ratio of rms to average of envelope voltage of the noise. This method is mostly based on previous studies by Crichlow et al. (1960) and Spaulding et al. (1962); the only exception is that we relax the relation between $V_d$ and $L_d$ (35) by a small amount to obtain a smooth curve for the relation between some necessary parameters.

This method was implemented in computer subprograms. These subprograms were successfully used by the author (Akima et al., 1969; Akima, 1970) and proved to be useful. They are described in the appendix. A similar mathematical representation of the APD function for atmospheric radio noise has been used by various authors in system performance studies. See, for example, Conda (1965), Halton and Spaulding (1966), and Spaulding (1966).

4. ACKNOWLEDGMENTS

The author expresses his deep appreciation to Edwin L. Crow, Robert W. Hubbard, and Arthur D. Spaulding of the Institute for Telecommunication Sciences for their constructive criticisms on this report.
5. REFERENCES


APPENDIX

Computer Subprograms

User information and Fortran listings are given on two function subprograms, APDAN and RANAN. Each subprogram implements the method for numerically representing the amplitude-probability distribution (APD) function of atmospheric radio noise, described in the text. These subprograms are written in the CDC-3800 Fortran. (For detail of the CDC-3800 Fortran, see 3400/3600/3800 Computer Systems Fortran Reference Manual, Pub. No. 60132900, and 3600/3800 Computer Systems Library Functions, Pub. No. 60056400 B, Control Data Corporation, Palo Alto, Calif.)

The APDAN Function. This function gives a value of the APD function or its density function of atmospheric radio noise for a specified $V_d$ value and for a specified envelope voltage relative to its rms value.

This function has the form

$$\text{APDAN}(V_D, K, \text{DB})$$

where

- $V_D = \text{CCIR-NBS atmospheric noise parameter}$
  - $= \text{ratio of rms to average of envelope voltage expressed in decibels,}$
- $K = \text{1 for the APD function}$
  - $= \text{2 for the probability density function,}$
- $\text{DB} = \text{envelope voltage, in decibels relative to its rms value,}$
  - $\text{for which the APD or probability density function is to be computed.}$

This function occupies 478 locations on the CDC-3800 computer. Computation time required for this function on the same computer is
approximately equal to 1500 microseconds when this function is called with a VD value that is different from the one in the previous call, and to 500 microseconds when called with the same VD value as in the previous call.

The RANAN Function. Successive calls of this function generate a series of random numbers that follow the APD function of atmospheric radio noise having a specified \( V_d \) value and its rms value of unity.

This function has the form

\[
\text{RANAN}(V_D)
\]

where

\[
V_D = \text{CCIR-NBS atmospheric noise parameter}
\]

\[
= \text{ratio of rms to average of envelope voltage expressed in decibels.}
\]

This function occupies 406 locations on the CDC-3800 computer. Computation time required for this function on the same computer is approximately equal to 1600 microseconds when this function is called with a different VD value from the one in the previous call, and to 500 microseconds when called with the same VD value.
FUNCTION APDAN(VD,K,DB)  
AMPLITUDE-PROBABILITY DISTRIBUTION OF ATMOSPHERIC NOISE  
VD = CCIR-NSP NOISE PARAMETER  
V = RATIO OF RMS TO AVERAGE OF NOISE ENVELOPE IN DB  
K = 1 FOR AMPLITUDE-PROBABILITY DISTRIBUTION FUNCTION  
DB = LEVEL RELATIVE TO RMS IN DB  

C DECLARATION STATEMENTS  
DIMENSION VVD(24),BB1(24),BB2(24),MM2(24)  
REAL MM2,M1,M2,M3,M4,M1SQPl,M2SQPl  
DATA (VVD=  
1.0491, 1.1779, 1.3215, 1.4803, 1.6549, 1.8466,  
2.2831, 2.7973, 3.3941, 4.0796, 4.8675, 5.7218,  
6.6744, 7.7069, 8.8107, 9.9740, 12.9794, 16.0528)  
DATA (BB1=  
0.0000, -0.2929, -0.8909, -1.3751, -1.8867, -2.4269,  
-3.5983, -13.4448, -41.9901, -88.1=901, -120.4991, -160.8927,  
-190.5641, -220.6100, -250.6072, -280.7380, -310.0919, -360.6321,  
-44.94001, 57.0708, -69.2146, -81.3777, -93.6426, -105.8298)  
DATA (BB2=  
0.0000, -0.7529, -1.5309, -2.3351, -3.1667, -4.0269,  
-5.0000, -6.0000, -7.0000, -8.0000, -10.0000)  
DATA (MM2=  
0.5000, 0.6000, 0.7000, 0.8000, 0.9000, -1.0000,  
-1.2000, -1.4000, -1.6000, -1.8000, -2.0000, -2.2000,  
-2.4000, -2.6000, -2.8000, -3.0000, -3.2000, -4.0000,  
-5.0000, -6.0000, -7.0000, -8.0000, -9.0000, -10.0000)  
DATA (M1=0.5),(M1SQPl=1.25)  
1 (C1=0.2305258593),(C2=0.1151292546)  
2 (VDPV=0.0)  
EQUIVALENCE (I,DM,L),(IMN,VL,B1),(IMX,VZ,B2),(V3,M2),(V4,SF),  
(V43,XCl),(V42,YCl),(V41,DBMN),(V32,DMX1),(V31,R5Q),(V21,SMrT),  
(A1,X3,DX),(A2,Y3,DY),(A3,X4,PP),(A4,Y4)  
CHECK IF INPUT PARAMETERS ARE IN ERROR  
10 VDO=VD  
$ K0=K  
$ DB0=DB  
IF(VDO>LT=1.049) GO TO 90  
IF(KO=LT=1.OR.KO=GT=2) GO TO 91  
C CHECK IF VD IS THE SAME AS IN THE PREVIOUS CALL  
20 IF(VDO=GE=1.05) GO TO 22  
21 L=1  
PP=EXP(C1*DB0)$ GO TO 65  
GO TO 60  
C LOCATE VDO  
30 VDPV=VDO  
IF(VDPV>LT=VVD(3)) GO TO 35  
IF(VDPV>GE=VVD(22)) GO TO 36  
IMN=4$ IMX=22  
31 I=(IMN+IMX)/2  
IF(VDPV>GE=VVD(1)) GO TO 33  
IMX=I  
33 IMN=I+1  
34 IF(IMX,GT,IMN) GO TO 31  
I=IMX  
$ GO TO 40  
21
C  INTERPOLATION OF B1, B2, AND M2

40 V1=VVD(I-2)-VDO $ V2=VVD(I-1)-VDO $ V3=VVD(I) -VDO $ V4=VVD(I+1)-VDO

V43=V4-V3 $ V42=V4-V2 $ V32=V3-V2 $ V31=V3-V1 $ V21=V2-V1

A1=V41*V31*V21 $ A1= V4*V3*V2/A1 $ A2=V42*V32*V22 $ A2=-V4*V3*V1/A2

A3=V43*V33*V31 $ A3 = V4*V2*V1/A3 $ A4=V44*V34*V41 $ A4=-V3*V2*V1/A4


C  GEOMETRY

50 SF=M2/M1 $ M2SQPl=M2*M2+1.0 $ SM=M1+M2

X3=(B2-B1)/DM $ Y3=(M1*B2-M2*B1)/DM $ T=(I+0-M1*M2)/SM

DM=M1-M2 $ DM=M1-M3 $ DM=M3-M4 $ DM=M3-M4

X4=(B3-B1)/DM $ Y4=(M1*B3-M2*B1)/DM $ T=(I+0-M1*M3)/SM

M3=-T-SQRT<T*T+1.0> $ M4=-T-S QRT<T*T+1.0>

C  COMPUTATION OF THE FUNCTION

60 IF(DB0>GE,DBMX) GO TO 63

IF(DB0GT,DBMN) GO TO 62

61 L=1 $ PP=EXP(C1*(DB0-B1)) $ GO TO 65

L=2 $ DY=YC-DBO $ DX=SQR(T(RSQ-DY*DY)) $ GO TO 65

L=3 $ PP=EXP(C1*(DB0-B2)/SF) $ GO TO 65

L=4 $ PP=EXP(C1*(DB0-B2)/SF) $ GO TO 65

65 APDAN=EXP(-PP) IF(KO.EQ.1) RETURN

GO TO (69*67,68) + L $ DYN=0.5*DY*APDAN/EX $ GO TO 69

APDAN=APDAN/EX $ APDAN=APDAN*PP*C1 $ RETURN

C  ERROR EXIT

90 PRINT 2090 $ GO TO 95

91 PRINT 2091

95 PRINT 2095, VDO*,KO,DB0

RETURN

C  FORMAT STATEMENTS

2090 FORMAT(1X/21H *** VD TOO SMALL,/) 2091 FORMAT(1X/25H *** IMPROPER K VALUE,/) 2095 FORMAT(7H,VD =,E12.3*8X=3HK =,E13.8*4HDB =,E12.3/$

1 35H ERROR DETECTED IN ROUTINE APDAN)
FUNCTION RAN(VD)
C RANDOM NUMBER THAT follows the apd of atmospheric noise
C THIS FUNCTION GIVES A LEVEL IN DB RELATIVE TO RMS
C VD = CCR-NBS NOISE PARAMETER
C = RATIO OF RMS TO AVERAGE OF NOISE ENVELOPE IN DB
C
DECLAREMENT STATEMENTS

DIMENSION VVD(24), BB1(24), BB2(24), MM2(24)
REAL MM2, M1, M2, M3, M4, M150P1, M250P1
DATA (VVD) =
1 1.0491, 1.1779, 1.3215, 1.4803, 1.6549, 1.8466
2 2.2831, 2.7973, 3.3941, 4.0796, 4.8567, 5.7218
3 6.6744, 7.7069, 8.8107, 9.9740, 12.9794, 16.0528
4 22.1551, 28.2294, 34.2720, 40.2839, 46.2711, 52.2264
DATA (BB1) =
1 0.0000, -0.4329, -0.8909, -1.3751, -1.8867, -2.4269
4 -44.9001, -57.0708, -69.2146, -81.3777, -93.6426, -105.8298
DATA (BB2) =
1 0.0000, -0.7529, -1.5309, -2.3351, -3.1667, -4.0269
2 -5.8383, -7.7827, -9.8695, -12.1068, -14.4991, -17.0495
4 -56.6023, -80.8042, -104.0424, -133.0262, -160.6034, -183.8612
DATA (MM2) =
1 -0.5000, -0.6000, -0.7000, -0.8000, -0.9000, -1.0000
2 -1.2000, -1.4000, -1.6000, -1.8000, -2.0000, -2.2000
3 -2.4000, -2.6000, -2.8000, -3.0000, -3.2000, -3.4000
4 -5.0000, -6.0000, -7.0000, -8.0000, -9.0000, -10.0000
DATA (M1=-0.5)*(M150P1=1.25),(CO=8.685889638),(VDPV=0.0)
EQUIVALENCE (PROB, X), (I, OM),
1 (IM1 V1), (IMX V2), (V3, M2), (V4, SF),
2 (V43 XC), (V42 YC), (V41 XM), (V32 XM),
3 (V31 RS), (V21 SM),
3 (A1, X3), (A2, Y3), (A3, X4), (A4, Y4)
C CHECK IF INPUT PARAMETER IS IN ERROR
10 VDO=VD
   IF(VDO.LT.1.049) GO TO 90
C GENERATION OF A RANDOM X VARIABLE
15 PROB=RAN(-1)
   IF(PROB.LT.0.0) GO TO 15
   IF(PROB.GE.1.0) GO TO 15
   X=CO*ALOG(-ALOG(PROB))
C CHECK IF VD IS THE SAME AS IN THE PREVIOUS CALL
20 IF(VDO.GE.1.05) GO TO 22
21 RAN=MM1*X
   RETURN
22 IF(VDO.EQ.VDPV) GO TO 60
C LOCATE VDO
30 VDPV=VD
   IF(VDO.LT.VDO(3)) GO TO 35
   IF(VDO.GE.VDO(21)) GO TO 36
   IMN=IMN+1
   IMX=IMX+1
   IF(VDO.GE.VDO(11)) GO TO 33
32 IMX=IMX+1
   GO TO 34
33 IMN=IMN+1
34 IF(IMX.GT.IMN) GO TO 31
35 I=IMX $ GO TO 40
36 I=23

C INTERPOLATION OF B1, B2, AND M2

40 V1=VVD(I-2)-VDO $ V2=VVD(I-1)-VDO
V3=VVD(I) -VDO $ V4=VVD(I+1)-VDO
V43=V4-V3 $ V42=V4-V2
V32=V3-V2 $ V31=V3-V1
V21=V2-V1

A1=V41*V31*V21 $ A1= V4*V3*V2/A1
A2=V42*V32*V21 $ A2=-V4*V3*V1/A2
A3=V43*V32*V31 $ A3= V4*V2*V1/A3
A4=V43*V42*V41 $ A4=-V3*V2*V1/A4

C GEOMETRY

50 SF=M2/M1 $ M2SQP1=M2*M2+1.0
DM=M1-M2 $ SM=M1+M2
X3=(B2-B11)/DM $ Y3=(M1*B2-M2*B1)/DM
T=(1.0-M1*M2)/SM $ M3=-T-SQRT(T*T+1.0)
B3=Y3-M3*X3+1.5*(SF-1.0)
DM=M1-M3 $ SM=M1+M3
X4=(B3-B11)/DM $ Y4=(M1*B3-M2*B1)/DM
T=(1.0-M1*M3)/SM $ M4=-T-SQRT(T*T+1.0)
DM=M3-M4
X3=X3+M3*X3 $ X4=X4+M4*Y4
XC=(M3*X4-M4*X3)/DM $ YC=(X3-X4)/DM
XMX=(XC+M1*(YC-B11))/M1SQP1
XMN=(XC+M2*(YC-B21))/M2SQP1
RSQ=5.0*(XMX-XC)**2

C COMPUTATION OF THE FUNCTION

60 IF(X.LE.XMN) GO TO 63
61 RANAN=M1*X+B1 $ RETURN
62 RANAN=YC-SQRT(RSQ-(X-XC)**2) $ RETURN
63 RANAN=M2*X+B2 $ RETURN

C ERROR EXIT

90 PRINT 2090, VDO $ RETURN

C FORMAT STATEMENT

2090 FORMAT(1X/2)H *** VD TOO SMALL.//7H VD =E12.3/
1 35H ERROR DETECTED IN ROUTINE RANAN)

END